## Introduction to Major and

Minor in Mathematics

(Advanced Courses)

Dr. Tak Kwong Wong Department of Mathematics, HKU Wednesday, April 3rd, 2019

#### Major in Mathematics

Most math majors will concentrate on one of the following three main themes:

- (Pure) Mathematics
- Computational Mathematics, Logistics/ Operations Research
- Mathematics, Economics and Finance

### Advanced Level Courses

- MATH3401 Analysis I [Disciplinary Core Course]
- + at least 2 of the following courses:
- MATH3301 Algebra I
- MATH3403 Functions of a Complex Variable
- MATH3601 Numerical Analysis
- MATH3603 Probability Theory
- MATH3904 Introduction to Optimization

+ other Disciplinary Electives (*including at least 2 non-capstone courses from MATH4XXX or MATH7XXX level*)

+ at least 1 Capstone Course.

## MATH 3401 Analysis I

- Study basic properties of metric spaces: openness; closedness; accumulation point; boundary point; compactness; completeness; continuity; connectedness, etc.
- Prerequisite of almost all pure mathematics courses.
- Pass in MATH2211
   Multivariable calculus
- Teacher: Prof. W.S. Cheung (*Harvard*)



## MATH3301 Algebra I

- Introduction to groups, rings and fields.
- Useful to MATH4302 Algebra II, MATH6501 Topics in Algebra and MATH6502 Topics in Applied Discrete Mathematics.
- Pass in MATH2101 Linear algebra I.
- Teacher: Dr. Y.K. Lau (HKU)



## MATH3403 Functions of a Complex Variable

- Do calculus on the complex plane.
- Study of "differentiable" functions of a complex variable which have many nice and interesting properties.
- Useful to the studies of almost all branches of pure mathematics and theoretical physics, like quantum mechanics.



- Pass in MATH2211 Multivariable calculus and MATH2241 Introduction to mathematical analysis.
- Teacher: Prof. N.M. Mok (Stanford)

#### **MATH3601 Numerical Analysis**

- Study numerical methods/ algorithms for solving equations.
- Important knowledge in many realworld applications, e.g., weather prediction, safety engineering, stock market simulations, etc.
- Pass in MATH2101 Linear algebra I and MATH2211 Multivariable calculus.
- Teacher: Dr. Z. Zhang (Tsinghua)



## **MATH3603 Probability Theory**

- Introduction to probability models and their applications.
- Numerous applications in daily-life and science, e.g., risk assessment, determine pricing and trading strategy, statistical mechanics, etc.





• Teacher: Dr. Z. Qu (*Ecole Polytechnique*)

## MATH3904 Introduction to Optimization

- Study the theory and techniques of optimization.
- Prepare for further studies in operations research, mathematical economics and related subject areas.



- Pass in MATH2101 Linear algebra I and MATH2211 Multivariable calculus.
- Teacher: Prof. W. Zang (Rutgers)

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#### Pure Mathematics

**Recommended courses: MATH3303** Matrix Theory and its Applications **MATH3304** Introduction to Number Theory MATH4402 Analysis II MATH3302 Algebra II **MATH4404** Functional Analysis MATH4501 Geometry MATH4511 Introduction to Differentiable Manifolds **MATH4406** Introduction to Partial Differential **Equations MATH6101 Intermediate Complex Analysis** MATH6504 Geometric Topology

## MATH 3303 Matrix theory and Its Applications

- Second course in Linear Algebra
- Useful to MATH3601 Numerical Analysis, MATH4602 Scientific Computing and Operation Research courses
- Related to MATH4404 Functional Analysis
- Pass in MATH2101 Linear algebra I and MATH2102 Linear algebra II
- Not offered in 2017-18

- Matrix theory is extremely useful in the studies of applied and pure mathematics, statistics, economics, finance, engineering, etc.
- For example, it is used in image processing and search engines like Google.
- See the book, "Google's PageRank and beyond : the science of search engine rankings" by Amy N. Langville and Carl D. Meyer.







Google



#### Larry Page

Sergey Brin

"The heart of our software is PageRank™, a system for ranking web pages developed by our founders Larry Page and Sergey Brin at Stanford University."

Page, Brin, Motwani, Winograd, The PageRank Citation Ranking: Bringing Order to the Web. Stanford Digital Library Technologies Project, 1998.

### MATH4402 Analysis II

 Comprehensive and rigorous treatment of calculus in several variables, and a modern treatment of integration theory in the language of differential forms which is essential for more advanced studies in analysis and geometry.



