Index Coding: Old and New

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Workshop on Coding and Information Theory
University of Hong Kong
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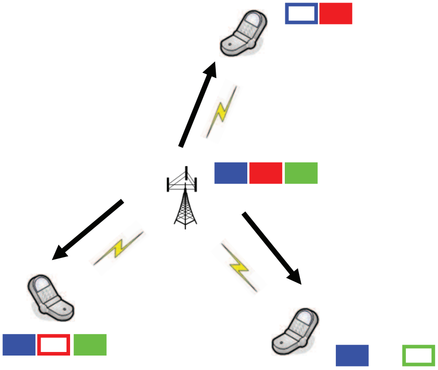
Joint work with Fatemeh Arbabjolfaei (UCSD), Bernd Bandemer (Bosch),
Eren Şaşoğlu (Berkeley), and Lele Wang (UCSD)

Index coding (Birk–Kol 1998)



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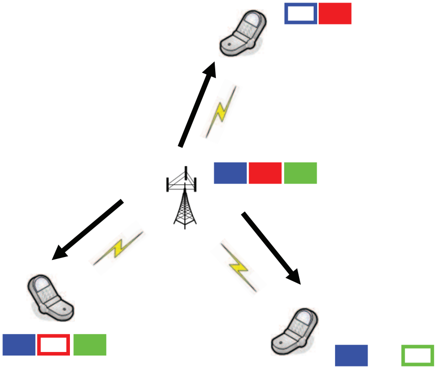

- What is the fundamental limit on the number of transmissions?

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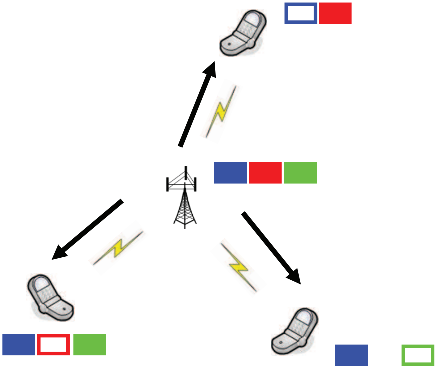
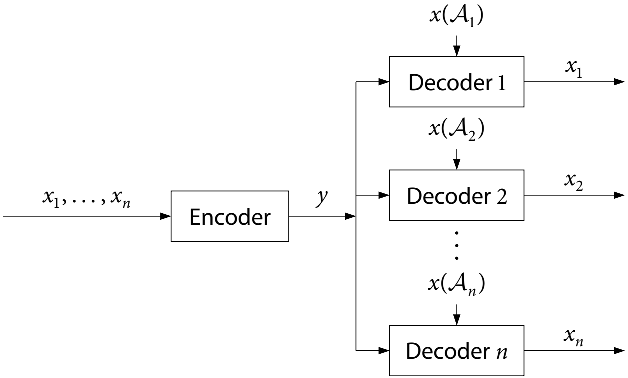


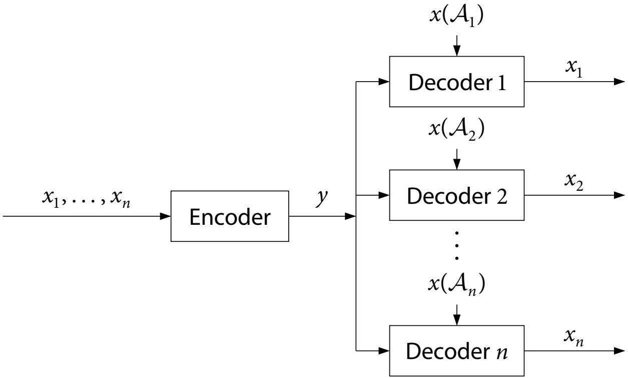
- What is the fundamental limit on the number of transmissions?
- Which coding scheme achieves the limit?

