Massive Access and Many-User Information Theory

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@ Conference on Applied Mathematics, The University of Hong Kong
August 25, 2016

Motivation

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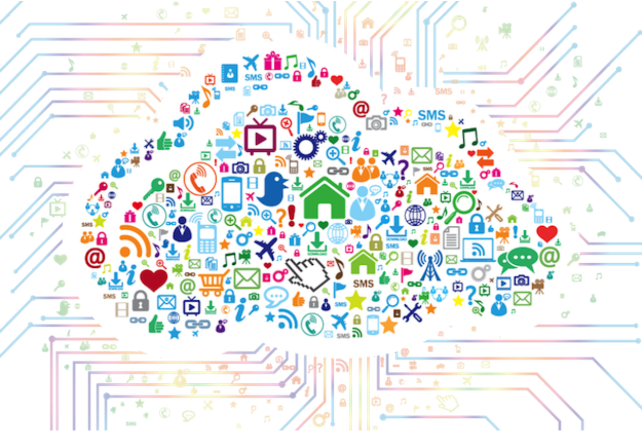
Internet of Things



Fundamental questions

1. ℓ devices in a cell; k of them are active.
2. ℓ and k are large numbers.
3. Massive grant free access in the uplink.
   Who transmitted?
   What are their messages?
4. Selective addressing in the downlink.
   Who should listen?
   What is the message for each receiver?

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Need a Many-User Information Theory

- Classical single-user Information Theory:
  1 user, coding blocklength $n \to \infty$.
- Multiuser Information Theory:
  k users (fixed, usually small), $n \to \infty$.
- Large-system analysis:
  $n \to \infty$ first, then $k \to \infty$.
- However, $k > n$ in many systems.
  E.g., large sensor networks, Internet of things.
- $n \to \infty$ for fixed k may be inaccurate and provide little insight.
- We propose a Many-User Information Theory:
  $k, n \to \infty$ simultaneously. For example:
  $k = \alpha n \to \infty$;
  $k = n^\alpha \to \infty$ (e.g., $k = 1,000,000$ devices, frame length $n = 1,000$).

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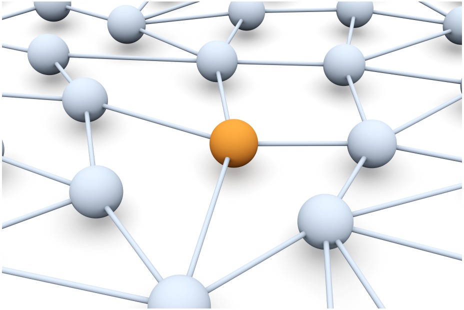
Outline

- (Almost practical) device identification
- Classical information theory
- Many-access channel
- Many-broadcast channel

Neighbor discovery/device identification



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- Prior art: random access. Each node sends its NIA repeatedly with random delay.

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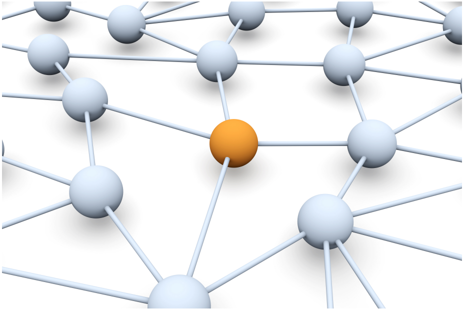

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Discovery is fundamentally sparse signal recovery

- b-bit address, $l = 2^b$ valid NIAs total.
- Node i sends signal $\boldsymbol{s}_i$.
- Multiaccess channel with path loss and fading:

$$\begin{aligned}\boldsymbol{Y} &= \sum_{i \in \text{neighborhood}} \boldsymbol{s}_i U_i + \boldsymbol{W} \\ &= \sum_{i=1}^{l} \boldsymbol{s}_i X_i + \boldsymbol{W} \\ &= \underline{\boldsymbol{S}}\boldsymbol{X} + \boldsymbol{W}\end{aligned}$$

- Given $\boldsymbol{Y}$, what is $\boldsymbol{X}$?
- $X_i \approxeq 0$ for all but a few neighbors.