- Useful to the studies of Differential and Algebraic Geometry and General Relativity, etc.
- Pass in MATH3401 Analysis I
- Teacher: Dr. Y.M. Chan (Oxford)

*Fundamental Theorem of Calculus (FTC)* The one-variable fundamental theorem:

$$\int_a^b f'(x)dx = f(b) - f(a).$$

Main goal: Generalize FTC to Stokes' theorem for manifolds:

$$\iint_{M} \operatorname{curl} \mathbf{F} \cdot \mathbf{n} \, d\sigma = \int_{\partial M} \mathbf{F} \cdot d \, \mathbf{x}$$

## **MATH4501 Geometry**

- Use multi-variable calculus to study geometry of curves and surfaces in 3-space.
- Useful to the studies of Differential and Algebraic Geometry and General Relativity, etc.
- Pass in MATH2101 Linear algebra I and MATH3401 Analysis I.



• Teacher: Dr. C.W. Wong (HKU)

# MATH4511 Introduction to differentiable manifolds

- Review of calculus in several variables.
- Do calculus on manifolds ("higher dimensional surface").
- Useful to the studies of Differential and Algebraic Geometry and General Relativity, etc.
- Pass in MATH4402 Analysis II and MATH4501 Geometry, or already enrolled in these courses
- Not offered in 2017-18

#### **MATH3541 Introduction to topology**

- This course aims at introducing students to fundamental knowledge in topology and some of its applications.
- Useful to the studies of Differential and Algebraic Geometry and General Relativity, etc.
- For recent important breakthrough in topology, see the book "The Poincaré conjecture: in search of the shape of the universe" by Donal O'Shea.
- Pass in MATH2101 and MATH2241. Students are recommended to have passed or already enrolled in MATH3301 and MATH3401.
- Teacher: TBA



#### **MATH3304 Introduction to number theory**

- Basic concepts about numbers, their properties and the arithmetic of congruence.
- The interplay between the multiplicative and additive properties of prime numbers
- Properties and the distribution of the prime numbers.
- Important applications of number theory to modern cryptography
- Pass in MATH3301 Algebra I, or already enrolled in this course.

#### Introduction to Number Theory

Dr. YK Lau (*HKU*)

Expert in modular forms and L-functions



Dr. B. Kane (Wisconsin)

Number theory and combinatorics, in particular, modular and automorphic forms Prof. K.M. Tsang (*Princeton*)

Expert in Riemann's zeta function and analytic number theory.

Student of Prof. Atle Selberg (1950 Fields-Medallist)

# Long tradition of number theory in our department



Prof. M.C.Liu

World expert in Goldbath's conjecture

#### Fields-Medallist, Prof. Alan Baker

Prof. K.M. Tsang

#### Computational Mathematics, Operations Research

**Recommended courses:** 

MATH3303 Matrix Theory and Its Applications
MATH3601 Numerical Analysis
MATH3603 Probability Theory
MATH3901 Operations Research I
MATh3904 Introduction to Optimization
MATH3905 Queuing Theory and Simulation
MATH3911 Game Theory and Strategy
MATH4602 Scientific Computing
MATH4903 Network Models in Operations Research

BUSI1003 Introduction to Management Information System COMP1117 Computer Programming COMP2119 Introduction to Data Structures and Algorithms ECON0701 Introductory Econometrics STAT4601 Time-series Analysis

# **Operations Research Group** Prof. W.K. Ching, Prof. W. Zang, Dr. Z Qu



#### Computational Mathematics, Operations Research

- Courses in Computational Mathematics and Operations Research are taught by experts in these two areas.
- For operations research: Prof. W.K. Ching (CUHK) Prof. W. Zhang (Rutgers) Dr. Z. Qu (Ecole Polytechnique)
- For computational mathematics: Prof. X. Yuan (*City U*)
   Dr. G.Y. Han (*Notre Dame*)
   Dr. Z.W. Zhang (*Tsinghua*)

The operations research courses mainly study different types of the constraint optimization problems.

# Main problem: Optimization with constraints

- Max/Min f(x<sub>1</sub>,...,x<sub>n</sub>) under the constraints g<sub>i</sub>(x<sub>1</sub>,...,x<sub>n</sub>)=0, i=1,...,m.
- OR I and II study this optimization problem when all functions are linear.
- IO studies the same optimization problem when some of these functions are nonlinear by using multi-variable calculus.
- Some numerical methods are needed to solve the problems.

