Block Markov Superposition Transmission: A Simple and Flexible Method for Constructing Good Codes

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Outline

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The Channel Coding Theorem

Theorem (Shannon 1948)

1. *For a channel, all rates below capacity C are achievable. Specifically, for every rate $R < C$, there exists a sequence of $(2^{nR}, n)$ codes with maximal probability of error $\lambda^{(n)} \to 0$.*
2. *Conversely, any rate above capacity C cannot be achievable. Equivalently, any sequence of $(2^{nR}, n)$ codes with $\lambda^{(n)} \to 0$ must have $R \leq C$.*

Capacity for AWGN Channels

A channel with additive white Gaussian noise (AWGN) is characterised by $y_t = x_t + w_t$, where x_t, y_t and w_t are input, output and noise, respectively. For AWGN channels, the capacity per dimension is given by [Shannon 1948]

$$C = \frac{1}{2} \log(1 + \mathrm{SNR}),$$

where SNR is the signal-to-noise ratio (SNR).

Capacity curves for AWGN Channels

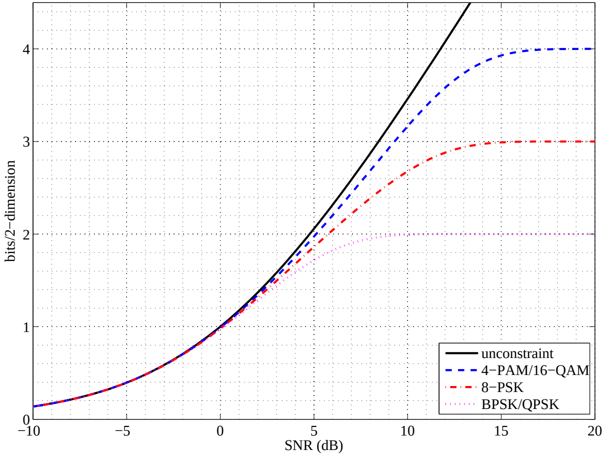


Figure: Capacity curves for AWGN channels and the i.u.d. capacity limits for several constellations (BPSK, 4-PAM, QPSK, 8-PSK, 16-QAM).

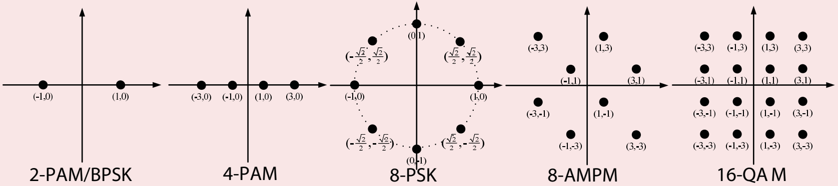
Existing Good Codes

- Turbo codes:
  parallel concatenated convolutional codes (PCCC) and serial concatenated convolutional codes (SCCC);
- Low-density parity-check (LDPC) codes (either random construction or algebraic construction): From decoding aspect, they can be viewed as serially concatenated repetition codes with single parity-check codes;
- Turbo/LDPC-like codes:
  (irregular) repeat-accumulate (RA) codes;
  accumulate-repeat-accumulate (ARA) codes;
  concatenated zigzag codes;
  precoded concatenated zigzag codes;
- Polar codes: Concatenation of a series of simple transformation;
- Spatially coupled codes: Convolutional LDPC codes; braided block/convolutional codes; stair-case codes;
- $\cdots$
- Non-binary, BICM, $\cdots$

Question

Is there a *universal* procedure to construct codes with

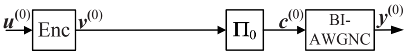
- any given (rational) code rate R, say $\frac{119}{911}$;
- any given signal constellation $\mathcal{A}$ (with moderate size);

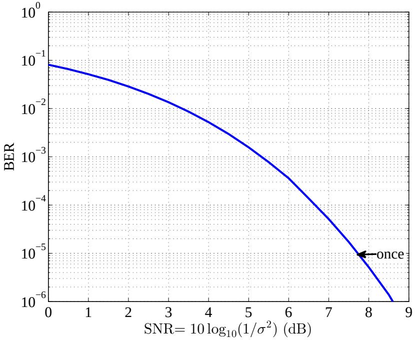


- any given target error performance (of interest), say, 10^{-4}, 10^{-6}, or 10^{-15}.

Outline

Principle of Block Markov Superposition Transmission (BMST)

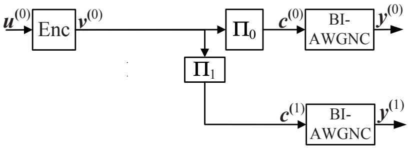


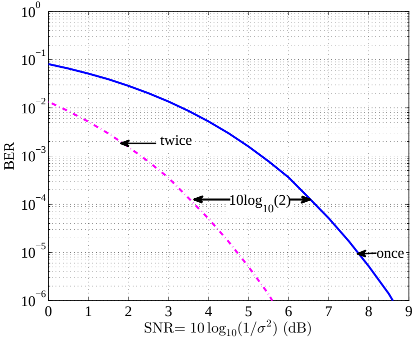


Repetition Increases Reliability

- Consider a basic code $\mathscr{C} = [N, K]^B$
  - B-fold Cartesian product of a short block code $[N, K]$.
- The codeword is transmitted once.
  - Performance curve in terms of BER versus SNR is shown.

Principle of Block Markov Superposition Transmission (BMST)

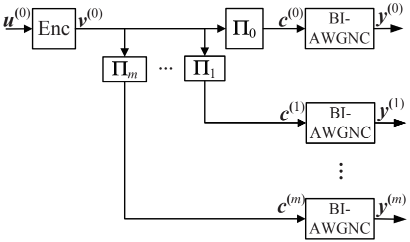


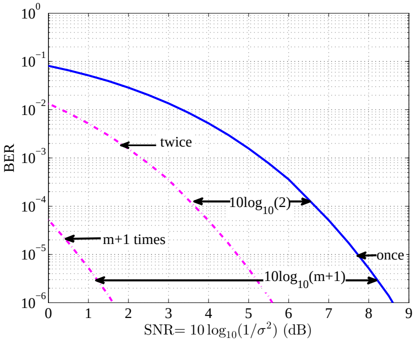


Repetition Increases Reliability

- The same codeword is transmitted twice.
- The performance curve shifts to the left by $10\log_{10} 2 = 3$ dB.

Principle of Block Markov Superposition Transmission (BMST)

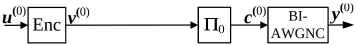


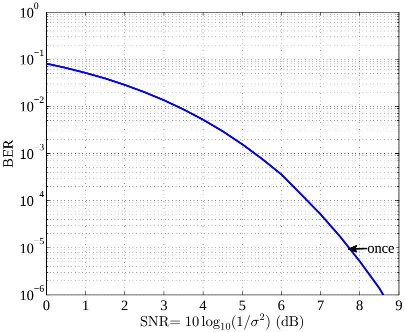


Repetition Increases Reliability

- The same codeword is transmitted $m + 1$ times.
- The performance curve shifts to the left by $10 \log_{10}(m + 1)$ dB.
- Repetition increases reliability but decreases efficiency (code rate).

Principle of Block Markov Superposition Transmission (BMST)





Superposition Increases Efficiency

- In the first transmission:
  - The transmitter sends a codeword $\boldsymbol{v}^{(0)}$ from the code $\mathscr{C}$ that corresponds to the first data block.

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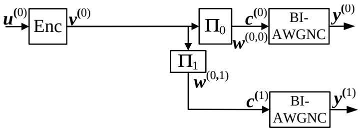
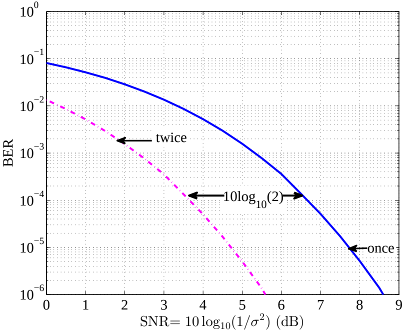
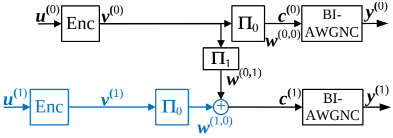


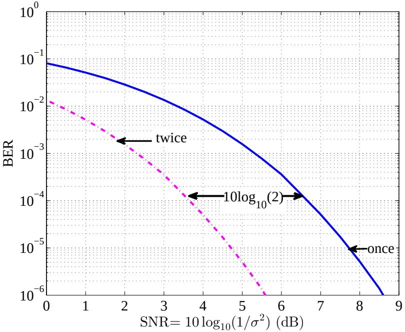


Superposition Increases Efficiency

- In the second transmission:
  - The transmitter generates the codeword $v^{(0)}$ (interleaved version) one more time;

Principle of Block Markov Superposition Transmission (BMST)





Superposition Increases Efficiency

- In the second transmission:
  - The transmitter generates the codeword $\boldsymbol{v}^{(0)}$ (interleaved version) one more time;
  - In the meanwhile, a fresh codeword $\boldsymbol{v}^{(1)}$ from $\mathscr{C}$ that corresponds to the second data block is superimposed on the interleaved version of $\boldsymbol{v}^{(0)}$.

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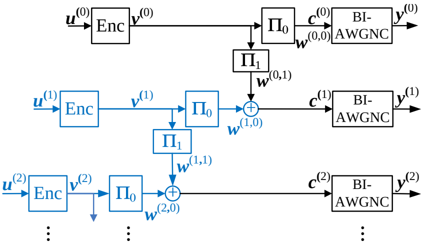
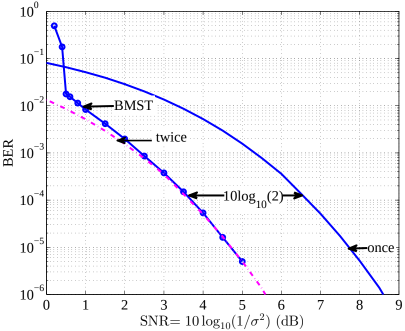
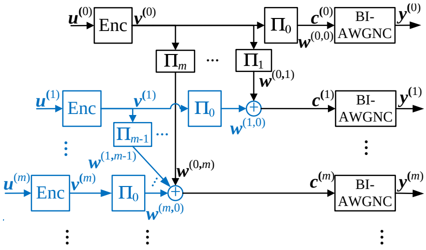


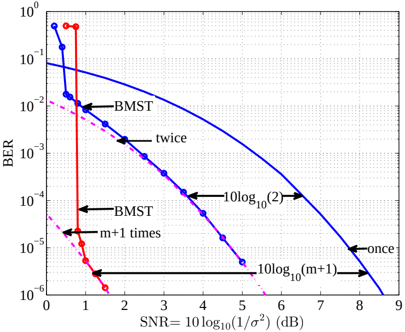


Superposition Increases Efficiency

- In the t-th transmission:
  - The current codeword $\boldsymbol{v}^{(t)}$ is superimposed on ("mixed into") the previous codeword $\boldsymbol{v}^{(t-1)}$ and then transmitted.
- We obtain a BMST code with memory 1.

Principle of Block Markov Superposition Transmission (BMST)

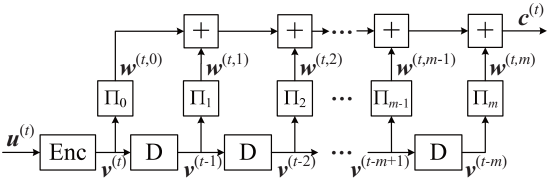




Superposition Increases Efficiency

- For a BMST code with memory m, the t-th transmission is a superposition of the current codeword and the m previous codewords, all randomly-interleaved.
- The high SNR performance can be predicted by shifting the BER curve to the left by $10\log_{10}(m+1)$ dB.

Principle of BMST – Encoding Structure



- A serially concatenated code:
  - Outer code (the *basic code*) introduces redundancy;
  - Inner code (a rate-one block-oriented feedforward convolutional encoder) introduces memory between transmissions.
- Termination procedure:
  - A tail consisting of m blocks of the all-zero vector is added;
  - Much simpler than for spatially coupled LDPC codes.
- Can be viewed as a class of spatially coupled codes
  - Generator matrix instead of the parity-check matrix is coupled.

Principle of BMST – Matrix Representation

$$\boldsymbol{G}_{\text{BMST}} = \begin{pmatrix} \boldsymbol{G\Pi}_0 & \boldsymbol{G\Pi}_1 & \cdots & \boldsymbol{G\Pi}_m & & & \\ & \boldsymbol{G\Pi}_0 & \boldsymbol{G\Pi}_1 & \ddots & \boldsymbol{G\Pi}_m & & \\ & & \ddots & \ddots & \ddots & \ddots & \\ & & & \boldsymbol{G\Pi}_0 & \cdots & \boldsymbol{G\Pi}_{m-1} & \boldsymbol{G\Pi}_m \end{pmatrix}_{Lk\times(L+m)n}$$

- L: length (in terms of blocks) of the transmitted data (coupling length).
- m: encoding memory (coupling width).
- $\boldsymbol{G}$: generator matrix of the basic code.
- $\boldsymbol{\Pi}_i (0 \leq i \leq m)$: $m+1$ randomly selected permutation matrices.
- Rate of the BMST code:

$$R_{\text{BMST}} = \frac{Lk}{(L+m)n} = \frac{L}{L+m} R,$$

where R is the rate of the basic code.

Principle of BMST – Decoding Algorithm

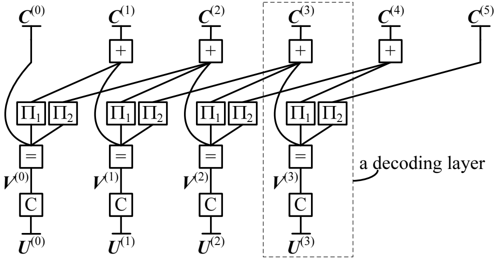


Figure: The normal graph of a BMST system with $L = 4$ and $m = 2$.

- An iterative sliding-window decoding (SWD) algorithm is used;
- Four types of nodes: C, =, +, and $\prod$;
- Messages are processed and passed through different decoding layers forward and backward over the normal graph.