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Network-wide discovery in one frame interval

- Each node transmits a single frame of signature.
- Synchronized transmissions (can be relaxed).
- One key challenge is decoding complexity (need to scale to 2^{20}–2^{48} NIAs).
- Second-order Reed-Muller codes + chirp decoding algorithm [Calderbank, Gilbert & Strauss '06], [Howard, Calderbank & Searle '08].
- In an ad hoc network with half-duplex transceivers, use symbol erasures to achieve rapid on-off division duplex (RODD).

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Second order Reed-Muller signature generation

- Signature length $n = 2^m$.
- $P_{m\times m}$ is a binary symmetric matrix, $x, t \in \mathbb{Z}_2^m$:

$$\varphi_{P,t}(x) = \left(\sqrt{-1}\right)^{x^\top P x + 2t^\top x}.$$

- Codebook size up to $2^{m(m+3)/2}$:
  - $m = 5$, $n = 2^5 = 32$, l up to 2^{20} codewords.
  - $m = 10$, $n = 2^{10} = 1{,}024$, l up to 2^{65};
  - $m = 12$, $n = 2^{12} = 4{,}096$, l up to 2^{90}.
- Introduce about 50% erasures in case of virtual full duplex.

1	1	j	-j	j	-j	-1	-1	1	1	-j	j	j	-j	1	1	1	-1	j	j	j	j	-1	1	-1	1	j	j	-j	-j	-1	1
1	1	j	-j	j	-j	1	1	1	-1	-j	-j	-j	-j	1	-1	j	-j	1	1	-1	-1	-j	j	j	j	-1	1	1	-1	-j	-j
1	1	j	j	j	-j	-1	1	j	j	1	1	-1	1	j	-j	j	-j	-1	1	1	1	j	j	1	-1	-j	j	-j	-j	-1	-1
1	-1	j	-j	j	j	1	1	j	j	1	1	1	-1	j	-j	1	1	-j	-j	-j	j	1	-1	j	-j	-1	1	-1	-1	j	j
1	-j	1	j	j	1	-j	1	1	j	-1	j	j	-1	j	1	j	-1	j	1	-1	-j	1	-j	j	1	-j	1	-1	j	-1	-j
1	-1	1	1	j	-j	-j	-j	1	1	-1	1	-j	-j	-j	j	j	j	-j	j	-1	-1	-1	1	-j	j	-j	-j	-1	1	1	1
1	j	-1	-j	j	-1	j	-1	j	1	j	1	-1	j	1	-j	1	-j	-1	j	-j	-1	-j	-1	j	-1	j	-1	1	j	-1	-j
1	1	-1	-1	j	j	j	j	j	-j	j	-j	1	-1	-1	1	1	-1	1	-1	-j	j	j	-j	-j	-j	j	j	1	1	1	1

