THE UNIVERSITY OF HONG KONG

Department of Mathematics

Y.C. Wong Lectures

Lectures on Discrepancy Theory

Professor William Chen

Emeritus Professor of Mathematics
Macquarie University, Australia

Abstract

Discrepancy theory, or irregularities of point distribution, originates from the theory of uniform distribution, and can sometimes be considered the quantitative analogue of uniform distribution. It owes its existence to the fundamental work of Klaus Roth and Wolfgang Schmidt, two of the greatest mathematicians of the twentieth century.

A point set in a given finite region is a discrete object, in the sense that the point count is an integer. Given a finite set of points in a finite region, the expected number of points in any given sub-region of the finite region depends on the area of the sub-region in question and often is not an integer. The discrepancy of the original point set in the sub-region, which is the difference between the actual point count in the sub-region and its expectation, therefore has a discrete part and a continuous part as the sub-region varies continuously. The purpose of discrepancy theory is to highlight this difference, as well as to seek ways to minimize it.

Monday	Thursday
	October 4, 2012
October 8, 2012	October 11, 2012
October 15, 2012	October 18, 2012
October 22, 2012	October 25, 2012
October 29, 2012	
Time:	10:00am - 12:00noon
Place:	Room 210, Run Run Shaw Bldg., HKU

All are welcome (especially graduate students)

The lectures assume only basic classical mathematical background and will be accessible to graduate students in any area of pure mathematics.

Brief summary of our proposed lectures

Chapter 1. Uniform Distribution. [Lecture Notes]
Here we discuss some very basic notions which historically gave rise to the subject.

Chapter 2. The Classical Discrepancy Problem. [Lecture Notes]
Roth's reformulation of the problem makes it somewhat geometric in nature and allows many generalizations and extensions. This is known as the classical discrepancy problem. We review the many known results and also highlight some difficult unsolved questions.

Chapter 3. Generalization of the Problem. [Lecture Notes]
We then introduce a few generalizations and extensions.

Chapter 4. Introduction to Lower Bounds. [Lecture Notes]
We first give a simple illustration, by discussing a result of Schmidt, of how we may blow up the small errors which inevitably arise. We then embark on a detailed study of Roth's lower bound technique as applied to the classical discrepancy problem, followed by Halász's variation of the technique as well as a modern Haar wavelet approach.

Chapter 5. Introduction to Upper Bounds. [Lecture Notes]
We discuss some upper bound techniques as applied to large discrepancy problems, and these involve some probability theory.

Chapter 6. Fourier Transform Techniques. [Lecture Notes]
The Fourier transform technique due to Beck is one of the most powerful techniques in the subject, and gives many lower bounds in large discrepancy problems. We illustrate this beautiful technique by studying a simple lower bound example. We also obtain some upper bounds.

Chapter 7. Upper Bounds in the Classical Problem. [Lecture Notes]
The classical discrepancy problem is an example of a small discrepancy problem. Here the upper bound questions are some of the hardest, but there is a very rich literature involving techniques from different areas of mathematics such as number theory, lattice theory, probability theory, analysis and group theory. We discuss some of these ideas.

Chapter 8. The Disc Segment Problem. [Lecture Notes]
The disc segment problem of Roth has a rather unusual setting and has led to some very interesting mathematics. Here we discuss a lower bound technique due to Alexander as well as an upper bound technique imported from the classical problem.

Chapter 9. Convex Polygons. [Lecture Notes]
Here we discuss an extension to the upper bound technique discussed in the last chapter, and use it to obtain some generalization of the classical problem!

Chapter 10. Fourier-Walsh Analysis. [Lecture Notes]
In this final chapter, we discuss some of the ideas in the work of the lecturer and Skriganov on the solution of the long-standing explicit construction question in the classical problem.