Outline

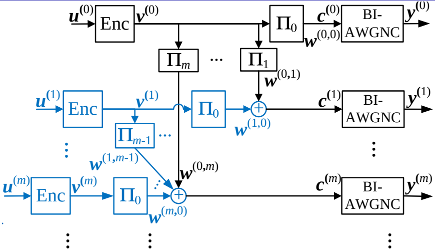


Figure: The BMST system.

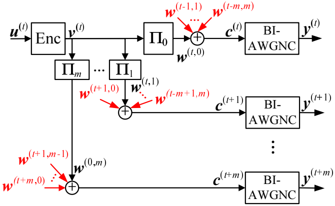


Figure: The genie-aided lower bound system.

Genie-Aided Lower Bound

- Imagine that $\mathbf{u}' = \{\mathbf{u}^{(i)}, t-m \le i \le t+m, i \ne t\}$ are known at the receiver.
- This is equivalent to transmitting $\boldsymbol{u}^{(t)}$ for $m+1$ times.
- The coding gain of the BMST can not be larger than

$$10\log_{10}(m+1) - 10\log_{10}(1+m/L) \text{ dB.}$$

- Noticing that $\Pr\{\boldsymbol{u}'|\boldsymbol{y}\} \approx 1$ in the low error rate region, we can expect that the maximal coding gain $10\log_{10}(m+1) - 10\log_{10}(1+m/L)$ dB.

Upper Bound

- The input-output weight enumerating function (IOWEF) of the BMST system can be computed from that of the basic code.
- The BER can be upper-bounded by an improved union bound.
- Notice that an incomplete (truncated) IOWEF is sufficient for upper bounds. (See Xiao Ma, Jia Liu and Baoming T-COMM 2013).

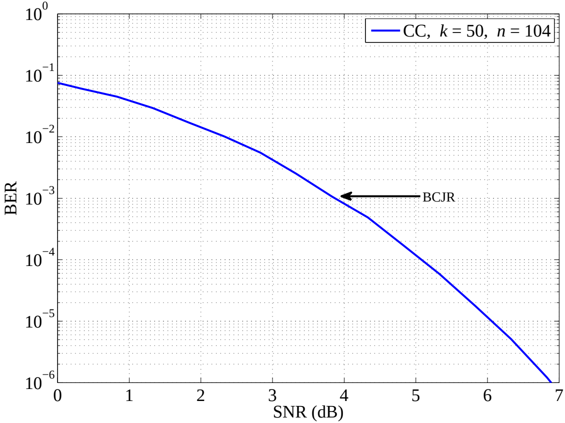
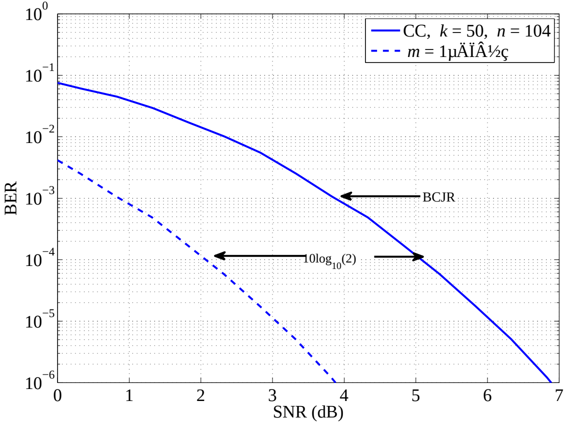


Figure: Coding gain analysis of the BMST system. The basic code is a terminated convolutional code (CC) with the polynomial generator matrix $[1, \frac{1+D+D^2}{1+D^2}]$. The coding parameters of the BMST system are $m = 1$, $L = 19$, $d = 19$, and $I_{\max} = 18$.

Performance Bounds of BMST – Example



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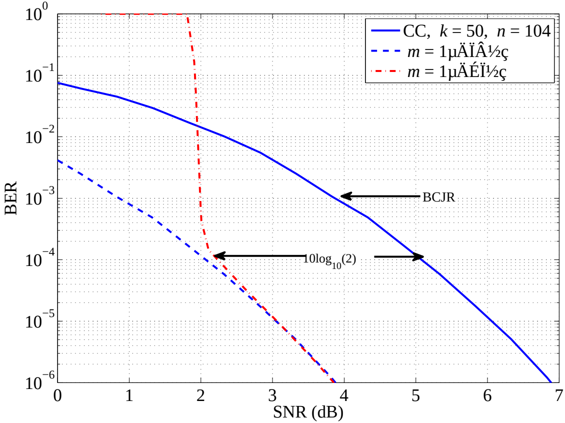

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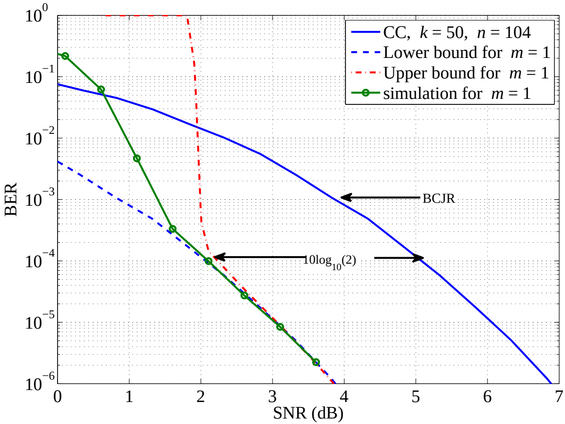

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A General Procedure of Designing BMST

With the genie-aided lower bound, to construct a BMST system of a given rate R with a target BER of p_{target}, we can perform the following steps.

1. Take a code $[N, K]^B$ with the given rate R as the basic code. In order to approach the channel capacity, we set the code length $n = NB \geq 10000$ in our simulations;
2. Find the performance curve $f_{\text{basic}}(\gamma_b)$ of the basic code. From this curve, find the required SNR $(\frac{1}{\sigma^2})$ to achieve the target BER. That is, find γ_{target} such that $f_{\text{basic}}(\gamma_{\text{target}}) \leq p_{\text{target}}$;
3. Find the Shannon limit for the code rate, denoted by γ_{lim};
4. Determine the encoding memory by $10\log_{10}(m+1) \geq \gamma_{\text{target}} - \gamma_{\text{lim}}$. That is,

$$m = \left\lfloor 10^{\frac{\gamma_{\text{target}}-\gamma_{\text{lim}}}{10}} - 1 \right\rceil,$$

   where $\lfloor x \rceil$ stands for the integer that is closest to x.
5. Generate $m+1$ interleavers randomly.

Construction Examples – BMST with Different Code Rates over Binary-Input AWGN Channels (BI-AWGNC)

Table: The encoding memories required to approach the corresponding Shannon limits using BMST systems for different code rates at given target BERs

Basic codes	p_{target}	γ_{target} (dB)	γ_{lim} (dB)	$\gamma_{\text{target}} - \gamma_{\text{lim}}$ (dB)	m
RC $[8,1]^{1250}$	10^{-3}	0.77	–7.23	8.00	6
RC $[8,1]^{1250}$	10^{-6}	4.51	–7.23	11.74	14
RC $[4,1]^{2500}$	10^{-3}	3.78	–3.80	7.58	5
RC $[4,1]^{2500}$	10^{-6}	7.52	–3.80	11.32	13
RC $[2,1]^{5000}$	10^{-3}	6.79	0.19	6.60	4
RC $[2,1]^{5000}$	10^{-6}	10.53	0.19	10.34	10
SPC $[4,3]^{2500}$	10^{-3}	7.62	3.39	4.23	2
SPC $[4,3]^{2500}$	10^{-6}	10.91	3.39	7.52	5
SPC $[8,7]^{1250}$	10^{-3}	8.18	5.27	2.91	1
SPC $[8,7]^{1250}$	10^{-6}	11.20	5.27	5.93	3

A Construction Example – BMST with Rate-$1/2$ over BI-AWGNC

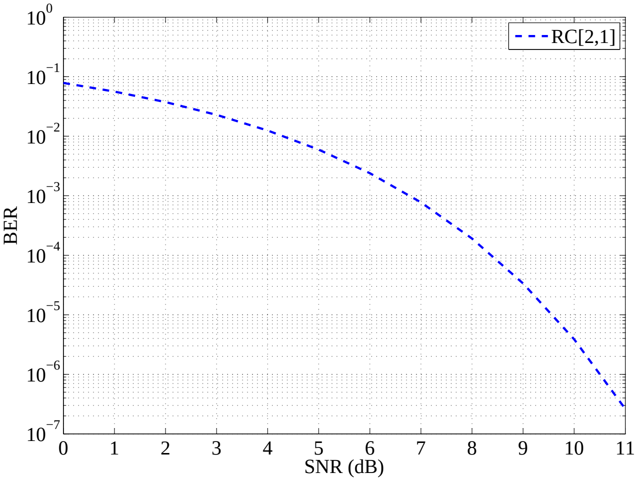

Figure: Performance of the BMST systems with the RC $[2,1]^{5000}$ as the basic code. The target BERs are 10^{-3} and 10^{-6}. The systems encode $L = 100000$ sub-blocks of data and decode with the SWD algorithm of a maximum iteration $I_{\max} = 18$.

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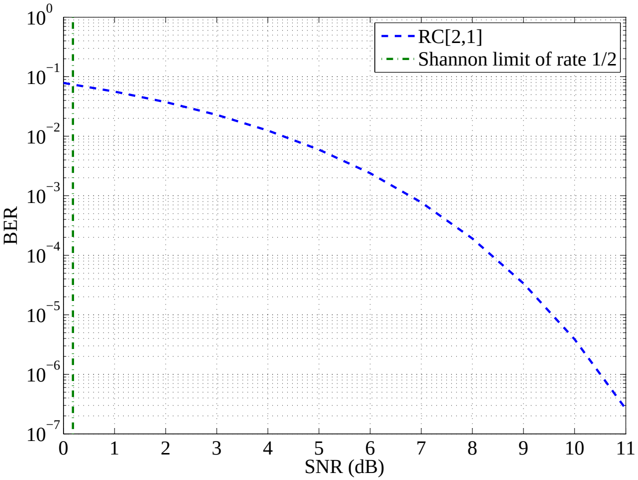

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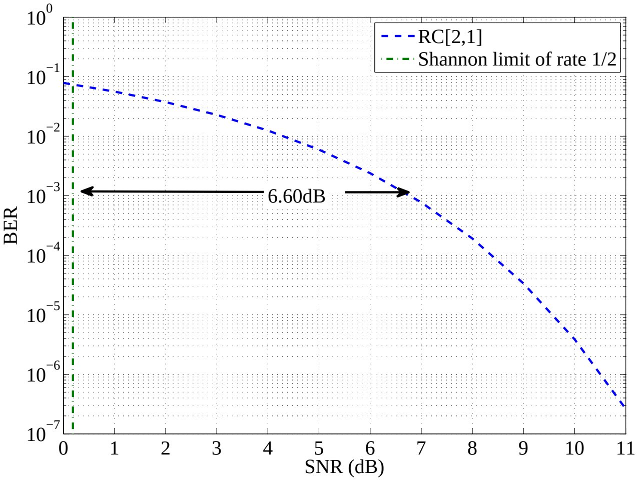

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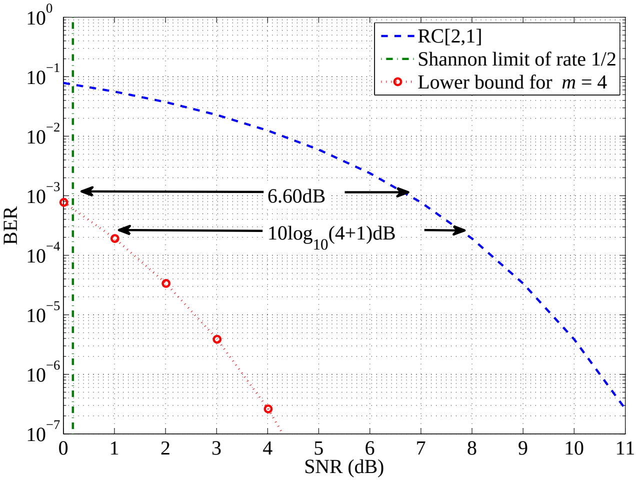

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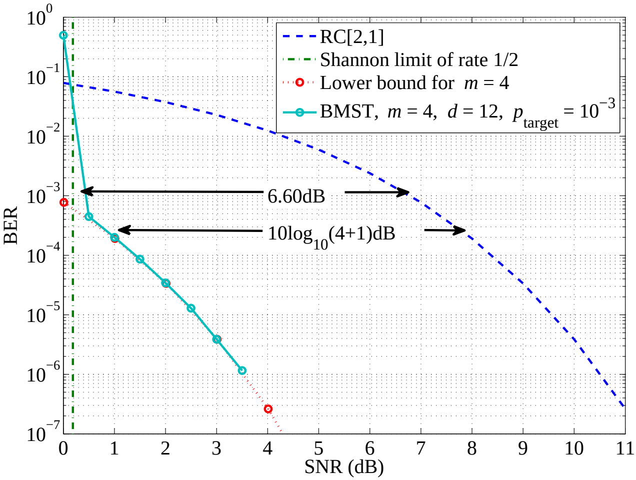

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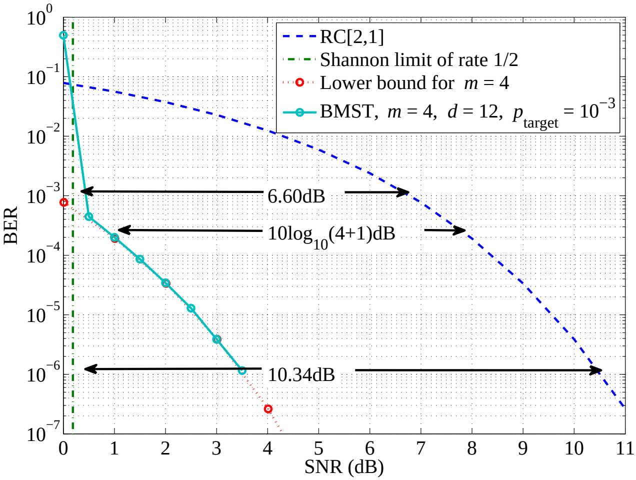