Index coding



- Message $x_j \in \{0,1\}^t$, $j \in [n]$
- Side information $x(\mathcal{A}_j)$, $\mathcal{A}_j \subseteq [n] \setminus \{j\}$ at receiver $j \in [n]$
- Codeword $y \in \{0,1\}^r$

Index coding



- Optimal broadcast rate

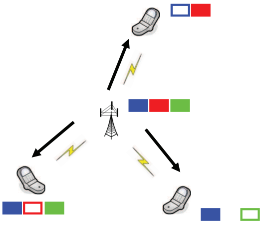
$$\beta^* = \inf_t \inf_{\mathcal{C}} \frac{r}{t} = \lim_{t\to\infty} \inf_{\mathcal{C}} \frac{r}{t}$$

- Zero error probability = small error probability

Representations

- Side information

$$\mathcal{A}_1 = \{2\},\ \mathcal{A}_2 = \{1,3\},\ \mathcal{A}_3 = \{1\}$$



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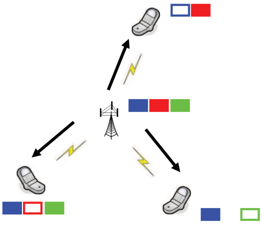
- Compact form

$$(1|2),\ (2|1,3),\ (3|1)$$



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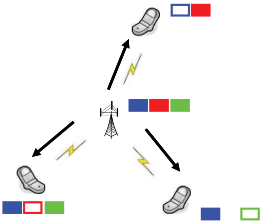

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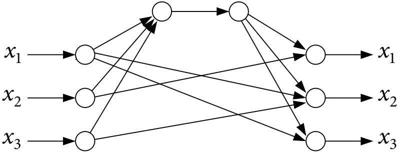
- Multiple unicast network coding



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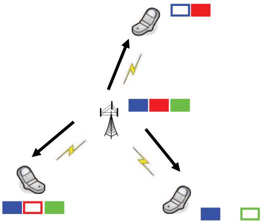

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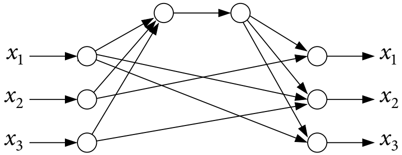
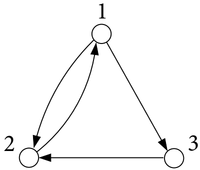


- Side information graph $\mathcal{G}$



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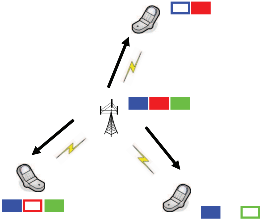

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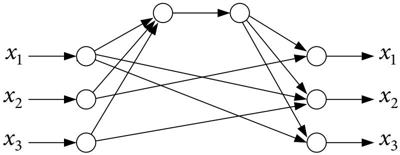

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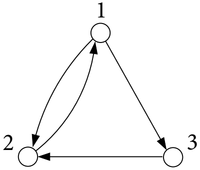

- # of n-message index coding problems = # of n-node directed graphs

$$1,\ 3,\ 16,\ 218,\ 9608,\ 1540944,\ 882033440,\ 1793359192848, \ldots$$

Motivations

- Applications
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  - Multimedia distribution
  - Distributed caching
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- Lotus, bamboo, …

Approaches

Maslow's axiom (1966)

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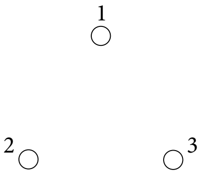
- Communication theory: interference alignment

Example

(1), (2), (3)



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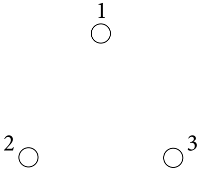
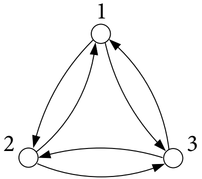