## AlphaGo uses the search-andoptimization idea

- Monte-Carlo tree search and deep convolutional neural networks
- Hidden Markov Chain
- Viterbi Algorithm (a Dynamic Programming algorithm for the formulation of computing the optimal hidden sequence).



 Reference: Michael Nielsen, 'Is AlphaGo Really Such a Big Deal?", Quanta Magazine (March 29, 2016)

# MATH7503 Topics in mathematical programming and optimization

- Special topics in mathematical programming or optimization.
- Useful to the studies of Operations Research, AI, machine learning, or related subject areas, etc.
- Pass in MATH3901 Operations research I, MATH3904 Introduction to optimization and MATH4902 Operations research II.



• Teacher: Prof. X. Yuan (*City U*)

#### Mathematics, Economics and Finance

**Recommended courses:** MATH3906 Financial Calculus MATH4907 Numerical Methods for Financial Calculus **BUSI1002** Introduction to Accounting **ECON2210 Microeconomics Theory ECON2220** Macroeconomics Theory **FINA2802** Investments and Portfolio Analysis **ECON0701** Introductory Econometrics Major in Mathematics Minor in Economics or

Finance or Risk.

Management !



Students can prepare the level I examination of Chartered Financial Analyst by taking suitable courses in this category.

#### Major in Mathematics, Minor in Economics / Finance / Risk Management



Mathematics is the common language in these areas and it is easier to learn math when you are young.

#### Major in Mathematics – Economics/Financ

Nobel Prize in Economics: Out of the 50 most recent Laureates,

24 of them have degrees in mathematics:

2007 Eric S. Maskin (BS in math) 2005 Robert J. Aumann (BS, MS, PhD in math) 2004 Edward C. Prescott (BA in math, MS in OR) 2003 Clive W. J. Granger (BA in math) 2002 Daniel Kahneman (BA in math and psychology) 2001 Michael Spence (BA, MA in math) 2000 James J. Heckman (BA in math) 1998 Amartya Sen (BA minor in math) **1997** Robert C. Merton (BS, MS in applied math) **1996 James A. Mirrlees (MA in math) 1996 William Vickrey (BS in math)** 1994 John F. Nash Jr. (PhD in math) **1994 Reinhard Selten (PhD in math) 1992 Gary S. Becker (BA in math)** 



#### **MATH 3911 Game Theory and Strategy**

2018 Paul Romer (BS in math and phys)
2016 Bengt Holmström (BS in math, MS in OR)
2016 Oliver Hart (BA in math)
2015 Angus Deaton (Read math for two years)
2014 Jean Tirole (doctorat de troisieme cycle in decision mathematics)
2013 Lars Peter Hansen (BS in math)
2012 Alvin E. Roth (BS, MS, PhD in OR)
2012 Lloyd Shapley (BA, PhD in math)
2011 Christopher A. Sims (BA in math)
2010 Peter A. Diamond (BA in math)

#### Major in Mathematics – Economics/Finance

Luo Guannan and Jian Bo (both Math major) was/is doing a PhD in Economics at Northwestern University and University of Chicago respectively.

If you plan to do a master or PhD in economics or finance, the following courses are recommended:

MATH3401 Analysis I MATH3901/3902 OR I and II MATH3904 Introduction to Optimization MATH3911 Game Theory and Strategy MATH3906 Financial Calculus MATH4907 Numerical Methods for Financial Calculus STAT4601 Time-series Analysis STAT6010 Advanced Probability





Many investment banks employ mathematicians to do risk management or computational finance

HSBC



Dr. Lau Chi Fong (PhD in number theory) Former Head of Market Risk Management, HSBC Global Markets

#### Computational/Quantitative Finance

- Usually need a PhD in Mathematics/Physics/Computer Sciences/Statistics to do quantitative finance in investment banks.
- Develop and analyze new financial products.
- Write programs to evaluate the price of financial derivatives.
- Mathematical modeling skill.
- Computing skills, e.g., Visual Basic.

#### To learn more about quantitative finance, can read the following books

- Options, futures, and other derivatives / John C. Hull
- An Elementary Introduction to Mathematical Finance/ Sheldon M
- Heard on the street : quantitative questions from Wall Street job interviews / Timothy Falcon Crack.