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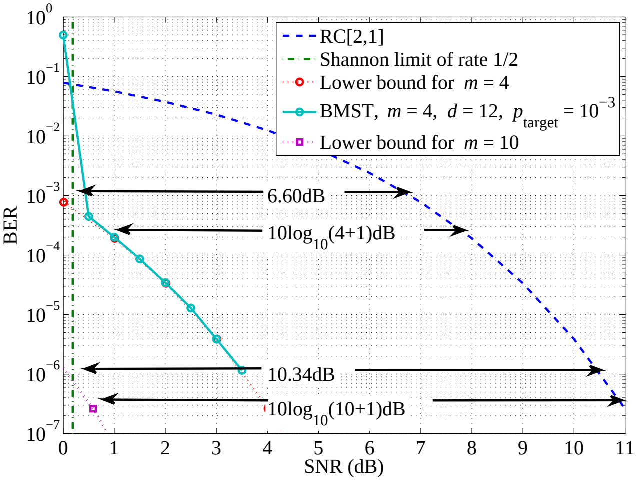

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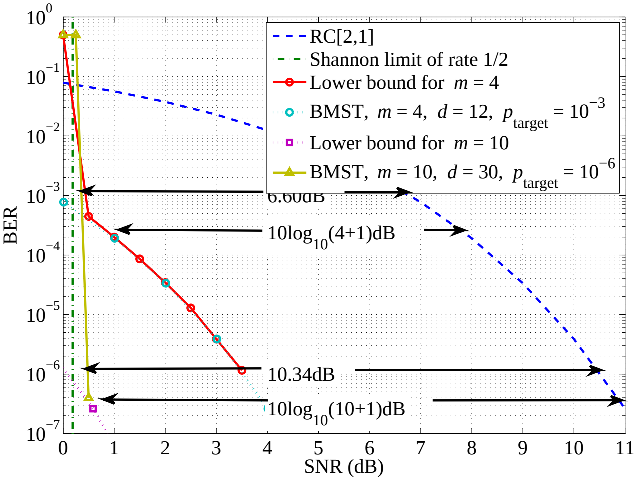

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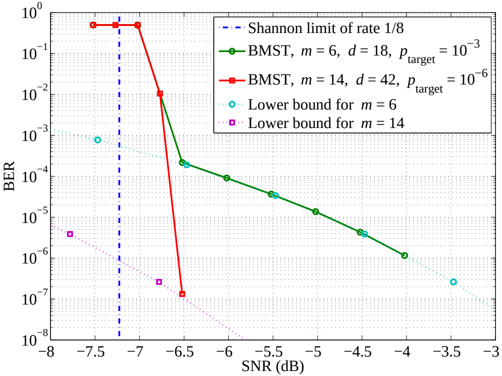

Figure: Performance of the BMST systems with the RC $[8,1]^{1250}$ as the basic code. The target BERs are 10^{-3} and 10^{-6}. The systems encode $L = 100000$ sub-blocks of data and decode with the SWD algorithm of a maximum iteration $I_{\max} = 18$.

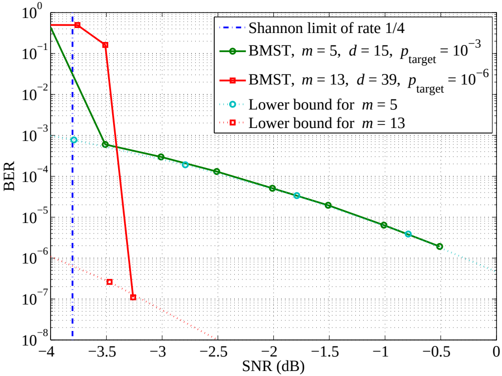

Figure: Performance of the BMST systems with the RC $[4,1]^{2500}$ as the basic code. The target BERs are 10^{-3} and 10^{-6}. The systems encode $L = 100000$ sub-blocks of data and decode with the SWD algorithm of a maximum iteration $I_{\max} = 18$.

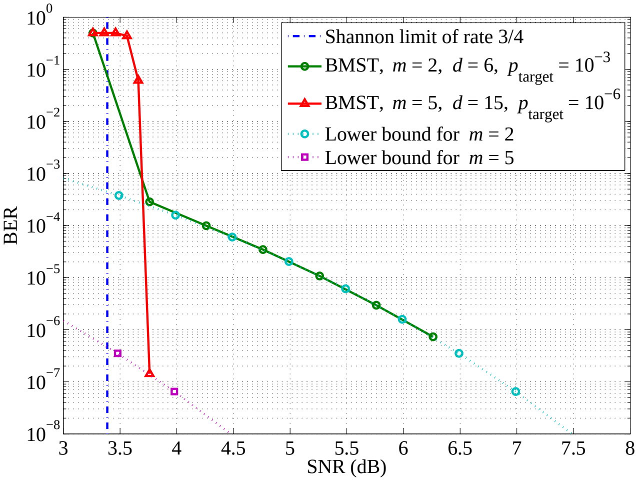

Figure: Performance of the BMST systems with the SPC $[4, 3]^{2500}$ as the basic code. The target BERs are 10^{-3} and 10^{-6}. The systems encode $L = 100000$ sub-blocks of data and decode with the SWD algorithm of a maximum iteration $I_{\max} = 18$.

Construction Examples – BMST with Rate-7/8 over BI-AWGNC

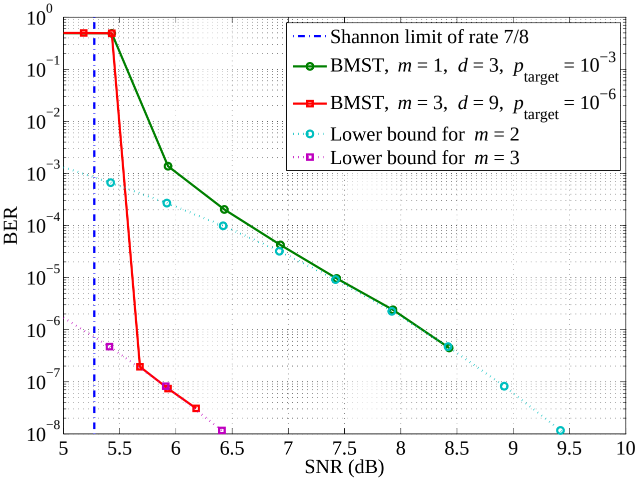

Figure: Performance of the BMST systems with the SPC $[8, 7]^{1250}$ as the basic code. The target BERs are 10^{-3} and 10^{-6}. The systems encode $L = 100000$ sub-blocks of data and decode with the SWD algorithm of a maximum iteration $I_{\max} = 18$.

Construction Examples – BMST with Different Code Rates over BI-AWGNC

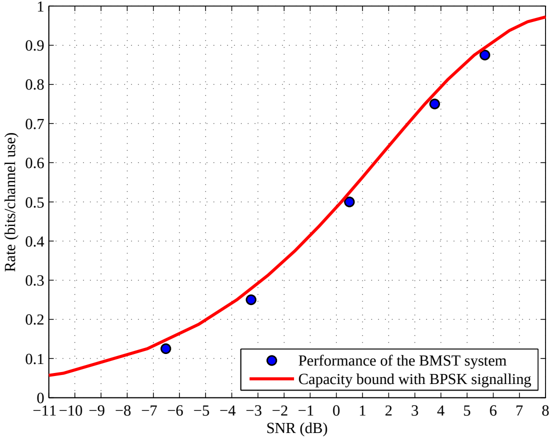

Figure: The required SNRs $(1/\sigma^2)$ for the BMST system using repetition codes and single-parity-check codes to achieve the BER of 10^{-6} over the BI-AWGNC.

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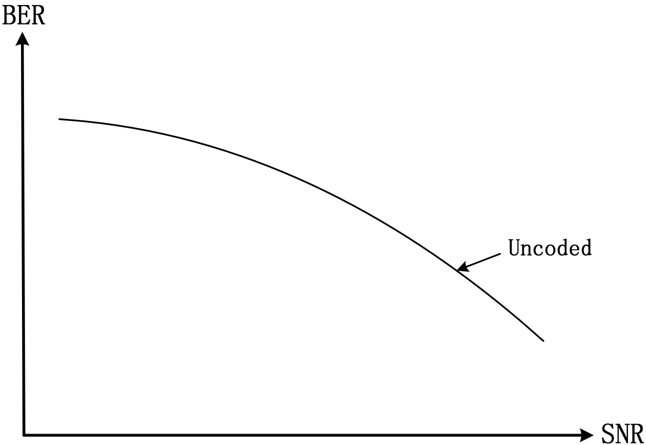

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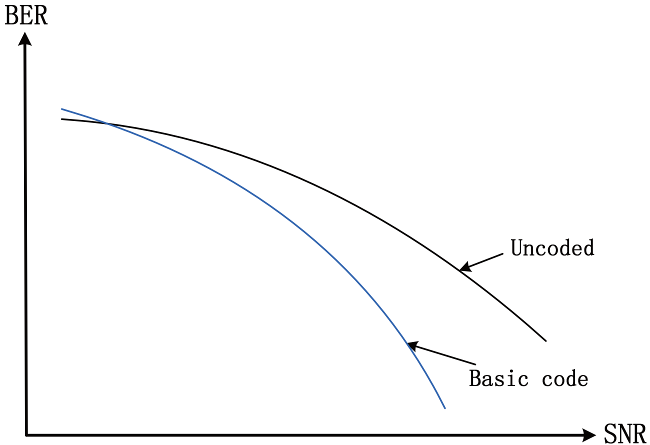

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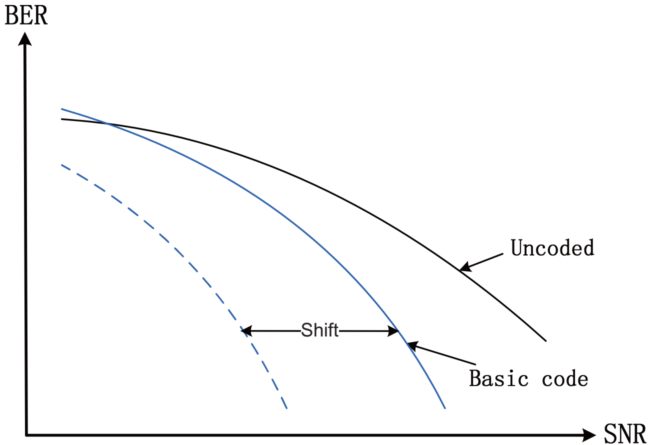

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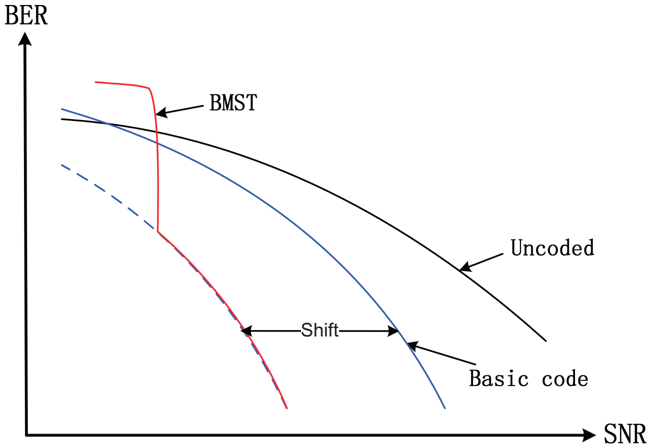

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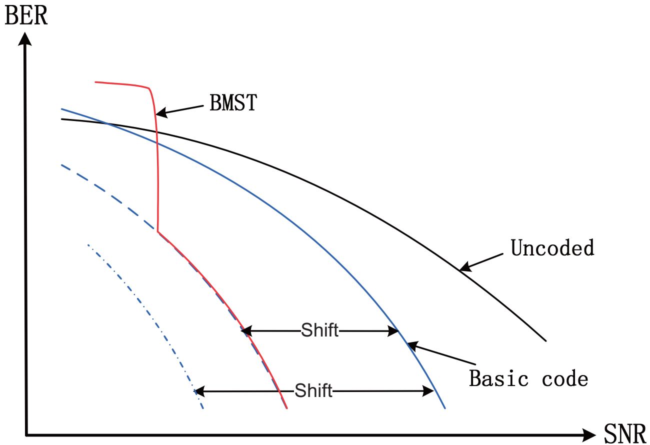

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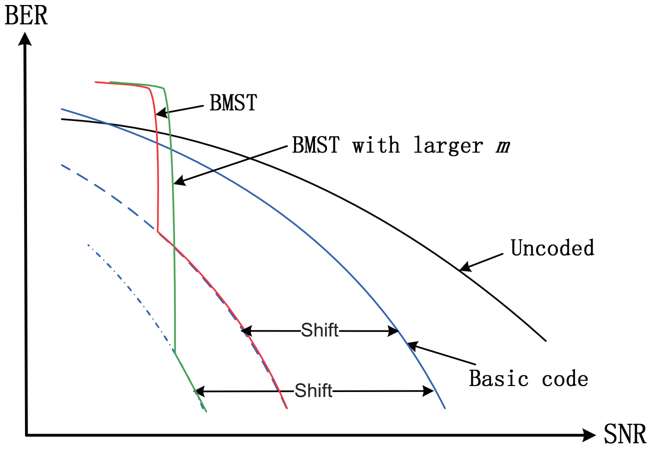

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- What do we really care about?

What we really care about is whether or not the basic code has efficient encoding/decoding algorithms.

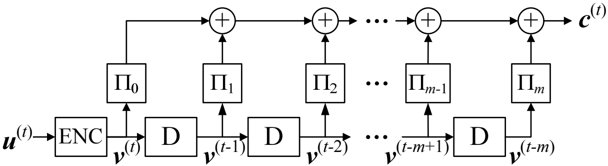


Figure: Encoding of BMST with memory m.