- Send $y = (x_1, x_2, x_3)$
- $\beta^* = 3$

Example

$$(1|2,3),\ (2|1,3),\ (3|1,2)$$



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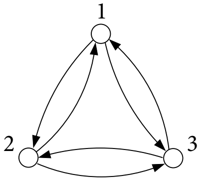

- Send $y = x_1 + x_2 + x_3$
- $\beta^* = 1$

Example

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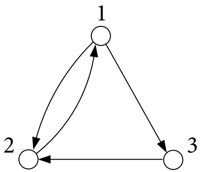

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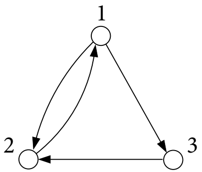

- Send $y = (x_1 + x_2,\ x_3)$
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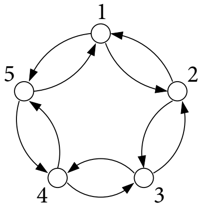

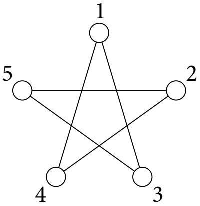

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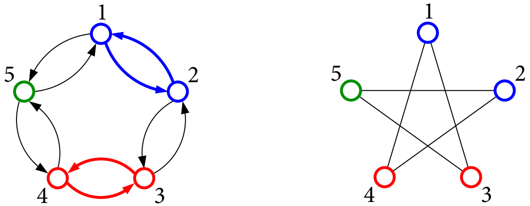

$$\mathrm{cc}(\mathcal{G}) = \chi(\bar{\mathcal{G}}) = 3$$

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- Integer programming characterization

$$\begin{aligned} \text{minimize} \quad & \sum_{\mathcal{S} \in \mathcal{K}} \rho_{\mathcal{S}} \\ \text{subject to} \quad & \sum_{\mathcal{S} \in \mathcal{K} : j \in \mathcal{S}} \rho_{\mathcal{S}} \ge 1, \quad j \in [n], \\ & \rho_{\mathcal{S}} \in \{0, 1\}, \quad \mathcal{S} \in \mathcal{K} \end{aligned}$$

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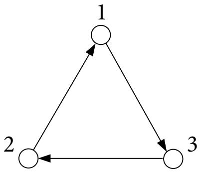

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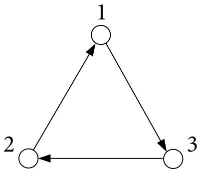

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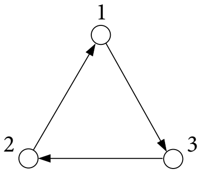

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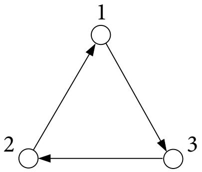

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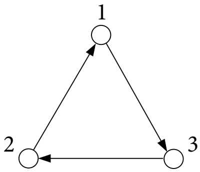

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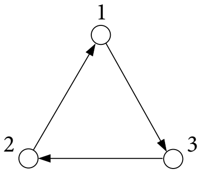

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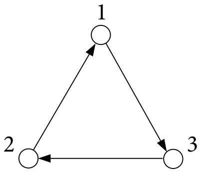

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Partial clique covering

- Partial cliques $\mathcal{G}_1, \ldots, \mathcal{G}_m$ of parameters $k_1, \ldots, k_m$ partitioning $\mathcal{G}$

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- Alternative characterization

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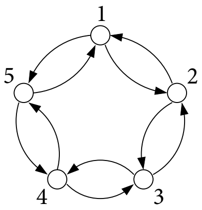
where $\mathcal{G}|_{\mathcal{S}}$ is a $k_{\mathcal{S}}$-partial clique for $\mathcal{S} \subseteq [n]$

Example

$(1|2,5),\ (2|1,3),\ (3|2,4),\ (4|3,5),\ (5|1,4)$



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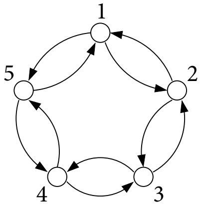