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1	1	j	j	j	-j	-1	1	j	j	1	1	-1	1	j	-j	j	-j	-1	1	1	1	j	j	1	-1	-j	j	-j	-j	-1	-1
1	-1	j	-j	j	j	1	1	j	j	1	1	1	-1	j	-j	1	1	-j	-j	-j	j	1	-1	j	-j	-1	1	-1	-1	j	j
1	-j	1	j	j	1	-j	1	1	j	-1	j	j	-1	j	1	j	-1	j	1	-1	-j	1	-j	j	1	-j	1	-1	j	-1	-j
1	-1	1	1	j	-j	-j	-j	1	1	-1	1	-j	-j	-j	j	j	j	-j	j	-1	-1	-1	1	-j	j	-j	-j	-1	1	1	1
1	j	-1	-j	j	-1	j	-1	j	1	j	1	-1	j	1	-j	1	-j	-1	j	-j	-1	-j	-1	j	-1	j	-1	1	j	-1	-j
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1	1	j	$-j$	j	$-j$	1	1	1	-1	$-j$	$-j$	$-j$	$-j$	1	-1	j	$-j$	1	1	-1	-1	$-j$	j	j	j	-1	1	1	-1	$-j$	$-j$
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1	-1	j	$-j$	j	j	1	1	j	j	1	1	1	-1	j	$-j$	1	1	$-j$	$-j$	$-j$	j	1	-1	j	$-j$	-1	1	-1	-1	j	j
1	$-j$	1	j	j	1	$-j$	1	1	j	-1	j	j	-1	j	1	j	-1	j	1	-1	$-j$	1	$-j$	j	1	$-j$	1	-1	j	-1	$-j$
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1	j	-1	$-j$	j	-1	j	-1	j	1	j	1	-1	j	1	$-j$	1	$-j$	-1	j	$-j$	-1	$-j$	-1	j	-1	j	-1	1	j	-1	$-j$
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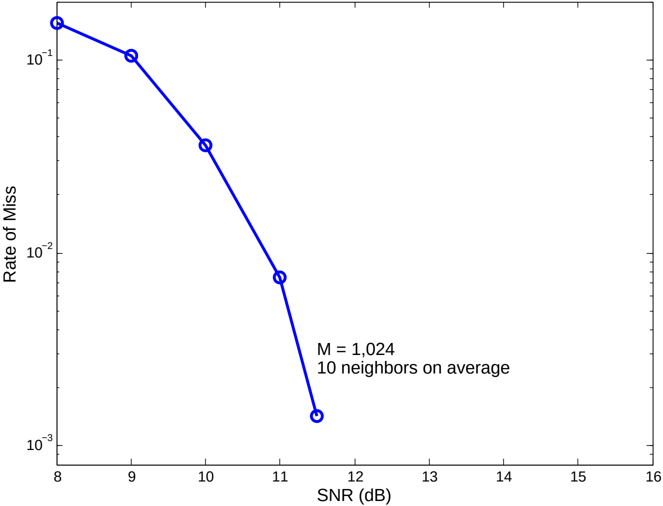
1	1	0	0	j	0	-1	0	0	0	0	1	-1	1	j	-j	j	-j	0	0	1	0	j	0	0	0	0	j	-j	-j	-1	-1
0	0	0	-j	0	0	0	1	j	j	0	1	1	0	0	0	0	0	0	-j	0	0	0	-1	j	-j	0	1	-1	0	0	0
0	0	0	0	0	1	0	0	1	j	0	0	j	0	0	0	0	0	0	0	0	-j	0	0	j	1	0	0	-1	0	0	0
1	-1	1	1	j	-j	-j	-j	1	1	-1	0	0	0	0	0	j	j	-j	j	-1	-1	-1	1	-j	j	-j	0	0	0	0	0
1	0	0	0	0	-1	j	0	0	0	0	0	-1	j	1	-j	1	0	0	0	0	-1	-j	0	0	0	0	0	1	j	-1	-j
0	1	0	-1	j	0	j	0	j	-j	0	-j	0	-1	-1	1	0	-1	0	-1	-j	0	j	0	-j	-j	0	j	0	1	1	1

Error rate vs. SNR

2^{20} nodes, path loss exponent $= 3$, Rayleigh fading



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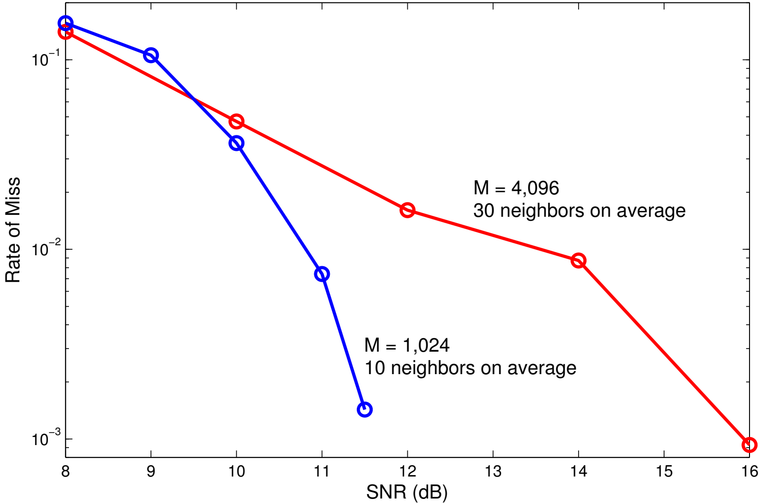

Comparison with random access

- $l = 2^{20}$ nodes, on average 10 neighbors, SNR $= 11.5$ dB

  Target $P_e = 0.002$

	Random access	RODD
# of frames	194	1
# of symbols	$\geq$ 194×20=3,880	1,024

- In addition, significant reduction of per-frame overhead.
- More results in:
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Comparison with random access

- $l = 2^{20}$ nodes, on average 10 neighbors, SNR $= 11.5$ dB
  Target $P_e = 0.002$

	Random access	RODD
# of frames	194	1
# of symbols	$\geq$ 194×20=3,880	1,024

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What are the fundamental limits?