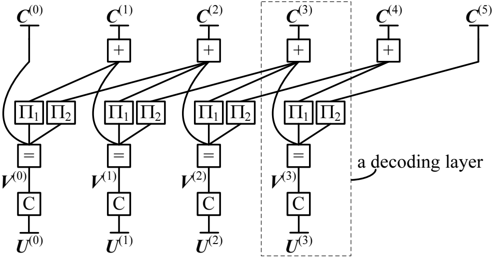


Figure: Sliding-window decoding over the normal graph.

Multiple-Rate Codes over BI-AWGNC – Hadamard Transform (HT) Coset Codes

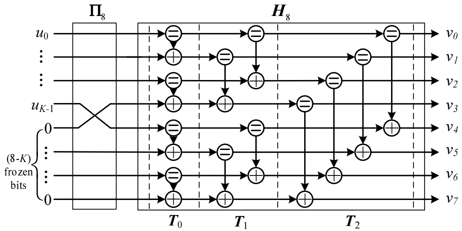


Table: The Memory Required for Each Code Rate Using the BMST of HT-coset Codes with $N = 8$ to Approach the Shannon Limit at the BER of 10^{-5}

Rate $R = K/8$	1/8	2/8	3/8	4/8	5/8	6/8	7/8
γ_K^* (dB)	-7.2	-3.8	-1.5	0.2	1.8	3.4	5.3
γ_K (dB)	3.6	6.8	7.2	8.0	9.9	10.4	10.6
Gap $\gamma_K - \gamma_K^*$ (dB)	10.8	10.6	8.7	7.8	8.1	7.0	5.3
Memory m_K	11	10	6	5	5	4	2

Multiple-Rate Codes over BI-AWGNC – BMST-HT Codes

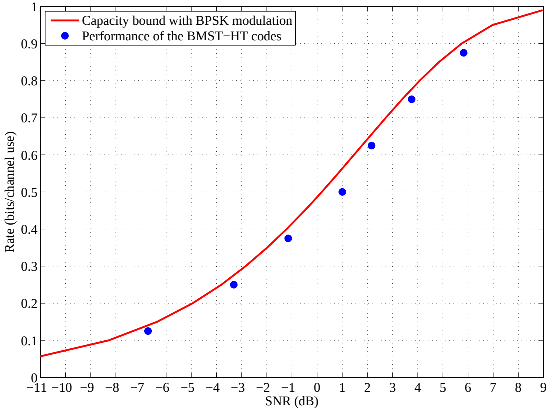


Figure: The required SNR for the BMST-HT codes $[8, K]^{1250}(1 \leq K \leq 7)$ to achieve the BER of 10^{-5} with BPSK signalling over AWGN channels.

Repetition (R) Codes And Single-Parity-Check (SPC) Codes

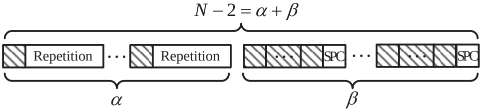


Figure: The form of a codeword in an RSPC code, where the locations for information bits are shaded. The code rate can be varied from $1/N$ to $(N-1)/N$ by setting $\beta = 0, 1, \cdots, N-2$.

Table: The Memories Required for the BMST-RSPC Codes with $N = 10$ to Approach the Shannon Limit at the BER of 10^{-5}

Rate K/N	$\frac{1}{10}$	$\frac{2}{10}$	$\frac{3}{10}$	$\frac{4}{10}$	$\frac{5}{10}$	$\frac{6}{10}$	$\frac{7}{10}$	$\frac{8}{10}$	$\frac{9}{10}$
γ_K^*	−8.3	−4.9	−2.8	−1.2	0.2	1.5	2.7	4.1	5.8
γ_K	2.6	10.4	10.4	10.5	10.5	10.5	10.5	10.5	10.5
Memory m_K	11	33	20	14	10	7	5	3	2

Multiple-Rate Codes over BI-AWGNC – BMST-RSPC Codes

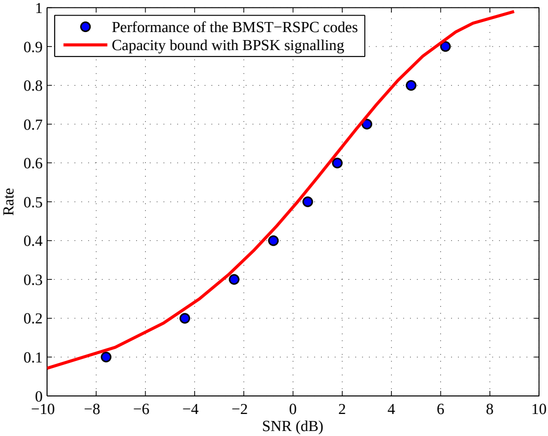


Figure: The required SNR for the BMST-RSPC codes with $N = 10$ to achieve the BER of 10^{-5} with BPSK signalling over AWGN channels.

Outline

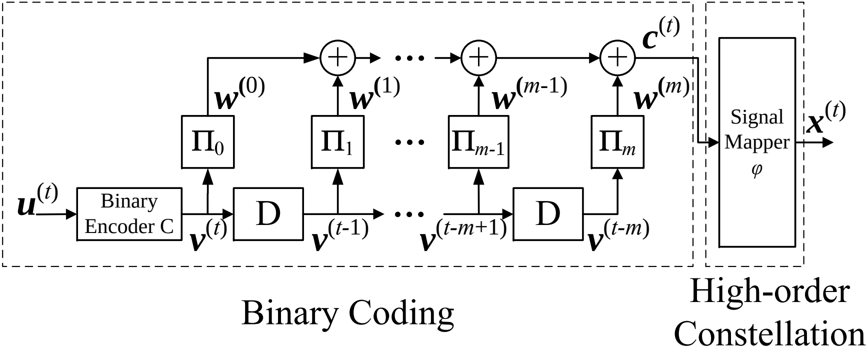


Figure: Binary BMST with high-order constellations.

BMST over High-Order Constellations – Binary BMST + 8-PSK

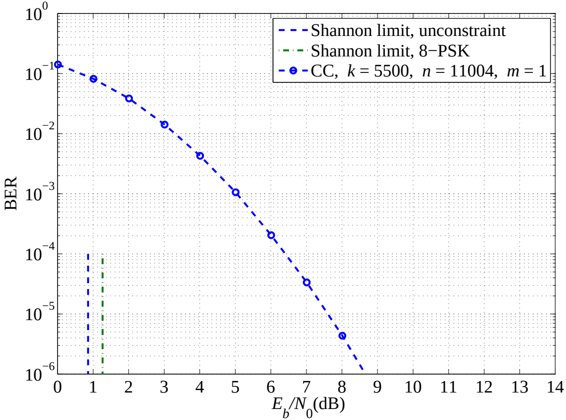
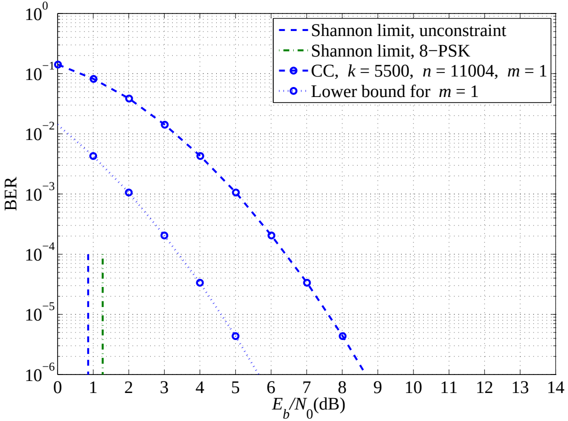


Figure: The basic code is a terminated 4-state $(2, 1, 2)$ convolutional code defined by the polynomial generator matrix $G(D) = [1 + D^2, 1 + D + D^2]$ with $k = 5500$ and $n = 11004$. Signals are transmitted using 8-PSK modulation with Gray mapping over AWGN channels. The system encodes $L = 1000$ sub-blocks of data and the iterative sliding-window decoding algorithm is performed, where the encoding memories and the decoding delays are specified in the legends.



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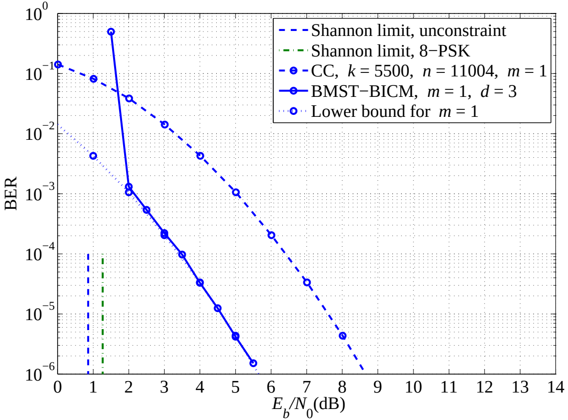
BMST over High-Order Constellations – Binary BMST + 8-PSK



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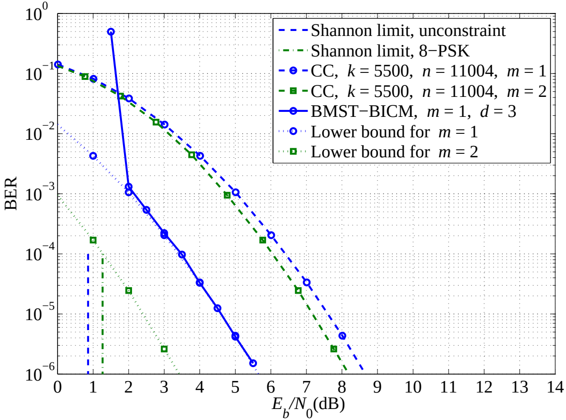

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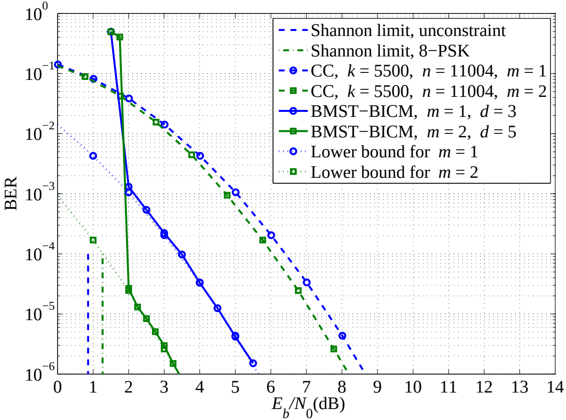

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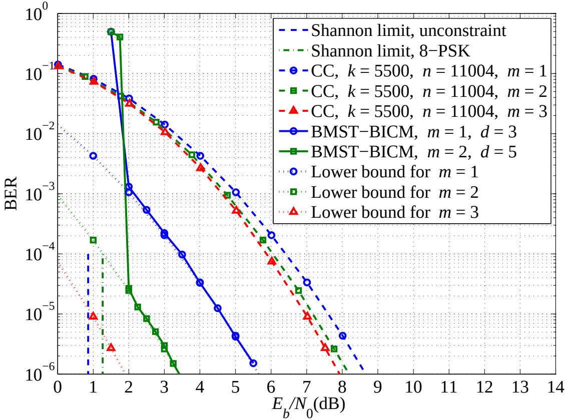

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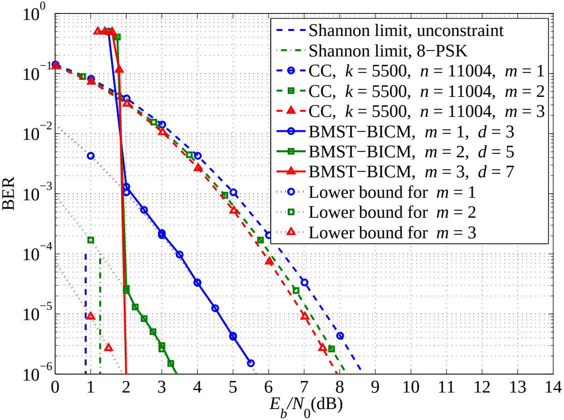

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BMST over High-Order Constellations – Nonbinary Codes + Nonbinary Constellations

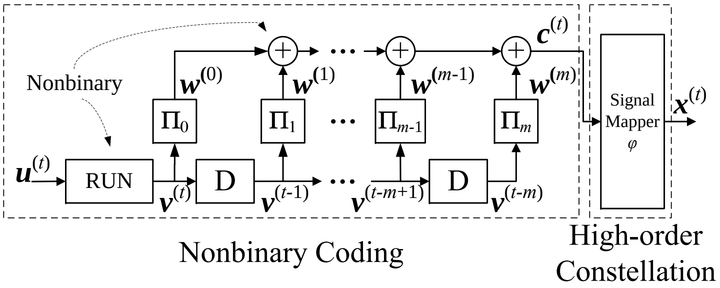


Figure: Nonbinary BMST with high-order constellations. $\boxed{\text{RUN}}$ stands a nonbinary code over groups.

Table: Construction Examples with 8-PSK Constellations over AWGN Channels

$\mathcal{A}$	$\frac{P}{Q}$	$\left(\frac{1}{N+1}, \frac{1}{N}\right)$	α	p_{target}	γ_{lim} (dB)	m
8-PSK	$\frac{1}{5}$	$\left(\frac{1}{6}, \frac{1}{5}\right)$	0	10^{-4}	-2.8	19
8-PSK	$\frac{2}{5}$	$\left(\frac{1}{3}, \frac{1}{2}\right)$	$\frac{1}{2}$	10^{-4}	1.3	17
8-PSK	$\frac{3}{5}$	$\left(\frac{1}{2}, 1\right)$	$\frac{2}{3}$	10^{-4}	4.7	15
8-PSK	$\frac{4}{5}$	$\left(\frac{1}{2}, 1\right)$	$\frac{1}{4}$	10^{-4}	8.1	7

BMST over High-Order Constellations – Nonbinary BMST + 8-PSK

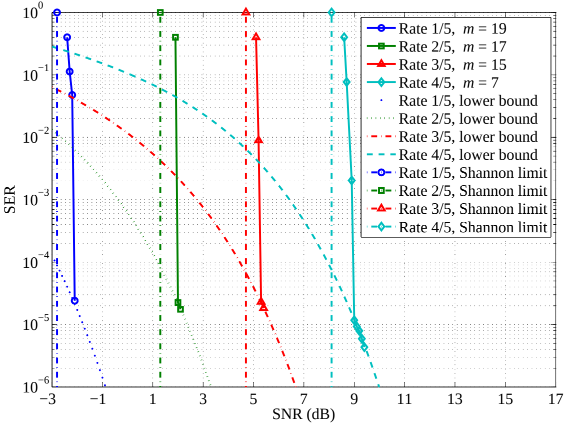


Figure: Performance of the BMST-RUN codes with the codes $\mathscr{C}_{\mathrm{RUN}}[Q,P]^{150}(\frac{P}{Q}=\frac{1}{5},\cdots,\frac{4}{5})$ as basic codes defined with 8-PSK modulation over AWGN channels.