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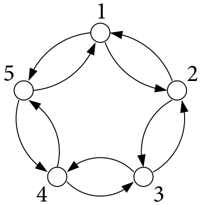

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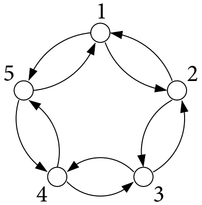

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- Time sharing over subproblems $\{1,2\}, \{2,3\}, \{3,4\}, \{4,5\}, \{5,1\}$
- Fractional partition $f : 2^{[n]} \setminus \{\emptyset\} \to [0,1]$ with $\sum_{\mathcal{S}: j \in \mathcal{S}} f(\mathcal{S}) = 1, j \in [n]$

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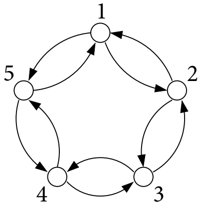

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  - Partition: $f(\mathcal{S}) \in \{0,\ 1\}$

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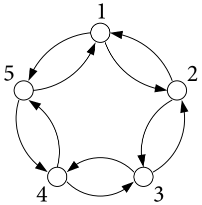

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- $\beta^* = 5/2$
- Time sharing over subproblems $\{1,2\}, \{2,3\}, \{3,4\}, \{4,5\}, \{5,1\}$
- Fractional partition $f : 2^{[n]} \setminus \{\emptyset\} \to [0,1]$ with $\sum_{\mathcal{S}: j \in \mathcal{S}} f(\mathcal{S}) = 1, j \in [n]$
  - Partition: $f(\mathcal{S}) \in \{0,\ 1\}$
  - $f(\mathcal{S}) = 1/2, \mathcal{S} = \{1,2\}, \{2,3\}, \{3,4\}, \{4,5\}, \{5,1\}$

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- Linear programming relaxation of clique covering

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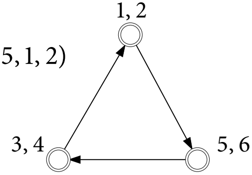
Example

$(1|2,3,4),\ (2|1,3,4),\ (3|4,5,6),\ (4|3,5,6),\ (5|6,1,2),\ (6|5,1,2)$



- $\mathrm{fcc}(\mathcal{G}) = 3$ (send $y = (x_1 + x_2,\ x_3 + x_4,\ x_5 + x_6)$)

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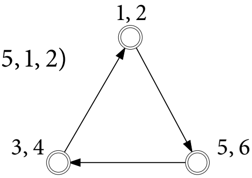

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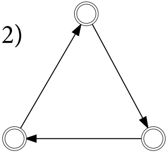

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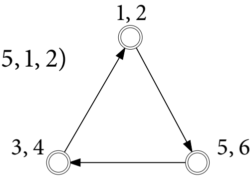

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- Combine with fractional covering (Shanmugam–Dimakis–Langberg 2013)

Fractional local partial clique covering

- Combine everything we have seen so far

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Fractional local partial clique covering bound (Arbabjolfaei–K 2013)

$$\beta^* \le \mathrm{flpcc}(\mathcal{G})$$

where $\mathrm{flpcc}(\mathcal{G})$ is the solution to the linear programming

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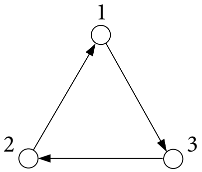
- Can we do better? Unfortunately, yes

Example (revisited)

$$(1|2),\ (2|3),\ (3|1)$$



- Let M be a 3-by-3 matrix such that

$$M_{ii} \neq 0 \quad \text{and} \quad M_{ij} = 0 \text{ if } i \notin \mathcal{A}_j$$

and send $y = (x_1, x_2, x_3)\bar{M}$, where $\bar{M}$ consists of independent columns of M

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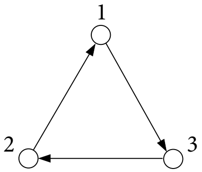

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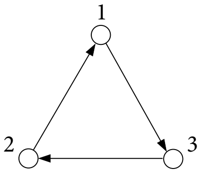

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Alternative approach

Maslow's axiom (1966)

If all you have is a hammer,

everything looks like a nail.