Outline

- (Almost practical) neighbor discovery
- Classical information theory: a digression
- Many-access channel
- Many-broadcast channel

Classical (single-user) information theory

- [Shannon–MacMillan–Breiman '48, '60] for discrete stationary ergodic sequence,

$$-\frac{1}{n}\log p_{X_1,\dots,X_n}(X_1,\dots,X_n) \xrightarrow{\text{a.s.}} \mathcal{H}$$

- Typical set

$$T_\epsilon^{(n)} = \left\{(x_1,\cdots,x_n) : \left| -\frac{1}{n}\log p_{X_1,\cdots,X_n}(x_1,\dots,x_n) - \mathcal{H}\right| \le \epsilon\right\}$$

- Asymptotic equipartition property:
  - Almost all sequences that occur are typical, $\lim_{n\to\infty} P\left\{T_\epsilon^{(n)}\right\} = 1$;
  - There are about $2^{n\mathcal{H}}$ of them, $|T_\epsilon^{(n)}| \approx 2^{n\mathcal{H}}$.
- Hence Shannon's lossless source coding theorem.
- A joint AEP is the workhorse for Shannon's channel coding theorem and rate distortion theorem.

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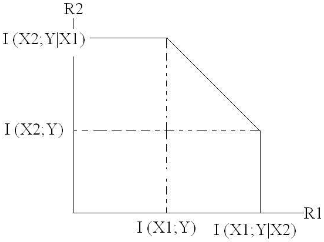
Classical multiaccess channel

$$\begin{array}{l} w_1 \longmapsto (X_{11}, \ldots, X_{1n}) \searrow \\ \qquad\qquad\qquad\qquad\qquad\quad \boxed{P_{Y|X_1,X_2}} \longrightarrow (Y_1, \ldots, Y_n) \longmapsto (\hat{w}_1, \hat{w}_2) \\ w_2 \longmapsto (X_{21}, \ldots, X_{2n}) \nearrow \end{array}$$

The capacity region is due to Ahlswede (1971) and Liao (1972). It can be achieved by random coding, joint typicality decoding, and time sharing.



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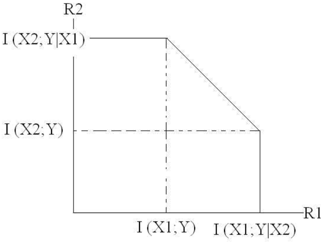

Jointly typical set

$$
\begin{aligned}
T_\epsilon^{(n)} = \Big\{ (\boldsymbol{x}_1, \boldsymbol{x}_2, \boldsymbol{y}) : & \left| \frac{1}{n} \log \frac{1}{p_{\boldsymbol{X}_1}(\boldsymbol{x}_1)} - H(X_1) \right| < \epsilon \\
& \left| \frac{1}{n} \log \frac{1}{p_{\boldsymbol{X}_2}(\boldsymbol{x}_2)} - H(X_2) \right| < \epsilon \\
& \left| \frac{1}{n} \log \frac{1}{p_{\boldsymbol{Y}}(\boldsymbol{y})} - H(Y) \right| < \epsilon \\
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& \left| \frac{1}{n} \log \frac{1}{p_{\boldsymbol{X}_2\boldsymbol{Y}}(\boldsymbol{x}_2, \boldsymbol{y})} - H(X_2, Y) \right| < \epsilon \\
& \left| \frac{1}{n} \log \frac{1}{p_{\boldsymbol{X}_1\boldsymbol{X}_2}(\boldsymbol{x}_1, \boldsymbol{x}_2)} - H(X_1, X_2) \right| < \epsilon \\
& \left| \frac{1}{n} \log \frac{1}{p_{\boldsymbol{X}_1\boldsymbol{X}_2\boldsymbol{Y}}(\boldsymbol{x}_1, \boldsymbol{x}_2, \boldsymbol{y})} - H(X_1, X_2, Y) \right| < \epsilon \Big\}
\end{aligned}
$$

The "empirical entropy" converges to the entropy for all subsets of (X_1, X_2, Y).