Outline

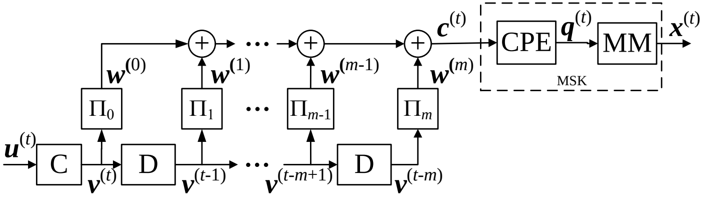


Figure: The BMST combined with minimum shift keying (MSK) modulation.

BMST Codes over Other Scenarios – Continuous Phase Modulation (CPM) over AWGN channels

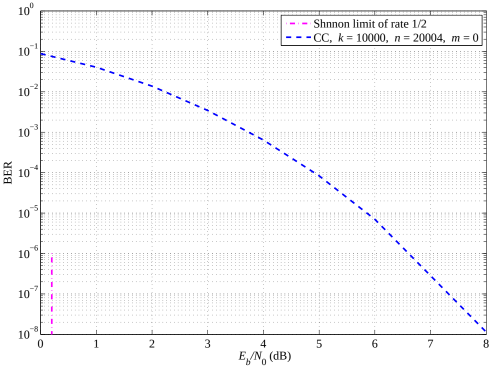
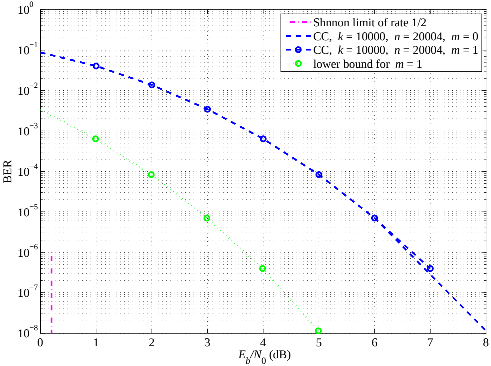


Figure: The basic code is a terminated 4-state $(2, 1, 2)$ convolutional code defined by the polynomial generator matrix $G(D) = [1 + D^2, 1 + D + D^2]$ with $k = 10000$ and $n = 20004$. Signals are transmitted using non-recursive MSK modulation over AWGN channels. The system encodes $L = 1000$ sub-blocks of data and the iterative sliding-window decoding algorithm with $d = 7$ and $I_{\max} = 18$ is performed, where the encoding memories are specified in the legends.

BMST Codes over Other Scenarios – Continuous Phase Modulation (CPM) over AWGN channels



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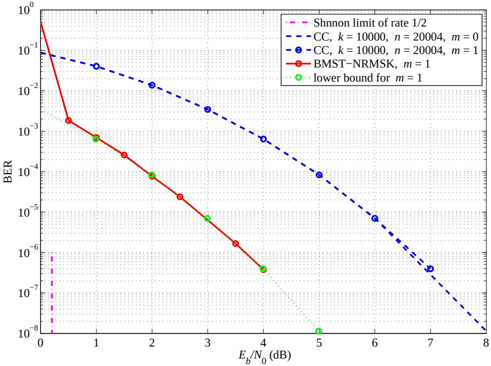

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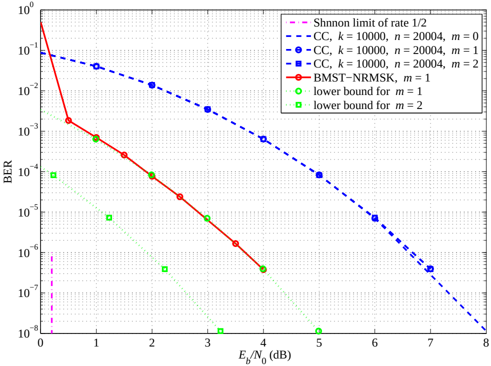

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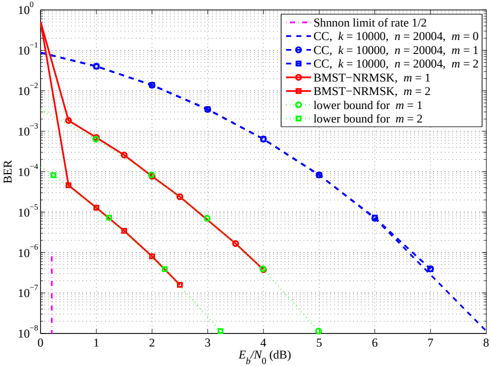

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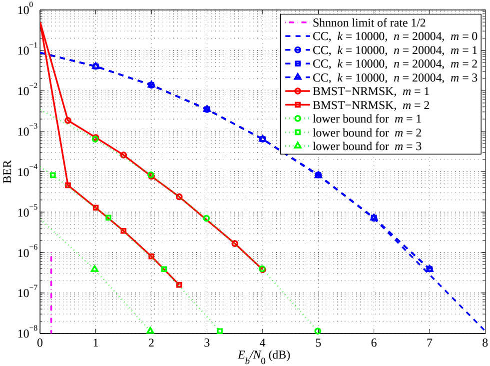

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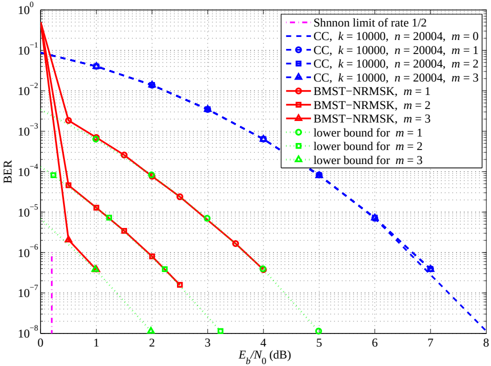

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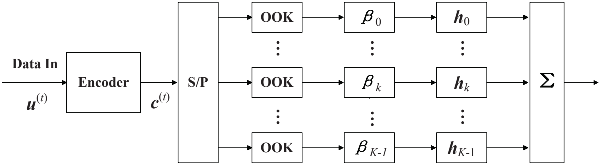


Figure: The VLC transmission.

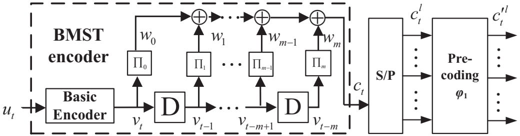


Figure: BMST combined in VLC transmission.

BMST Codes over Other Scenarios – Binary + Visible Light Communication (VLC)

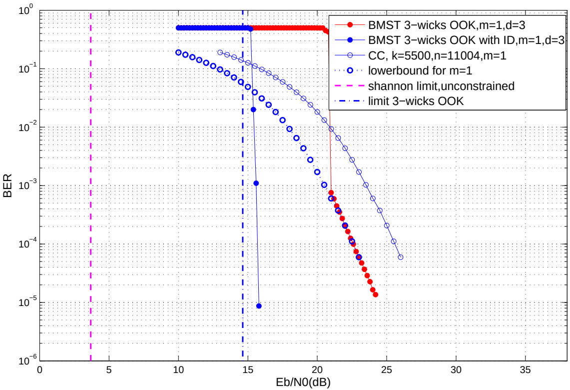


Figure: Performances of BMST systems with and without iterative demapping over AWGN Channels

BMST Codes over Other Scenarios – Nonbinary + Visible Light Communication (VLC)

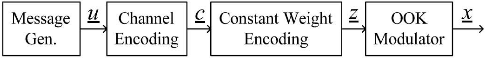


Figure: Block diagram of a VLC system.

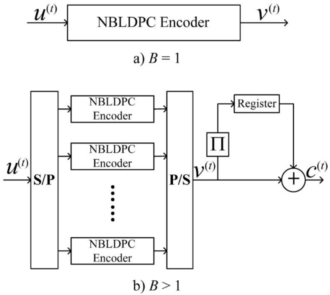


Figure: The nonbinary BMST encoder for the VLC system.

BMST Codes over Other Scenarios – Nonbinary + Visible Light Communication (VLC)

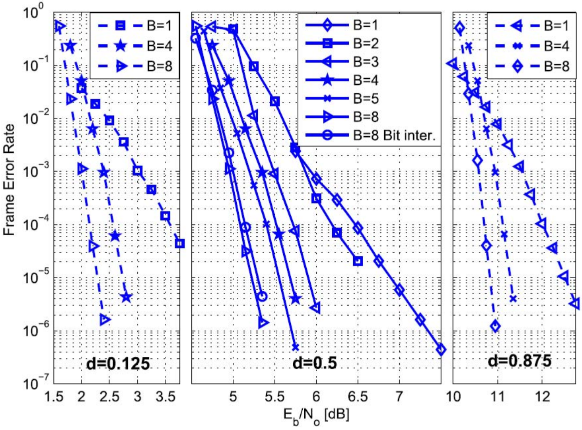


Figure: Error performances of the nonbinary BMST scheme under different delay requirements and dimming targets: OOK modulation and the nonbinary LDPC code $C_{64}[20, 10]$.

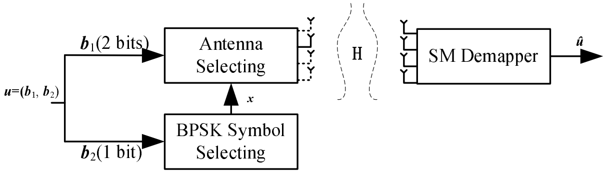
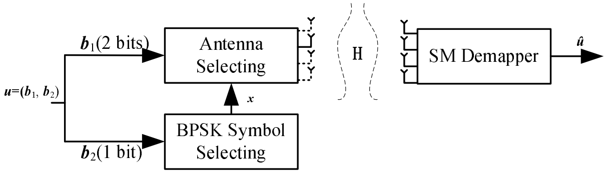


Figure: The spatial modulation with 4 transmitter antennas and 4 receiver antennas using BPSK modulation. Only one antenna is active for each transmission.



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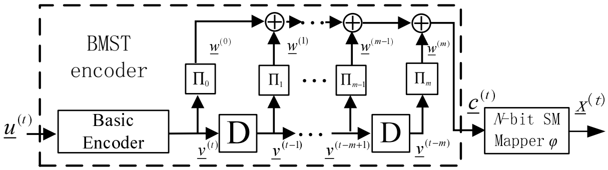

Figure: The BMST combined with spatial modulation.

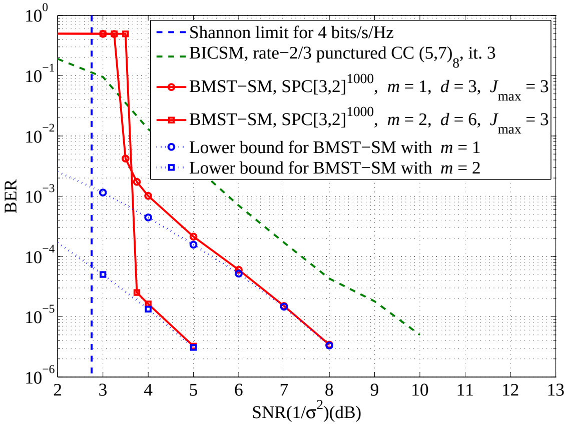

Figure: Comparison of the BMST-SM scheme with the BICSM scheme at 4 bits/s/Hz spectral efficiency.

BMST Codes over Other Scenarios – Two-Layer Coded Spatial Modulation (SM) over Rayleigh Fading Channels

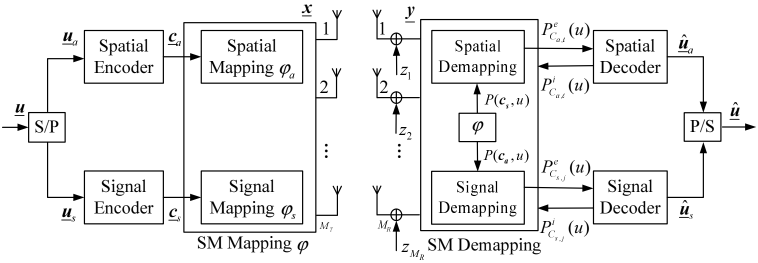

Figure: The block diagram of the two-layer coded spatial modulation system.

BMST Codes over Other Scenarios – Two-Layer Coded Spatial Modulation (SM) over Rayleigh Fading Channels

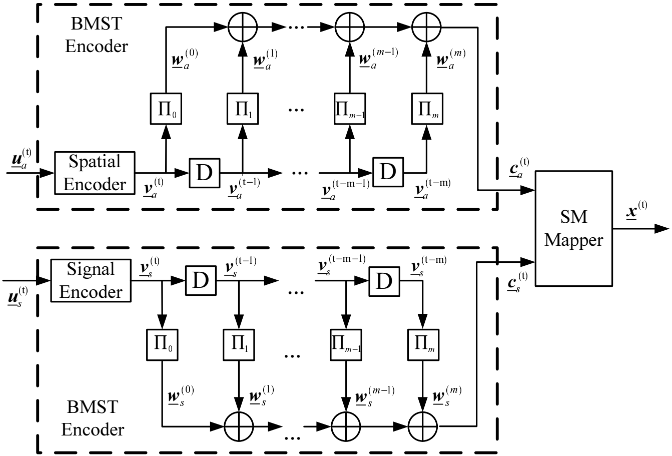

Figure: The block diagram for the encoding and mapping of the two-layer scheme using BMST codes.