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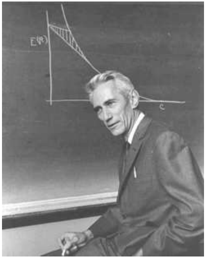

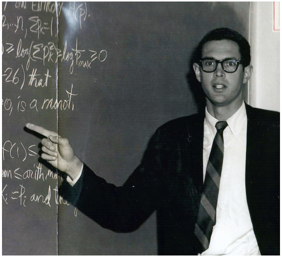

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- Our hammer: Shannon's random coding (Cover's random binning)
- Flat coding (= partial clique covering)
- Dual index coding
- Composite coding

Flat coding

$$(1|2),\ (2|1,3),\ (3|1)$$

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- Codebook generation:
  - For each (x_1, x_2, x_3), generate a Bern(1/2) sequence $y(x_1, x_2, x_3)$
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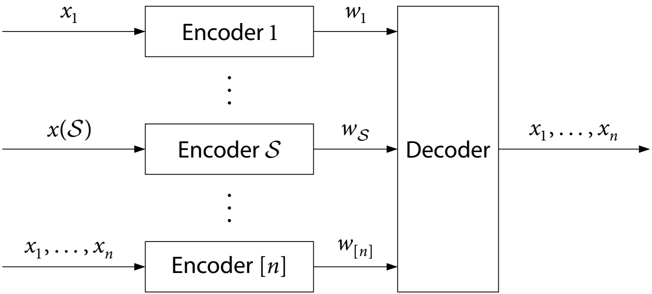
- Can be combined with local time sharing (not optimal in general)

Interlude: Dual index coding



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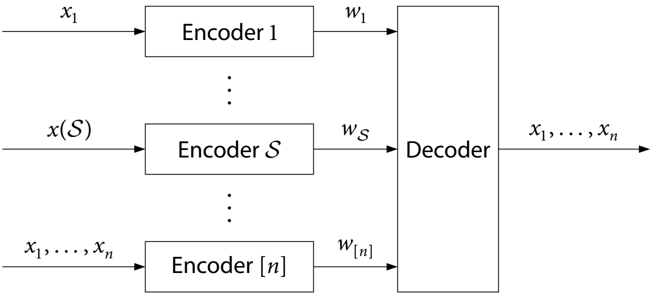

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- A special case of the general multiple access channel with correlated messages (Slepian–Wolf 1973, Han 1979)

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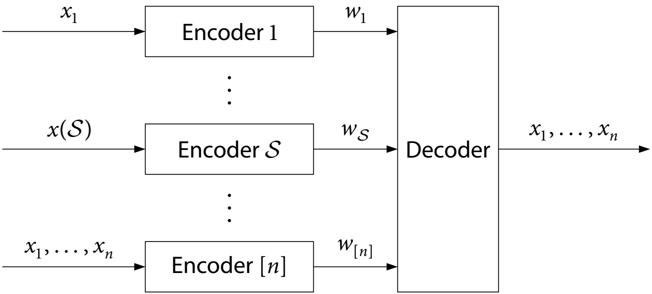

Optimal condition for reliable communication

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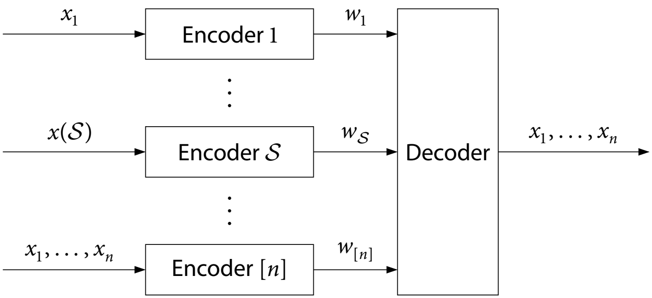