Proof of multiaccess channel capacity

- Two users transmit $\boldsymbol{X}_1(w_1)$ and $\boldsymbol{X}_2(w_2)$ from random codebooks.
- Receiver puts out the first $(\hat{w}_1, \hat{w}_2)$ satisfying $(\boldsymbol{X}_1(\hat{w}_1), \boldsymbol{X}_2(\hat{w}_2), \boldsymbol{Y}) \in T_\epsilon^{(n)}$.
- Let $E_{w_1 w_2} = \{(\boldsymbol{X}_1(w_1), \boldsymbol{X}_2(w_2), \boldsymbol{Y}) \in T_\epsilon^{(n)}\}$. Then

$$
\begin{aligned}
P_e \le\ & \underbrace{P(E_{11}^c)}_{\to 0 \text{ by AEP}} \\
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The large-system limit is ill-suited for many-user

- Example: The sum rate of the k-user Gaussian multiaccess channel:

$$C_{\text{sum}} = \frac{1}{2}\log(1+k\gamma) \to \infty$$
$$\frac{1}{k}C_{\text{sum}} = \frac{1}{2k}\log(1+k\gamma) \to 0$$

- *"When the total number of senders is very large, so that there is a lot of interference, we can still send a total amount of information that is arbitrary large even though the rate per individual sender goes to 0."* —Cover & Thomas, Elements of Information Theory.

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- (Almost practical) neighbor discovery
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Gaussian many-access channel (MnAC)

$$\boldsymbol{Y} = \sum_{j=1}^{\ell} \boldsymbol{S}_j(w_j) + \boldsymbol{Z}$$

- Many (ℓ) transmitters, each active w.p. $\alpha \in (0, 1]$ in a block.
- Average number of active users:

$$k = \alpha\ell.$$

- Message of user j: w_j, corresponding codeword $\boldsymbol{S}_j(w_j)$.
- User j is silent if $w_j = 0$, $\boldsymbol{S}_j(0) = \boldsymbol{0}$.

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New challenges

- Time sharing is not good in general. If $n = 1,000$, $k = 2,000$, an average user has half a channel use!
- Classical joint typicality does not apply as $n, \ell \to \infty$:

$$\begin{bmatrix} X^n_{1,1} & X^n_{1,2} & \dots & X^n_{1,n} \\ X^n_{2,1} & X^n_{2,2} & \dots & X^n_{2,n} \\ \vdots & \vdots & & \vdots \\ X^n_{\ell,1} & X^n_{\ell,2} & \dots & X^n_{\ell,n} \end{bmatrix}$$

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Identification code for the memoryless Gaussian MnAC

1. Let ℓ denote the total number of users.
2. If user j is inactive ($W_j = 0$), it transmits $\mathbf{0}$;
3. If user j is active ($W_j = 1$), it transmits $\boldsymbol{s}_j$,

$$\frac{1}{n_\ell}\sum_{i=1}^{n_\ell} s_{ji}^2 \le \gamma.$$

4. On average, k_ℓ users are active with i.i.d. activities, $P\{W_j = 1\} = \frac{k_\ell}{\ell}$.
5. Average identification error probability:

$$p_\ell = P\left\{\mathcal{D}(\boldsymbol{Y}) \neq (W_1, \dots, W_\ell)\right\}.$$

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Main result 1: identification cost

Theorem

Let

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For every $\epsilon \in (0,1)$, as $\ell \to \infty$, arbitrarily reliable identification ($p_\ell \to 0$) is achievable with $(1+\epsilon)n_\ell$ channel uses; whereas $p_\ell \to 0$ is not achievable with $(1-\epsilon)n_\ell$ channel uses. Here

$$H_2(\alpha) = \alpha \log \frac{1}{\alpha} + (1-\alpha) \log \frac{1}{1-\alpha}.$$