BMST Codes over Other Scenarios – Two-Layer Coded Spatial Modulation (SM) over Raleigh Fading Channels

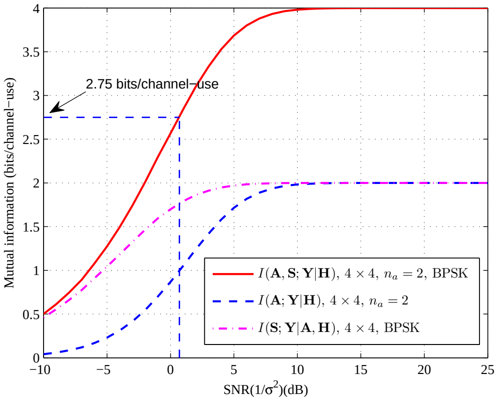

Figure: Mutual information for the 4×4, $n_a=2$ BPSK setup.

BMST Codes over Other Scenarios – Two-Layer Coded Spatial Modulation (SM) over Rayleigh Fading Channels

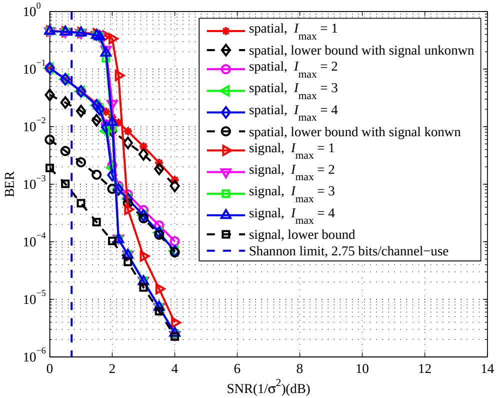

Figure: BER performance of the BMST-SM scheme with $m_1 = m_2 = 1$ and $L_1 = L_2 = 100$ under the 4×4, $n_a = 2$ BPSK setup, where the spectral efficiency is 2.75 bits/channel-use and $I_{\max}$ is the number of iterations between the two layers.

BMST Codes over Other Scenarios – Coded OFDM System over High-Mobility Channels

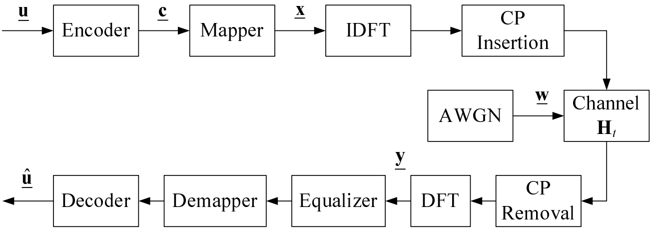

Figure: The block diagram of the coded OFDM system.

The receive vector can be written as

$$\mathbf{y} = \mathbf{F}\mathbf{H}_t\mathbf{F}^H\mathbf{x} + \mathbf{F}\mathbf{w}.$$

Let the frequency-domain matrix $\mathbf{H}_f = \mathbf{F}\mathbf{H}_t\mathbf{F}^H$, then the receive vector can be rewritten as

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BMST Codes over Other Scenarios – Coded OFDM System over High-Mobility Channels

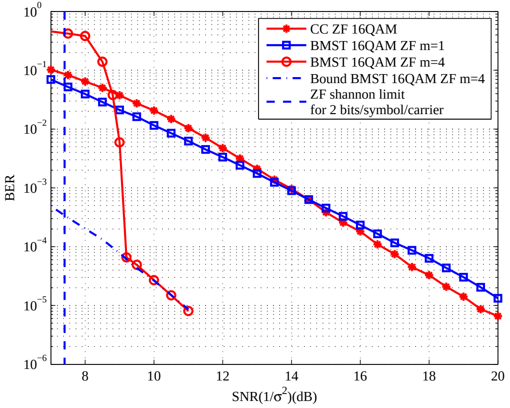


Figure: Comparison of the BMST scheme with the CC for OFDM system at 2 bits/symbol/carrier spectral efficiency. 16-QAM is used over the high-mobility channel with 360 km/h. The Shannon limit is based on ZF equalization.

BMST Codes over Other Scenarios – OFDM with Index Modulation (OFDM-IM) System over High-Mobility Channels

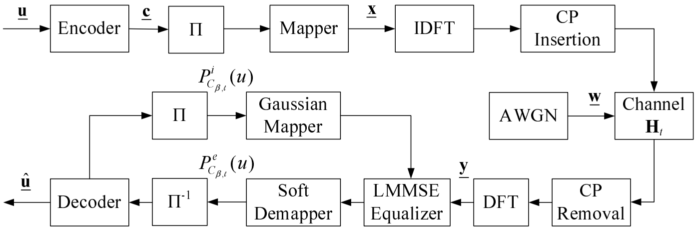


Figure: The block diagram of the coded OFDM-IM system.

The receive vector can be written as

$$\mathbf{y} = \mathbf{F}\mathbf{H}_t\mathbf{F}^H\mathbf{x} + \mathbf{F}\mathbf{w}.$$

Let the frequency-domain matrix $\mathbf{H}_f = \mathbf{F}\mathbf{H}_t\mathbf{F}^H$, then the receive vector can be rewritten as

$$\mathbf{y} = \mathbf{H}_f\mathbf{x} + \mathbf{w}_f.$$

Table: Simulation Parameters

Number of Subcarriers (N)	128
Number of Occuppied Subcarriers	96
Subcarrier Spacing F_c	15 KHz
Carrier Frequency (f_c)	2 GHz
Number of Multipaths (N_{tap})	9
Cyclic Prefix Length (N_{cp})	8
Velocity	360 km/h
Speed of Light (c_0)	3×10^8 m/s

The power-delay profile (PDP) is $P_i = \alpha e^{-0.6i}, 0 \leq i \leq N_{tap} - 1$, where α is a normalization constant. For IM system, we assume that one group has 4 subcarriers, i.e., we have $\binom{4}{2} = 6$ possible combinations of the selected subcarriers, and we choose $\mathcal{I} = \{(1,1,0,0),(0,1,1,0),(0,0,1,1),(1,0,0,1)\}$ as the *index constellation*.

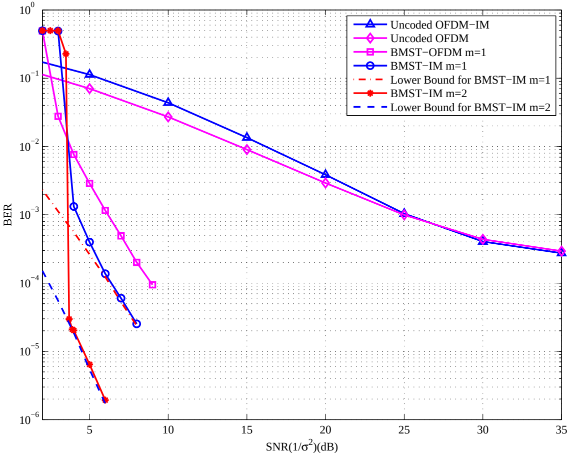


Figure: Comparison of the BMST-IM, BMST-OFDM scheme and the uncoded system under BPSK.

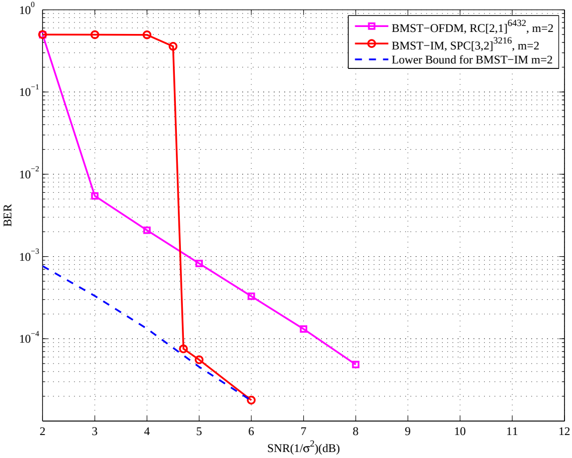
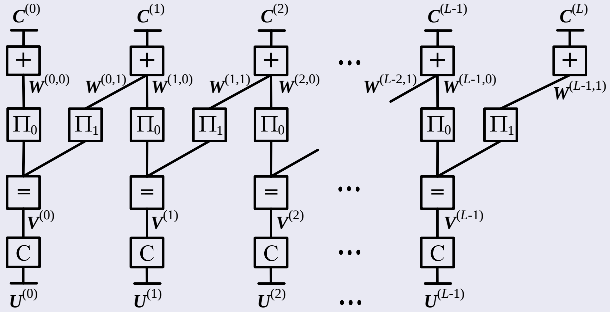


Figure: Comparison of the BMST-IM, BMST-OFDM scheme at 1 bits/symbol/carrier spectral efficiency under QPSK.

Outline

Drawbacks of BMST Codes

Drawbacks



- Neither rate-compatible nor systematic;
- Do not perform well over block fading channels due to error propagation.

Recent Focus

Rate-compatible systematic BMST codes

- Support a wide range of code rates;
- Maintain essentially the same encoding/decoding hardware structure.

Drawbacks of BMST Codes

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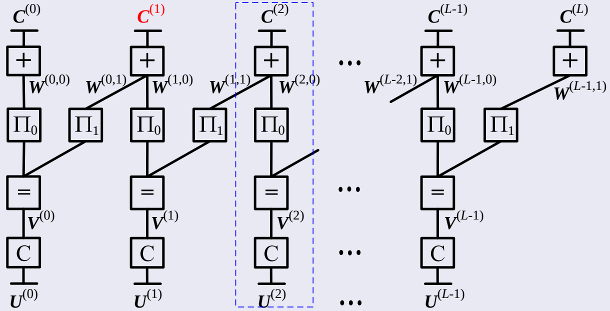

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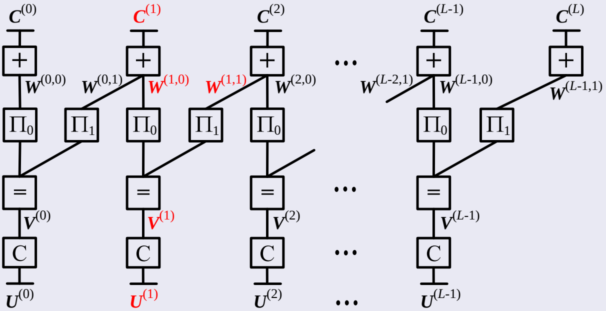

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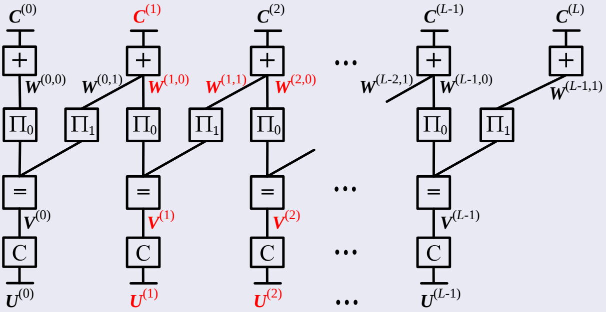

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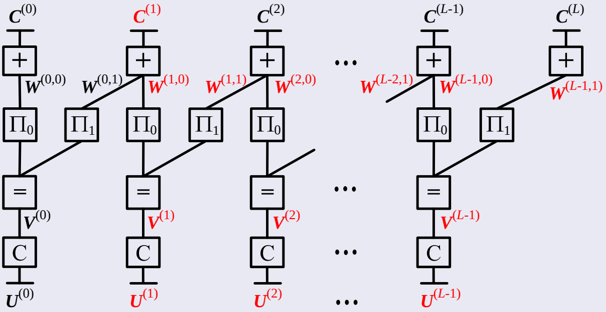

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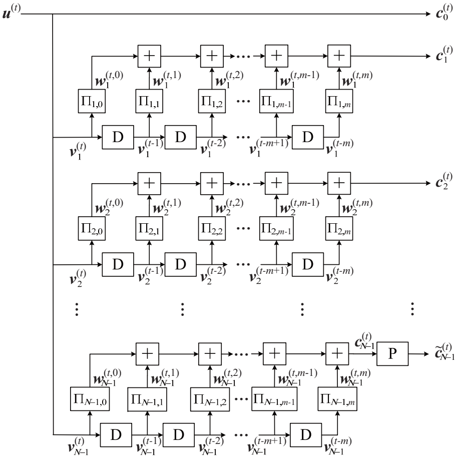


Figure: Encoder of a systematic BMST-R code with repetition degree N and encoding memory m.

Encoding of Systematic BMST-R Codes

1. **Initialization:** For $t < 0$ and $1 \le i \le N-1$, set $\boldsymbol{v}_i^{(t)} = \mathbf{0} \in \mathbb{F}_2^K$.
2. **Loop:** For $t \ge 0$,
   - Repeat $\boldsymbol{u}^{(t)}$ N times such that $\boldsymbol{c}_0^{(t)} = \boldsymbol{u}^{(t)} \in \mathbb{F}_2^K$ and $\boldsymbol{v}_i^{(t)} = \boldsymbol{u}^{(t)} \in \mathbb{F}_2^K$ for $1 \le i \le N-1$;
   - For $1 \le i \le N-1$,
     1. For $0 \le j \le m$, interleave $\boldsymbol{v}_i^{(t-j)}$ into $\boldsymbol{w}_i^{(t,j)}$ using the (i,j)-th interleaver $\mathbf{\Pi}_{i,j}$;
     2. Compute $\boldsymbol{c}_i^{(t)} = \sum_{0 \le j \le m} \boldsymbol{w}_i^{(t,j)}$.
   - Puncture randomly K_p of K bits in $\boldsymbol{c}_{N-1}^{(t)}$, resulting in $\widetilde{\boldsymbol{c}}_{N-1}^{(t)}$;
   - Take $\boldsymbol{c}^{(t)} = \{\boldsymbol{c}_0^{(t)}, \boldsymbol{c}_1^{(t)}, \boldsymbol{c}_2^{(t)}, \cdots, \widetilde{\boldsymbol{c}}_{N-1}^{(t)}\}$ as the t-th block of transmission.
3. **Termination:** For $t = L,\ L+1,\ \cdots,\ L+m-1$,
   - Set $\boldsymbol{u}^{(t)} = \mathbf{0} \in \mathbb{F}_2^K$, compute $\boldsymbol{c}^{(t)}$ following **Loop**;
   - Take the redundant check part of $\boldsymbol{c}^{(t)}$ as the t-th block of transmission.

