THE UNIVERSITY  OF HONG KONG

Department of Mathematics

Numerical Mathematics and Applied Analysis Group Seminar (NMAA)

On Some Research Problems in Mathematical Finance

Dr. LI Xun

Department of Mathematics

National University of Singapore

on Monday, December 12, 2005 at 2:30p.m.

in Room 517, Meng Wah Complex

Abstract

This presentation is to first introduce mean-variance portfolio selection problems in continuous-time under the constraint that short-selling of stocks is prohibited. The problem is formulated as a stochastic optimal linear-quadratic (LQ) control problem. However, this LQ problem is not a conventional one in that the control (portfolio) is constrained to take nonnegative values due to the no-shorting restriction, and thereby the usual Riccati equation approach (involving a "completion of squares") does not apply directly. In addition, the corresponding Hamilton-Jacobi-Bellman (HJB) equation inherently has no smooth solution. To tackle these difficulties, a continuous function is constructed via two Riccati equations, and then it is shown that this function is a viscosity solution to the HJB equation. Solving these Riccati equations enables one to explicitly obtain the efficient frontier and efficient investment strategies for the original mean-variance problem.

Next, we discuss a continuous-time market where an agent, having specified an investment horizon and a targeted terminal mean return, seeks to minimize the variance of the return. The optimal portfolio of such a problem is called mean-variance efficient Markowitz. It is shown that, under very mild conditions, a mean-variance efficient portfolio realizes the (discounted) targeted return on or before the terminal date with a probability greater than 0.8072. This number is universal irrespective of the market parameters, the targeted return, and the length of the investment horizon.

Third, we will simply introduce a new efficient State-Space Partitioning Algorithm (SSPA) to pricehigh-dimensional American options and present an alternative semi-analytic approximation method for pricing European options on underlying assets with mean-reverting prices, time-dependent correlations, and stochastic volatility.

All are welcome