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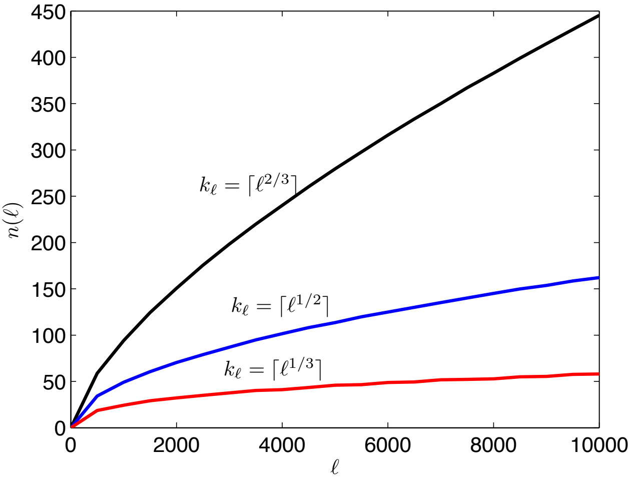
Identification cost vs. user number

$\gamma = 10$ dB



Identification and channel code for the Gaussian MnAC

An (M, n) symmetric code:

1. Encoders $\mathcal{E}_k : \{0, \dots, M\} \to \mathcal{S}_k^n$ yields codewords $\boldsymbol{s}_k(0), \cdots, \boldsymbol{s}_k(M)$.

$$\frac{1}{n}\sum_{i=1}^{n} s_{ki}^2(w) \leq \gamma, \quad \forall w \in \{1, \cdots, M\}.$$

$\boldsymbol{s}_k(0) = \mathbf{0}$.

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Identification and channel code for the Gaussian MnAC

An (M,n) symmetric code:

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Capacity

- $(v(n))_{n=1}^{\infty}$ with $v(n) > 1$ is a sequence of asymptotically achievable message lengths for the MnAC if there exists a sequence of $(\lceil \exp(v(n)) \rceil, n)$ codes such that $P_e^{(n)}$ vanishes as $n \to \infty$.
- $\mathcal{B} = (B(n))_{n=1}^{\infty}$ is said to be *a* symmetric capacity of the MnAC channel if, for every $\epsilon \in (0,1)$, $((1-\epsilon)B(n))$ is asymptotically achievable but $((1+\epsilon)B(n))$ is not.
- If $(B(n))$ is a capacity, then $(B(n) + o(B(n)))$ is also a capacity.
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Main result 2: capacity of Gaussian MnAC

Theorem (Symmetric capacity)

Suppose $k_n \to \infty$, $k_n = O(n)$, *and* $\ell_n = o(e^{\delta k_n})$, $\forall \delta > 0$.

$$B(n) = \left(\frac{n}{2k_n} \log(1 + k_n \gamma) - \frac{H_2(\alpha)}{\alpha} \right)^+$$
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- The capacity is $B_1(n)$ if $\alpha = 1$ or the set of active users is known.
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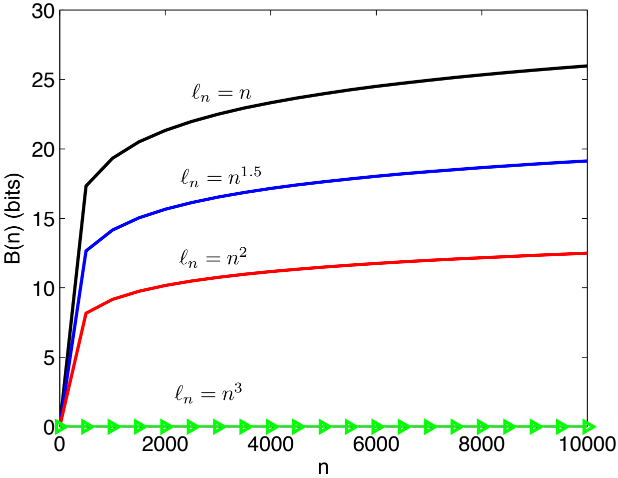
Capacity (message length) vs. blocklength

$\gamma = 10$ dB, $k_n = n/4$



Proof using an equivalent model

$$\boldsymbol{Y} = \underline{\boldsymbol{S}}\boldsymbol{X} + \boldsymbol{Z}$$

- Concatenated codebook

$$\underline{\boldsymbol{S}} = [\boldsymbol{s}_1(1), \cdots, \boldsymbol{s}_1(M), \ldots, \boldsymbol{s}_{\ell_n}(1), \cdots, \boldsymbol{s}_{\ell_n}(M)]_{n \times (M\ell_n)}$$