- Puncturing fraction $\theta \overset{\Delta}{=} \frac{K_p}{K}$;
- Rate: $R_L = \frac{1}{N-\theta+(N-1-\theta)m/L}$.

Window Decoding

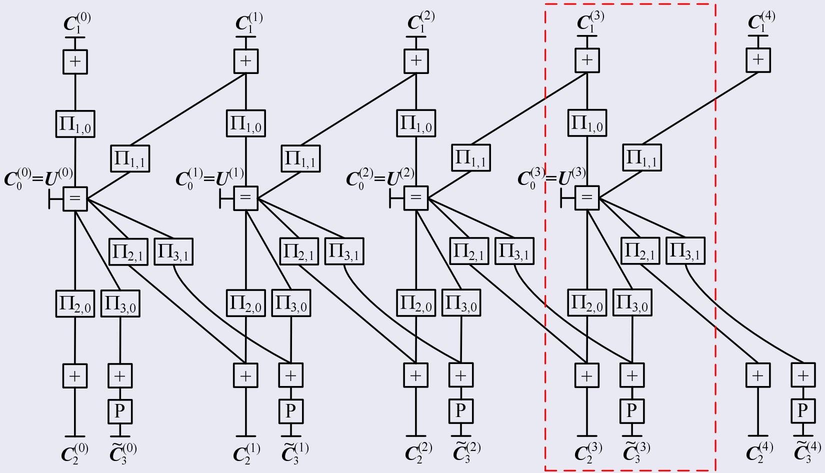
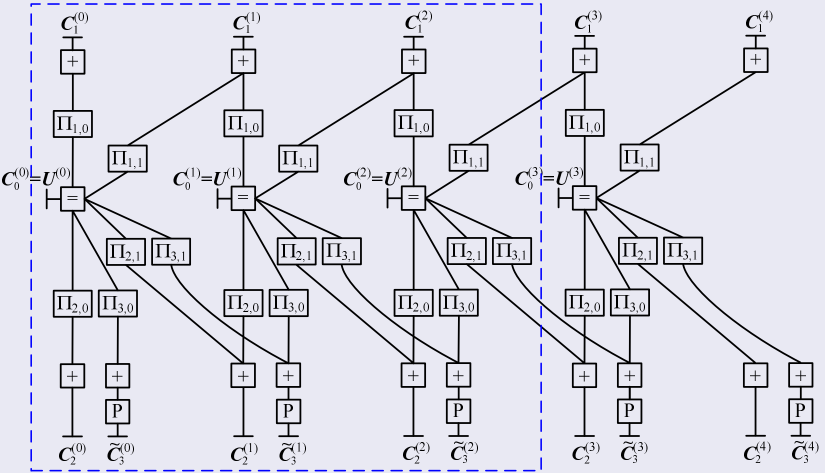


Figure: Window decoder with decoding delay $d = 2$ operating on the normal graph of a systematic BMST-R code with $N = 4$, $m = 1$ and $L = 3$.

Window Decoding



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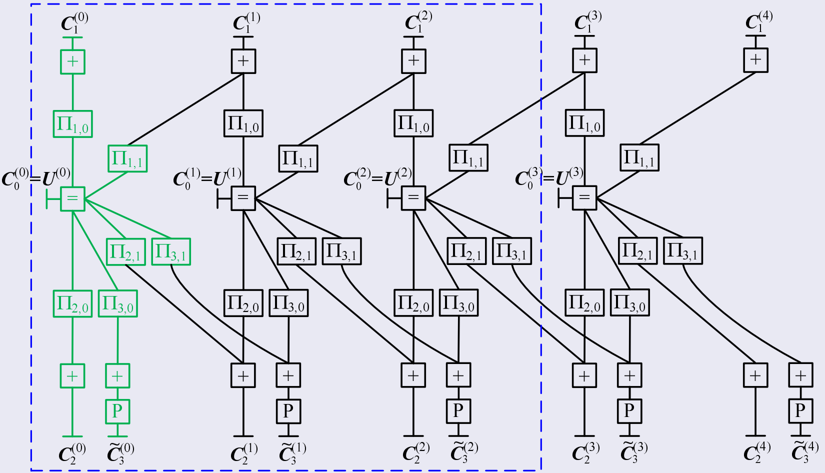

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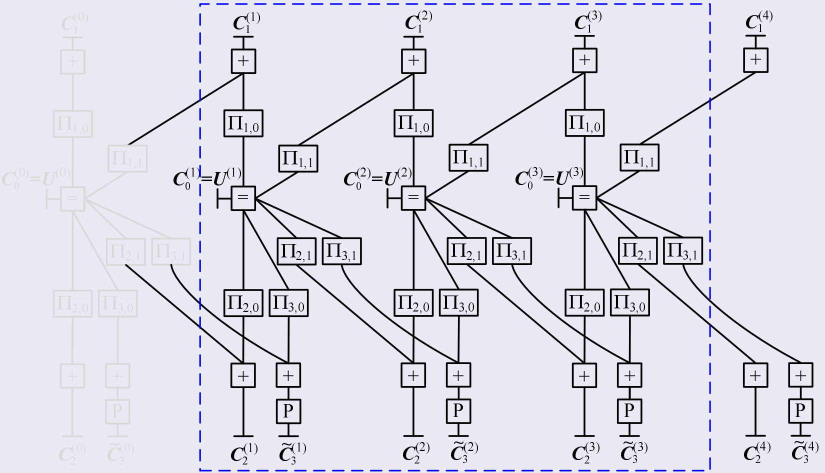


Figure: Window decoder with decoding delay $d = 2$ operating on the normal graph of a systematic BMST-R code with $N = 4$, $m = 1$ and $L = 3$.

Systematic BMST-R Codes – Relations with Existing Codes

- Systematic BMST-R codes resemble the classical rate-compatible punctured convolutional (RCPC) codes
  - Start from a rate $1/N$ systematic BMST-R code, where N is as large as required;
  - By puncturing, one can obtain all code rates of interest from $1/N$ to 1.
- The encoding of systematic BMST-R codes is block-oriented.
- The decoding is typically not implementable by the Viterbi algorithm.

- Systematic BMST-R codes can be viewed as a special class of spatially coupled codes.
- Similar to SC-LDPC codes, systematic BMST-R codes are decodable with a sliding window decoding algorithm.
- The encoding procedure for systematic BMST-R codes is simpler than for SC-LDPC codes.

- Different from existing codes, systematic BMST-R codes have a simple lower bound on the BER performance.

Systematic BMST-R Codes – Performance Bounds

Upper Bound on BER Performance

- Assuming that we know the truncated input-redundancy weight enumerating function (IRWEF) $\{A_{i,j}, 0 \le i \le T\}$ of systematic BMST-R codes, the bit-error probability under MAP decoding can be upper-bounded by

$$\mathrm{BER}_{\mathrm{MAP}} \le \min_{0 \le r^* \le T/2} \left\{ \sum_{i \le 2r^*} \frac{i}{k} \left(\sum_j A_{i,j} \, Q\left(\frac{\sqrt{i+j}}{\sigma} \right) \right) + \sum_{i=r^*+1}^{k} \frac{\min\{i+r^*, k\}}{k} \binom{k}{i} \varepsilon^i (1-\varepsilon)^{k-i} \right\},$$

where $\varepsilon \triangleq Q\left(\frac{1}{\sigma}\right)$.

Lower Bound on BER Performance

- The bit-error probability of a systematic BMST-R code ensemble under MAP decoding can be lower-bounded by

$$\mathrm{BER}_{\mathrm{MAP}} \ge \sum_{\ell=0}^{m+1} \binom{m+1}{\ell} \theta^{m+1-\ell} (1-\theta)^{\ell} \, Q\left(\frac{\sqrt{N + m(N-2) - 1 + \ell}}{\sigma} \right),$$

where θ is the puncturing fraction.

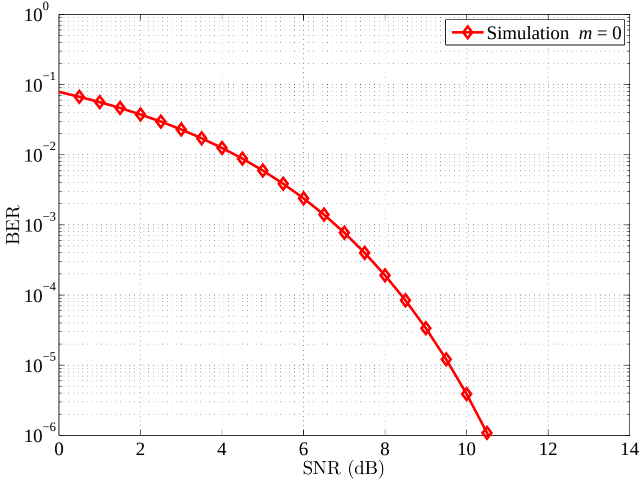
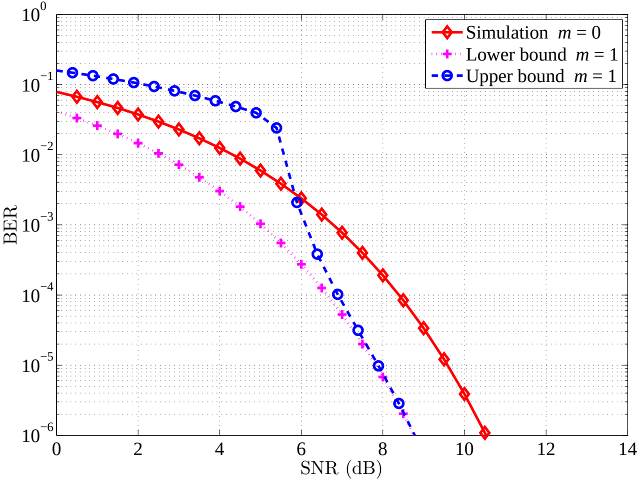


Figure: Performance of systematic BMST-R codes with $m = 0$, $m = 1$ and $m = 2$. BPSK modulation and AWGN channels. $L = 20$, $K = 30$, and $d = 3m$. The truncating parameter is set to $T = 60$.



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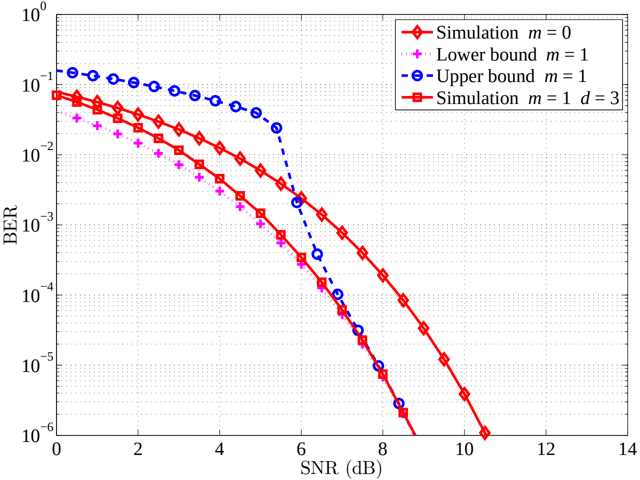
Systematic BMST-R Codes – Example: Performance Bounds



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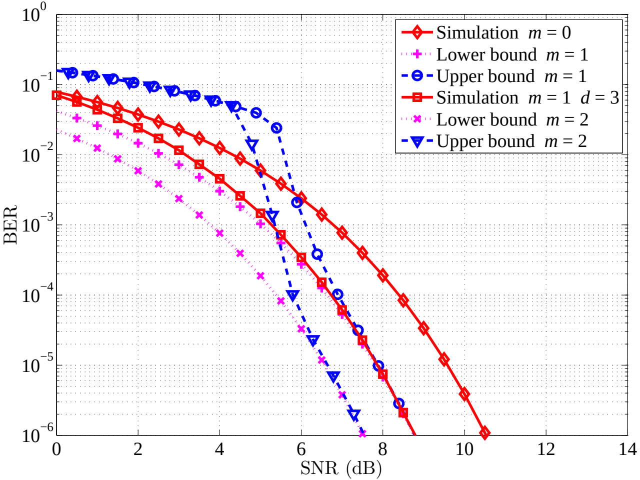

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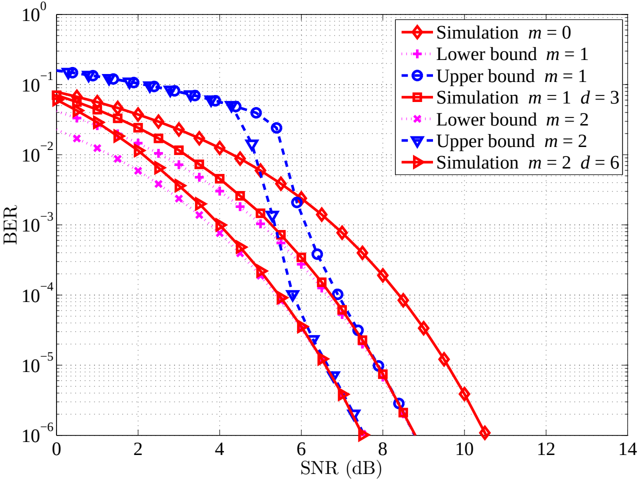

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Object

- Target code rate: $R \in (0, 1)$
- Target BER: p_{target}
- To construct a code with rate $R_L \approx R$, which can approach the Shannon limit at the target BER.
- Five parameters: repetition degree N, information subsequence length K, puncturing length K_p, data block length L, and encoding memory m.