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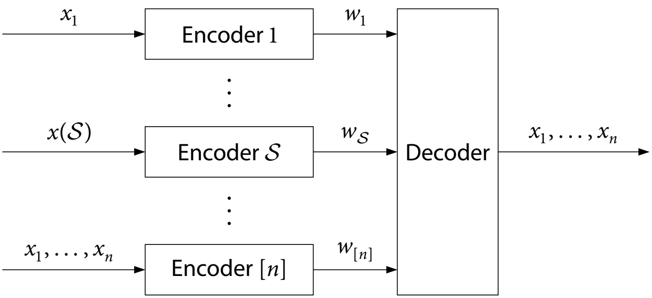

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- Extension $\mathscr{R}(\mathcal{D}|\mathcal{A})$: Demand $\mathcal{D}$ and side information $\mathcal{A}$ at the receiver

Composite coding

$$(1|4),\ (2|3,4),\ (3|1,2),\ (4|2,3)$$

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- Encoding (step 1): Introduce 2 "virtual" senders (cf. dual index coding)
  - Random coding of (x_1, x_4) into $w_{1,4}$ at rate $\gamma_{1,4} > 1$
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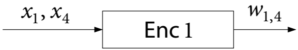

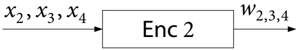

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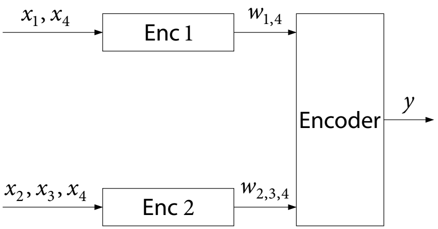

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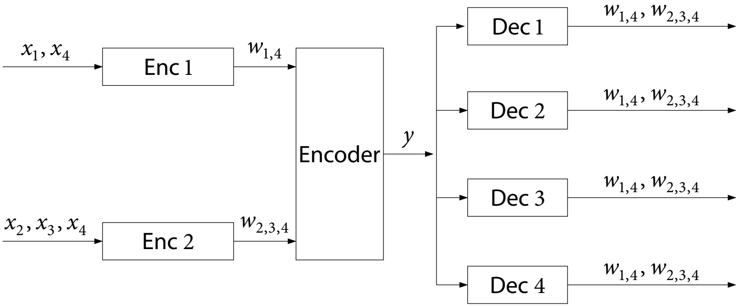

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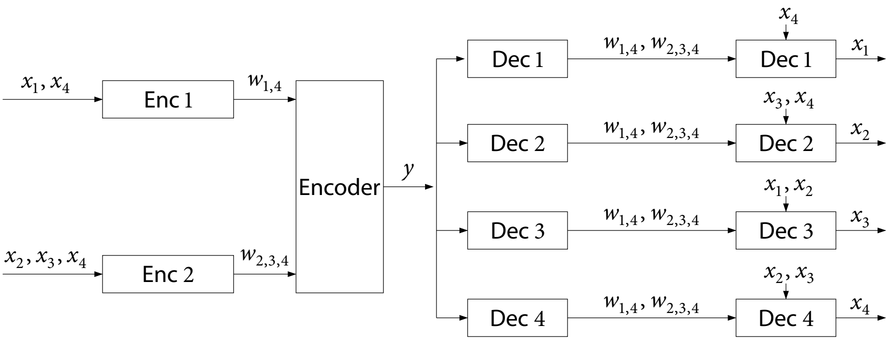

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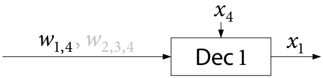

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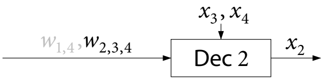

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    $$2 < \gamma_{1,4} + \gamma_{2,3,4},$$
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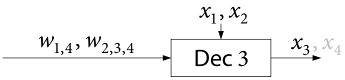