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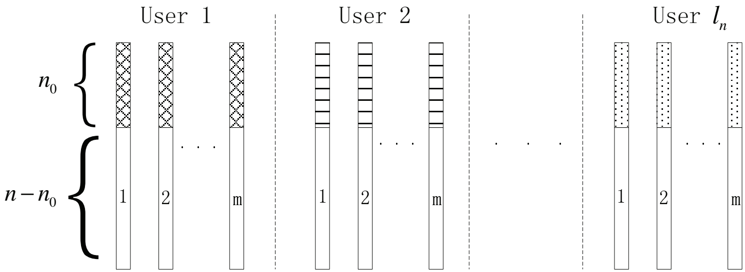

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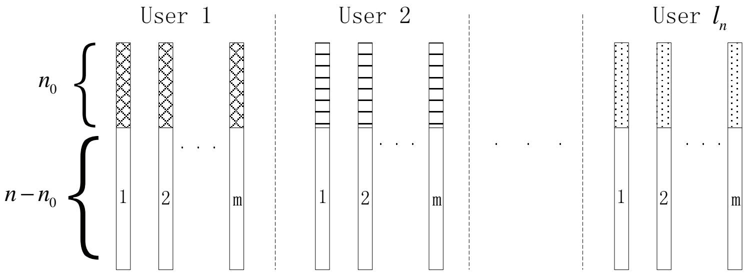

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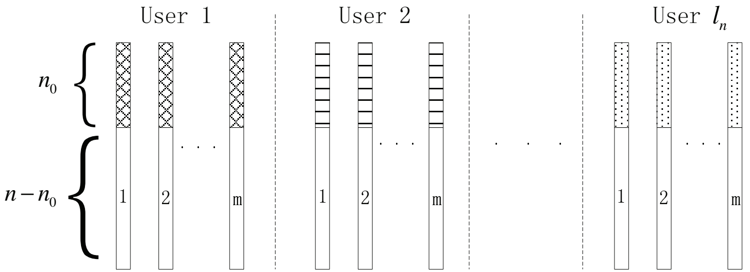

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Identification error probability

- Simple union bound fails for the exponential number of error events.
- The identification error event is decomposed to polynomial number of events (t_1 misses and t_2 false alarms):

$$\mathcal{E} = \bigcup_{t_1,t_2} \mathcal{E}_{t_1,t_2}$$

- Using techniques from Gallager '68 to upper bound the error

$$\mathsf{P}\left\{\mathcal{E}_{t_1,t_2} | \boldsymbol{X}^a = \boldsymbol{x}^a\right\} \leq e^{-nh_n(t_1,t_2)}$$

where for large enough n, $h_n(t_1,t_2) \geq c_0(\epsilon) > 0$, $\forall (t_1,t_2)$.

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- Characterized similarly by lower bounding the error exponents.
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Outline

- (Almost practical) neighbor discovery
- Classical information theory
- Many-access channel
- Many-broadcast channel

Classical degraded broadcast channels (BC)

- 2-user BC: $P_{Y_1 Y_2|X}$.
- Degraded if X–Y_1–Y_2 is Markov.
- The capacity region is

$$\bigcup_{P_{XU}:U-X-Y_1-Y_2} \left\{ (R_1, R_2) : \begin{array}{c} 0 \le R_2 \le I(U;Y_2) \\ 0 \le R_1 \le I(X;Y_1|U) \end{array} \right\}$$

- Generalizing to a k-user degraded BC:

$$R_j \le I(U_j; Y_j | U_{j+1}), \quad j = 1, \dots, k$$

where $(0 = U_{k+1})$–U_k–$\dots$–$(U_1 = X)$–Y_1–$\dots$–Y_k.

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where $(0=U_{k+1})$–U_k–$\dots$–$(U_1=X)$–Y_1–$\dots$–Y_k.