Construction Procedure

1. Determine N and θ such that $\frac{1}{N-\theta} = R$. Choose sufficiently large K and K_p such that $K_p/K \approx \theta$;
2. Find the Shannon limit for the given code rate R and target BER p_{target};
3. Determine the minimum m such that the lower bound of $\mathrm{BER}_{\mathrm{MAP}}$ at the Shannon limit is not greater than the preselected target BER p_{target};
4. Choose a L such that the rate loss (i.e., $R - R_L$) is small;
5. Generate $(m+1)(N-1)$ interleavers randomly.

Systematic BMST-R Codes – Example: Equal Decoding Latency

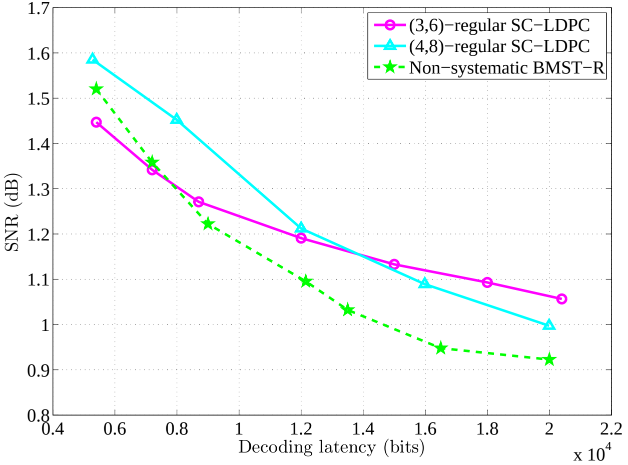
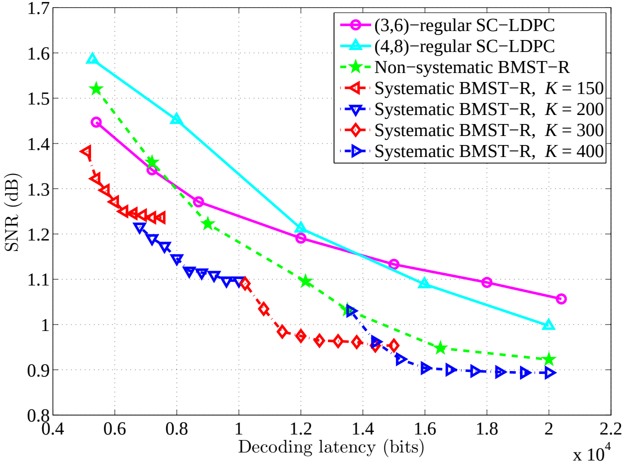


Figure: Required SNR to achieve a BER of 10^{-5} for finite-length systematic BMST-R codes, non-systematic BMST-R codes, $(3, 6)$-regular SC-LDPC codes, and $(4, 8)$-regular SC-LDPC codes as a function of decoding latency.



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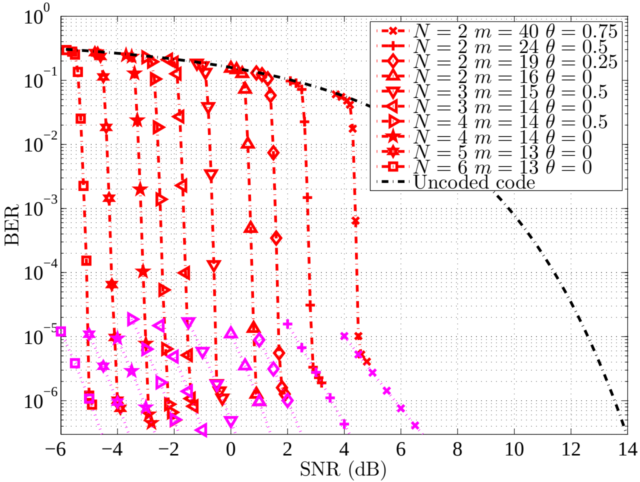


Figure: Simulated decoding performance of systematic BMST-R codes with $K = 500$ and $L = 500$. The rates corresponding to the BER curves from left to right are 0.1631, 0.1959, 0.2449, 0.2801, 0.3272, 0.3929, 0.4921, 0.5623, 0.6562, and 0.7874.

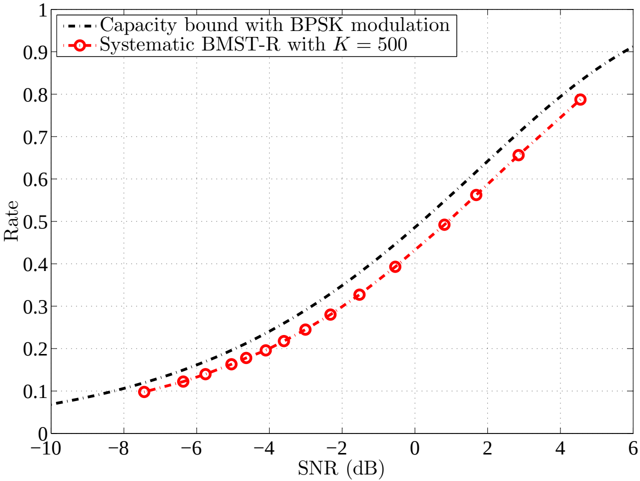
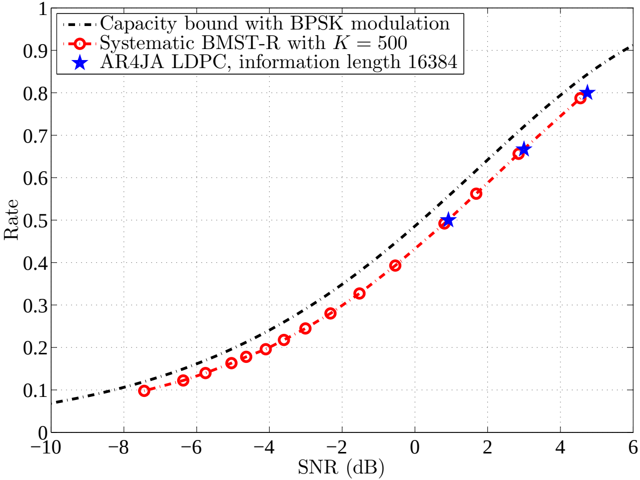


Figure: Required SNR to achieve a BER of 10^{-5} for systematic BMST-R codes. The performances of three AR4JA LDPC codes with code rates $1/2$, $2/3$ and $4/5$ in the CCSDS standard, and five PBRL LDPC codes with code rates $1/4$, $1/3$, $1/2$, $2/3$, and $4/5$, all of which have information length 16384, are also included.

Systematic BMST-R Codes – Example: Bandwidth Efficiency



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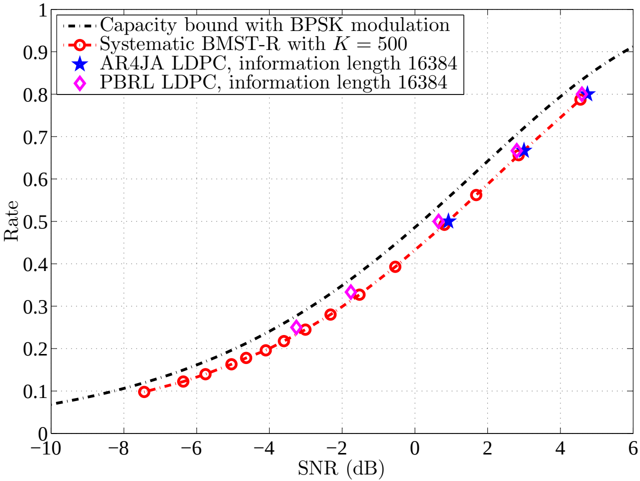

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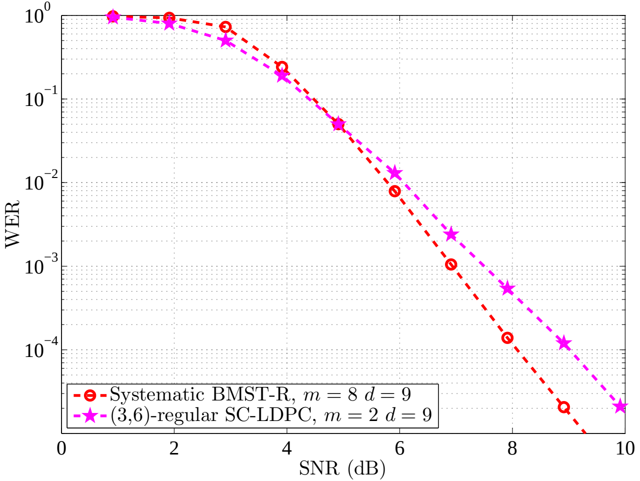

Figure: Performance comparison of the systematic BMST-R code and the SC-LDPC code with BPSK modulation over a block fading channel. The $(3, 6)$-regular SC-LDPC codes is constructed with the protograph lifting factor 100 and three component submatrices $\mathbf{B}_0 = \mathbf{B}_1 = \mathbf{B}_2 = [1\ 1]$. The decoding latencies of two codes are the same.

Three Ensembles of Low Density Generator Matrix (LDGM) Codes – Ensemble 1

Definition (Ensemble 1)

The generator matrix has the form $\mathbf{G} = [\mathbf{I}\ \mathbf{P}]$ of size $k \times n$, where

$$\mathbf{P} = \begin{pmatrix} P_{1,1} & P_{1,2} & \cdots & P_{1,n-k} \\ P_{2,1} & P_{2,2} & \cdots & P_{2,n-k} \\ \vdots & \vdots & \ddots & \vdots \\ P_{k,1} & P_{k,2} & \cdots & P_{k,n-k} \end{pmatrix}$$

and $P_{i,j}$ is generated independently according to the Bernoulli distribution with success probability $\Pr\{P_{i,j} = 1\} = \rho$.

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Theorem (Coding Theorem for Ensemble 1)

For any given $0 < \rho \leq 1/2$, Ensemble 1 is capacity-achieving in terms of BER in the following sense. Given a code rate $R < I(1/2)$. For any $\epsilon > 0$, there exist a sequence of codes $C_2[n, k]$ such that $\lim_{n\to\infty} k/n = R$ and BER is not greater than ϵ.

Three Ensembles of Low Density Generator Matrix (LDGM) Codes – Ensemble 2

Definition (Ensemble 2)

The generator matrix has the form $\mathbf{G} = [\mathbf{I}\ \mathbf{P}]$ of size $kB \times nB$ with $B > 1$, where

$$\mathbf{P} = \begin{pmatrix} \mathbf{P}_{1,1} & \mathbf{P}_{1,2} & \cdots & \mathbf{P}_{1,n-k} \\ \mathbf{P}_{2,1} & \mathbf{P}_{2,2} & \cdots & \mathbf{P}_{2,n-k} \\ \vdots & \vdots & \ddots & \vdots \\ \mathbf{P}_{k,1} & \mathbf{P}_{k,2} & \cdots & \mathbf{P}_{k,n-k} \end{pmatrix}$$

and $\mathbf{P}_{i,j}$ is a random matrix of size $B \times B$ with each column drawn independently and uniformly from $\mathcal{B} = \{v^B \in \mathbb{F}^B | W_H(v^B) \leq 1\}$, the collection of all binary *column* vectors of weight 0 or 1.

Three Ensembles of Low Density Generator Matrix (LDGM) Codes – Ensemble 2

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and $\mathbf{P}_{i,j}$ is a random matrix of size $B \times B$ with each column drawn independently and uniformly from $\mathcal{B} = \{v^B \in \mathbb{F}^B | W_H(v^B) \leq 1\}$, the collection of all binary *column* vectors of weight 0 or 1.

Theorem (Coding Theorem for Ensemble 2)

Ensemble 2 achieves the channel capacity as $k \to \infty$.

Three Ensembles of Low Density Generator Matrix (LDGM) Codes – Ensemble 3

Definition (Ensemble 3)

This ensemble is different from the above two ensembles, which is a convolutional code with memory mk and conveniently defined by an algorithm. The input to the encoder is a sequence of binary vectors $u^k(1), u^k(2), \cdots, u^k(t), \cdots$, where $u^k(t) \in \mathbb{F}^k$ for all $t \geq 1$. At time $t \geq 1$, the output from the encoder is $x^n(t) = (u^k(t), w^{(n-k)}(t))$ with

$$w^{(n-k)}(t) = \sum_{t-m \leq j \leq t} u^k(j)\mathbf{P}_{j,t},$$

where $u^k(t) = 0^k$ for $t < 1$ and $\mathbf{P}_{j,t}$ is a random matrix of size $k \times (n-k)$ with each column drawn independently and randomly at uniform from $\mathcal{K} = \{v^k \in \mathbb{F}_k | W_H(v^k) \leq 1\}$.

Three Ensembles of Low Density Generator Matrix (LDGM) Codes – Ensemble 3

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This ensemble is different from the above two ensembles, which is a convolutional code with memory mk and conveniently defined by an algorithm. The input to the encoder is a sequence of binary vectors $u^k(1), u^k(2), \cdots, u^k(t), \cdots$, where $u^k(t) \in \mathbb{F}^k$ for all $t \geq 1$. At time $t \geq 1$, the output from the encoder is $x^n(t) = (u^k(t), w^{(n-k)}(t))$ with

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Theorem (Coding Theorem for Ensemble 3)

Ensemble 3 achieves capacity as m goes to infinity.

Outline

Conclusions

Conclusions

- BMST codes are spatially coupled codes with simple encoding algorithm and construction method.
- BMST codes have predictable error floors (lower bound).
- BMST codes have near-capacity performance in the waterfall region.
- BMST codes have flexible construction: any basic code with SISO decoding, any rate, any signal constellation, any target BER.
- BMST codes have good performance over different scenarios (BICM, CPM, VLC, SM, OFDM, IM, Rayleigh fading channels, High-mobility channels).

Related Works

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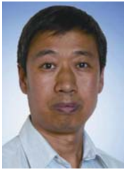
Related Peoples


Xiao Ma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Thank You for Your Attention!