THE UNIVERSITY OF HONG KONG

Department of Mathematics

Conference on Number Theory

November 23, 2009
Room 210, Run Run Shaw Bldg., HKU

9:00 – 9:05	**Opening Remark**
9:05 – 9:50	**Alain Togbe (Purdue University North Central)** *On families of Diophantine triples*
10:00 – 10:45	**Irina Rezvyakova (Steklov Mathematical Institute, Russia)** *On the zeros of Hecke L-functions and its linear combinations on the critical line*
11:00 – 11:45	**Kai-Man Tsang (The University of Hong Kong)** *An extension of the Brun-Titchmarsh inequality*
Lunch Break	
14:45 – 15:30	**Charles Li (The Chinese University of Hong Kong)** *Trace formulas on GL(2)*
15:40 – 16:25	**Yuk-Kam Lau (The University of Hong Kong)** *Sum of Fourier coefficients of Cusp Forms*
16:40 – 17:25	**Maxim Korolev (Steklov Mathematical Institute, Russia)** *Estimations of short Kloosterman sums*

Abstracts

Maxim Korolev (Steklov Mathematical Institute)

Estimations of short Kloosterman sums

1. Introduction.
   Definition of Kloosterman sums. Complete and incomplete Kloosterman sums. Classical result of H.D. Kloosterman and A. Weyl (without proofs.)
2. Karatsuba's method in the theory of Kloosterman sums (without proofs.)
   (a) Estimation of the number of the congruence

   $$p_1^* + \cdots + p_k^* \equiv p_{k+1}^* + \cdots + p_{2k}^* \pmod{m}$$

   in prime numbers $p_1, \ldots, p_{2k}$ such that $X < p_j \leq X_1$, $j = 1, \ldots, 2k$, where $kX_1^{2k-1} < m$.

   (b) Estimation of the double Kloosterman sum W,

   $$W = \sum_{X<p\leq X_1} \sum_{Y<q\leq Y_1} \exp\left(2\pi i \frac{ap^*q^* + bpq}{m}\right),$$

   for special X, X_1, Y, Y_1, and it's applications.
3. Estimation of ≪very short≫ Kloosterman sum of the type

   $$\sum_{n\leq x} \exp\left(2\pi i \frac{an^* + bn}{m}\right),$$

   where n runs through the interval $(1, x]$, $(n, m) = 1$, and

   $$\exp\left\{(\ln m)^{\frac{4}{5}+\varepsilon}\right\} \leq x \leq m^{\frac{4}{7}}.$$

   (a) Auxilliary assertions (five short lemmas).
   (b) Main theorem (sketch of the proof).

Yuk-Kam Lau (The University of Hong Kong)

Sum of Fourier coefficients of Cusp Forms

Let $t_\varphi(n)$ denote the nth normalized Fourier coefficient of a primitive holomorphic or Maass cusp form φ for the full modular group. Our interest is to understand its value in mean sense. Assuming the validity of suitable conjectures, we evaluate the high moments $\sum_{n\leq x} t_\varphi(n)^j$ and the mean value $\sum_{n\leq x} t_\varphi(n^j)$ over jth powers.

Charles Li (The Chinese University of Hong Kong)

Trace formulas on $GL(2)$

In this talk, I will use representation theory on the group $GL(2)$ to derive and describe several explicit relative trace formulas. Some applications of the formulas will also be discussed.

Irina Rezvyakova (Steklov Mathematical Institute)

On the zeros of Hecke L-functions and its linear combinations on the critical line

One of the most interesting questions in the theory of the Riemann zeta-function is the problem on distribution of zeros of the Riemann zeta-function and other Dirichlet L-series. In our talk we will consider this problem for

1) L-functions associated with holomorphic cusp forms $f \in S_k(\Gamma_0(D), \chi)$

and for

2) linear combinations of Hecke L-functions of an imaginary quadratic field with complex class group characters (which are also associated with holomorphic cusp forms of weight one).

For a given L-function associated with an automorphic cusp form, which is also an eigenfunction of all the Hecke operators, we prove an analogue of Selberg's theorem, i.e. that a positive proportion of its non-trivial zeros lie on the critical line. This results extends J.L. Hafner's work of 1983 where the same statement was proved in case of cusp forms for the full modular group with trivial character. Our theorem for linear combinations of Hecke L-functions is formulated as follows.

Theorem. *Let ψ_j be complex class group characters on ideals of imaginary quadratic field $\mathbb{Q}(\sqrt{-D})$ and let a_j be real numbers. We denote by $L(s, \psi_j)$ an L-function associated with the character ψ_j. Then for*

$$\mathfrak{f}(s) = \sum_j a_j L(s, \psi_j) \tag{1}$$

the following estimate is valid for the number of odd order zeros on the interval $\{s = \frac{1}{2} + it,\ 0 < t \leq T\}$ of the critical line provided that T is large enough:

$$N_0(T) \gg T(\ln T)^{2/h} e^{-c\sqrt{\ln \ln T}},$$

where $h = h(-D)$ is the class number and c is an absolute constant.

As we know from the works of H. Davenport and H. Heilbronn of 1936 and S.M.Voronin work of 1976, those linear combinations in general have zeros outside the critical line. Nevertheless, our theorem shows that the critical line is an exceptional set which contains "many" zeros of (1). This result is the improvement of the previous theorem of S.A. Gritsenko (1997) on the same subject and is an analogue of A.A. Karatsuba's result (1989) for the Davenport-Heilbronn function.

We hope to give a sketch of the main ideas of the proof and touch more closely Jutila's variant of circle method which we used to deal with so-called "non-diagonal" term in our problem.

Alain Togbe (Purdue University North Central)

On families of Diophantine triples

A set of m distinct positive integers $\{a_1, \ldots, a_m\}$ is called a Diophantine m-tuple if $a_i a_j + 1$ is a perfect square. In general, let n be an integer, a set of m positive integers $\{a_1, \ldots, a_m\}$ is called a Diophantine m-tuple with the property $D(n)$ or a $D(n)$-m-tuple (or a P_n-set of size m), if $a_i a_j + n$ is a perfect square. Diophantus studied sets of positive rational numbers with the same property, particularly he found the set of four positive rational numbers $\left\{\frac{1}{16}, \frac{33}{16}, \frac{17}{4}, \frac{105}{16}\right\}$. But the first Diophantine quadruple was found by Fermat. That is the set $\{1, 3, 8, 120\}$. Moreover, Baker and Davenport proved that the set $\{1, 3, 8, 120\}$ cannot be extended to a Diophantine quintuple. The problem of extendibility of P_n-sets is of a big interest.

In this talk, we will give a very quick survey of results obtained. Then we will expose how we extended a result of Dujella-Filipin-Fuchs on the $D(-1)$-triple $\{1; k^2+1, k^2+2k+2\}$ and its unique D(1)-extension. Finally, we will discuss other results obtained on Diophantine triples and our future work. This is a joint work with Bo He.

Kai-Man Tsang (The University of Hong Kong)

An extension of the Brun-Titchmarsh inequality

The celebrated Brun-Titchmarsh inequality

$$\pi(x; k, a) = \sum_{\substack{p \leq x \\ p \equiv a \pmod{k}}} 1 \ll \frac{x}{\varphi(x) \log \frac{x}{k}}$$

which holds uniformly in $k < x$ is a useful supplement to the Siegel-Walfisz theorem and the Bombieri-Vinogradov theorem. Generalizations of this to integers with given number of prime factors have been obtained by many authors. In this talk we will give a survey of the results and present our recent joint work with T.H. Chan and S.K.K. Choi.