#### **Computational Finance**

- MATH3906 Financial Calculus
- MATH4907 Numerical Methods for Financial Calculus
- MATH3603 Probability Theory
- MATH3601 Numerical Analysis
- MATH4602 Scientific Computing
- MATH4406 Introduction to PDEs
- STAT4601 Time-series Analysis
- STAT6010 Advanced Probability

#### **MATH3906 Financial Calculus**

- Modeling of financial derivatives, asset pricing and market risks
- Introduction to stochastic calculus
- Provide a solid background for future study in quantitative finance.
- Prerequisite: MATH3603 Probability Theory



- Good knowledge of partial differential equations and probability theory is needed for further studies.
- Teacher: Dr. S.P. Yung (Wisconsin)

## Major in Mathematics (**Requirement**)

Capstone Requirement (6 credits)\*
 At least 6 credits selected from the following courses:

MATH3999 Directed studies in mathematics (6 credits) MATH4910 Senior Mathematics Seminar (6 credits) MATH4966 Mathematics Internship (6 credits) MATH4911 Mathematics Capstone Project (6 credits) MATH4999 Mathematics Project (12 credits)

Students are not required to take Capstone if this Science major is taken as a second major on the condition that the capstone experience in the first major requires the integration or application of knowledge from both major disciplines.

If this is approved, a 6-credit advanced level course in the second major must<sub>38</sub> be taken to fulfill the credit requirement of the capstone experience.

*MATH4966 Mathematics Internship* This course aims to offer students the opportunities to gain work experience in the industry related to their major of study.

Students have to take on at least 160 hours of internship work either within the University or outside the University arranged by the department.

Details of internship will be recorded on the student's transcript.

Available in 1st, 2nd, and the summer semester.

Students (who take this course) are **expected** to have satisfactorily completed their Year 3 study.

**Assessment:** Required to submit a written report and to give an oral presentation on their internship experience. This course will be assessed on **Pass or Fail** basis.

- In the past years, our students interned at various organizations: for example
- AECOM: A Fortune 500 company, serves clients in more than 100 countries and provides professional technical and management support services to a broad range of markets, including transportation, facilities, environmental and energy.
- Hong Kong Observatory
- HKEx OTC Clearing Risk Management
- Education Bureau (EdB) of HKSAR
- Marshall Cavendish Education
- NITTAN CAPITAL ASIA LIMITED
- HKEx Group Credit & Quantitative Analysis Hong Kong Exchanges and Clearing Limited
- Times Publishing (HK) Limited Pearson Education Asia Limited
- Secondary schools
- Our teachers also offered internship in the summer.

Internships offered by our teachers:

- Channel Polarization in Polar Coding
- Development of Teaching and Learning Materials in Mathematics
- Efficient method for numerical upscaling
- Mastering 3D printing
- Mathematical Laboratory
- Sums of polygonal numbers
- Uncertainty quantification for computational cardiology
- What is Topology
- Finite Subgroups of PGL(2, F) and Related Topics
- Open Problems in Affine Algebraic Geometry and Noncommutative Algebra

#### Internships offered by Education Bureau of HKSAR:

- 1. The Design of Learning and Teaching Resources for promoting Science, Technology and Mathematics (STEM) education in primary Mathematics
- 2. The Design of Learning and Teaching Resources for promoting Science, Technology and Mathematics (STEM) education in secondary Mathematics
- 3. The Design of GeoGebra Applets for the Learning and Teaching of Selected Topics in the Senior Secondary Mathematics Curriculum
- 4. Analyzing the Data of Trends in International Mathematics and Science Study (TIMSS)
- 5. A study of Jiǔ Zhāng Suàn Shù (九章算術)
- 6. The Design of GeoGebra Applets for the Learning and Teaching of Geometry

- Usually, we will inform our students the internship opportunities around May and inform the students the results around June.
- You can also find a summer internship through CEDARS.
- After the completion of the project, you need to do a 10-15 minute oral presentation about your internship.
- Finally, need to submit a written report of at least 1000 words on what has been done and what has been learnt during the internship.

#### MATH4999 Mathematics Projects

The aim of the course is to provide students with opportunity to formulate and investigate, in depth, problems of practical interest and/or have a foretaste of mathematical research.

The work, to be done on an individual basis, is considered a highly desirable part of the training of a mathematician.

Pre-requisites: Pass in at least 24 credits of advanced level disciplinary core/elective mathematics courses (MATH3XXX, MATH4XXX, or MATH7XXX) in the Mathematics, and Mathematics/Physics.