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Gaussian degraded many-broadcast channel (MnBC)

- k_n channel outputs:

$$Y_j = X + \sigma_{n,j} Z_j, \quad j = 1, 2, \dots, k_n$$

  where Z_j i.i.d.$\sim \mathcal{N}(0,1)$ and $\sigma_{n,j} \le \sigma_{n,j+1}$.
- $k_n \to \infty$ monotonically. $k_n = O(n)$.
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- How many bits can one send to each user reliably?
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$$V = \begin{pmatrix} v_{1,1} & & & & \\ v_{2,1} & v_{2,2} & & & \\ v_{3,1} & v_{3,2} & v_{3,3} & & \\ & \vdots & & & \\ v_{n,1} & v_{n,2} & \cdots & \cdots & v_{n,k_n} \\ & \vdots & & \vdots & \end{pmatrix}$$

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Capacity results

- Let auxiliary variables $U_1, \ldots, U_{k_n}$ be jointly Gaussian.
- Power allocation described by a triangular array $(\alpha_{n,j} : n = 1, 2, \ldots; j = 1, \ldots, k_n)$, with $\sum_{j=1}^{k_n} \alpha_{n,j} = 1$.
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$$B_{n,j} = B_j(n) = \frac{n}{2} \log \left(1 + \frac{\alpha_{n,j}\gamma}{\sigma_{n,j}^2 + \sum_{i=1}^{j-1} \alpha_{n,i}\gamma} \right).$$

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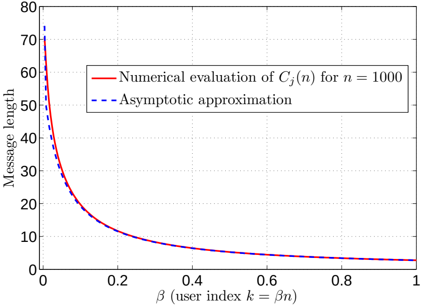
Gaussian MnBC: a numerical example

$n = 1000$, $\gamma = 20$, $k_n = 250$ (i.e., $c = 1/4$), and

$$\sigma_j^2 = \exp\left[-\frac{j}{(1+j)^2}\right].$$



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- Can send ≥ 1 bits reliably to all users;
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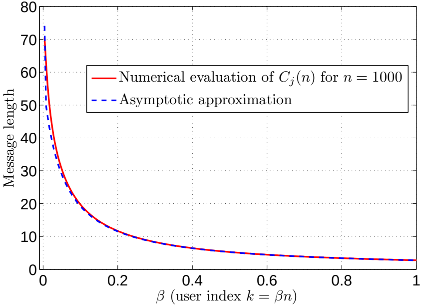

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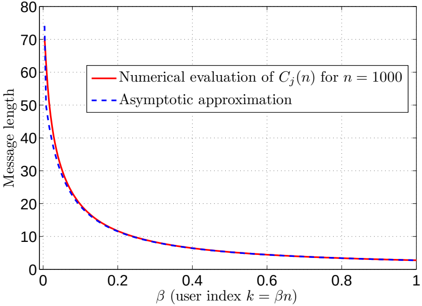

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- It's optimal to drop the group of qk_n least capable users.
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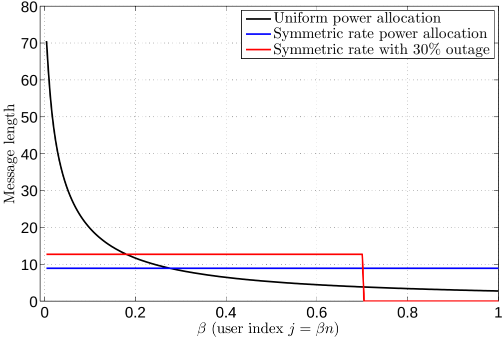

Many-Source coding?

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Conclusion

- Proposed a new many-user paradigm.
- Determined the minimum device identification cost.
- Determined the symmetric capacity of the Gaussian many-access channel with random user activities.
- Capacity results for the Gaussian degraded many-broadcast channel also develped (not shown here).
- large-system $\neq$ many-user.
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Remark: a related work

The only prior many-user model in the literature is the noiseless binary adder channel:

S.-C. Chang and E. Weldon, "Coding for t-user multiple-access channels," IEEE Trans. Inform. Theory, vol. 25, no. 6, pp. 684-691, 1979.

The number of users and blocklength taken to infinity simultaneously.

They studied uniquely decodable multiuser codes and the capacity.

Remark: large-system ≠ many-user

- Large-system analysis of CDMA and MIMO:
  Send to infinity the number of users and spreading factor simultaneously with fixed ratio; or the number of transmit and receive antennas with fixed ratio;
  Blocklength $n \to \infty$ before that.
  [Foschini & Gans '96, Telatar '99, Verdú & Shamai '99, Tanaka '02, Guo & Verdú '05, Huh, Tulino & Caire '12]
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