



COLLOQUIUM

Estimations of short Kloosterman sums I, II

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November 19, 2009 (Thursday) 2:00 – 3:00pm

November 20, 2009 (Friday) 2:00 – 3:00pm

Room 210, Run Run Shaw Bldg., HKU

Abstract

1. Introduction.

Definition of Kloosterman sums. Complete and incomplete Kloosterman sums. Classical result of H.D. Kloosterman and A. Weyl (without proofs.)

2. Karatsuba's method in the theory of Kloosterman sums (without proofs.)

(a) Estimation of the number of the congruence

$$p_1^* + \dots + p_k^* \equiv p_{k+1}^* + \dots + p_{2k}^* \pmod{m}$$

in prime numbers p_1, \dots, p_{2k} such that $X < p_j \leq X_1, j = 1, \dots, 2k$, where $kX_1^{2k-1} < m$.

(b) Estimation of the double Kloosterman sum W ,

$$W = \sum_{X < p \leq X_1} \sum_{Y < q \leq Y_1} \exp\left(2\pi i \frac{ap^*q^* + bpq}{m}\right),$$

For special X, X_1, Y, Y_1 , and it's applications.

3. Estimation of «very short» Kloosterman sum of the type

$$\sum_{n \leq x} \exp\left(2\pi i \frac{an^* + bn}{m}\right),$$

where n runs through the interval $(1, x], (n, m) = 1$, and

$$\exp\{(\ln m)^{\frac{4}{5} + \varepsilon}\} \leq x \leq m^{\frac{4}{7}}.$$

(a) Auxilliary assertions (five short lemmas).

(b) Main theorem (sketch of the proof).

All are welcome