#### MATH4999 Mathematics Projects

**Topics**:

- An introduction to differential topology and its applications
- Calculus of Variations
- Introduction to Algebraic Geometry
- Mathematical methods in option pricing
- Matrix Completion Problems
- Matrix groups and Poisson algebras
- Mean Values of Error Terms in Number Theory
- Modern optimization methods
- Multiplicative Number Theory
- Numerical methods for multiscale problems
- Probabilistic Boolean Networks
- Representations of integers by sums of polygonal numbers
- The Mathematics in Contract Bridge
- Topics in Information Theory
- Various Traveling Salesman Problems (Advanced Level)

#### MATH4999 Mathematics Projects

- Usually, we will inform our students the project topics around June/July and inform the students the results around July/August.
- This is a 12 credit course and students must achieve good standing and get the approval from both the prospective supervisor and the course co-ordinator to take this course.
- The student is expected to do approximately 200 hours of independent work and to attend meetings and seminars.
- Assessment Method: By dissertation (70% weighting) and continuous assessment which may include oral presentation (30% weighting)

#### MATH4910 Senior Mathematics Seminar

- This is a seminar style course intended for students who have very strong interests and good ability in mathematics and have inclination to mathematical research.
- Students will look at particular mathematical topics in depth, and will master the topics through reading, listening, discussing and writing
- Active participation in all the discussions is expected. The aim of the course is to let students learn how to initiate self/independent study in mathematics.
- Prerequisite is at least 24 credits of advanced level Mathematics courses (MATH3xxx, MATH4xxx, or MATH6xxx).
- Research report(50%), Dissertation(20%), Oral presentation(30%)

#### MATH4911 Mathematics Capstone Projects (6 credits)

- Students will work collaboratively in small groups on a project that requires integration or application of knowledge they have acquired.
- The project topic is not limited to academic context, but can also be extended to a community or corporate outreach project. Projects may take the form of a combination of literature research, survey, data analysis, creation of artifacts or media contents, exhibition, public lectures, etc.

Prerequisite is at least 24 credits of any advanced level Mathematics courses (MATH3xxx, MATH4xxx, MATH6xxx).

Research report(50%), Dissertation(20%), Oral presentation(30%)

Topics of MATH4966 and MATH4911 will be announced to students after the May examination. Enrollment in the courses requires approval by the Department.

#### MATH4911 Mathematics Capstone Project (2016-2017)

**Title**: What is "Algorithmic Tradings"?

Supervisor: Dr. S.P. Yung

**Quota:** 2-4 **Duration:**  $2^{nd}$  Semester

#### **Objective**:

Computer speed has been increased tremendously these years. This enables people to use computer programs to capture quick profits that flash and disappear in split seconds in the markets.

In this project, we shall look deep into this "Algorithmic Tradings" and see what they really are. By carrying out experiments on past data, we shall see whether they can be applied to the markets in Hong Kong and other places.

#### The Necessary Requirements of the Project:

Student is required to have a good background in Analysis and Linear Algebra.

Programming skill in Scilab (or Mathlab) is needed. A written report is required at the end of the project course.

Led by Dr. S.P. Yung, the Mathematics team has received the Merit Award the CASH Inter-University ALGO Trading Context 2016/17

#### Research Track: Better preparations for graduate studies

Students who have done well in MATH3999 Directed Studies in Mathematics, MATH4910 Senior Mathematics Seminar, MATH4999 Mathematics Projects, plus some oversea exchange and summer research experience have better chances to get into top graduate schools.



Boya Wen (B.Sc. 2016) Princeton



Yongquan Zhang (B.Sc. 2016) Harvard





Ruoxuan Yang (B.Sc. 2017) MIT

Shuangping Li (B.Sc. 2017) Princeton

#### MATH3002 Mathematics Seminar [NOT a Capstone Course]

- This is a seminar style course intended for those who have very strong interests and good ability in mathematics.
- Students will be given book chapters and elementary research articles for private study and then make presentations in front of the whole class.
- Pass in MATH2012, MATH2101, MATH2211 and MATH2241.

(This course is for second year BSc students only.)

Further Enquiries : Course Selection Advisers Dr. C.W. Wong (cwwongab@hkusua.hku.hk) Dr. G. Han (ghan@maths.hku.hk) Dr. H. Zhang (hyzhang@maths.hku.hk) Dr. K.H. Law (lawkaho@maths.hku.hk) Dr. Y.M. Chan (ymchan@maths.hku.hk) http://hkumath.hku.hk/web/info/math\_student.html All information (e.g. teaching arrangement) in this document is tentative, and may be subject to change. Please check the departmental and faculty websites for the most updated information and details. The End