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  - Decoder 4 uses $w_{2,3,4}$ to recover x_4

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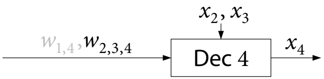

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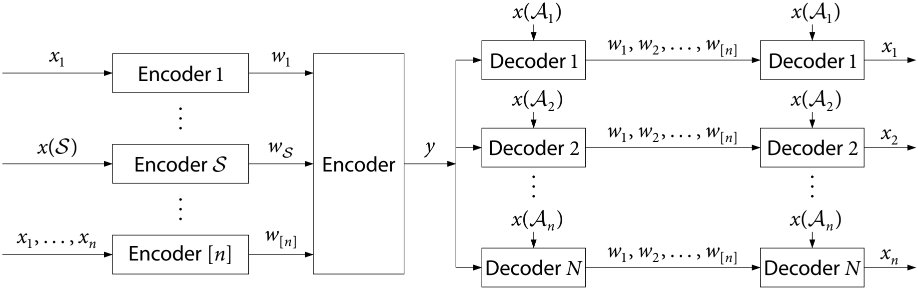

- $(2^n - 1)$ virtual senders to encode n messages

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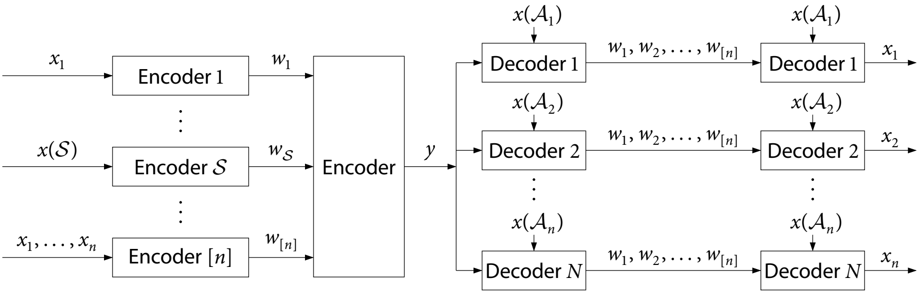

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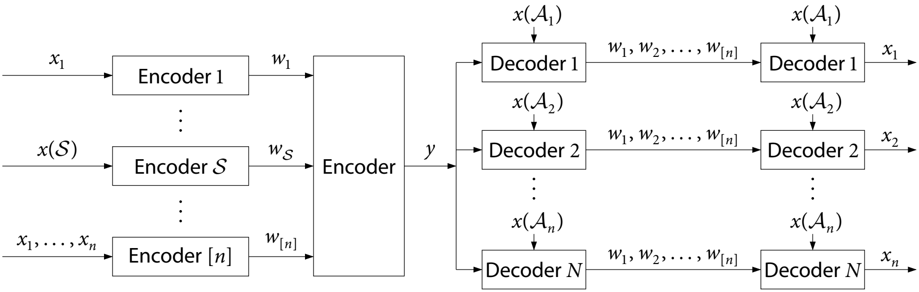

- $(2^n - 1)$ virtual senders to encode n messages
- Flat coding of the composite indices using side information
- Optimal (simultaneous nonunique) decoding of the desired message

Composite coding

Composite coding bound

$$\beta^* \leq \mathrm{comp}(\mathcal{G})$$

where $\mathrm{comp}(\mathcal{G})$ is the solution to the optimization problem

$$\begin{aligned}
\text{minimize} \quad & \max_{j \in [n]} \sum_{\mathcal{S} \subseteq [n] : \mathcal{S} \not\subseteq \mathcal{A}_j} \gamma_{\mathcal{S}} \\
\text{subject to} \quad & \min_{\mathcal{T} \subseteq \mathcal{D}_j \setminus \mathcal{A}_j} \frac{1}{|\mathcal{T}|} \sum_{\mathcal{S} \subseteq \mathcal{D}_j \cup \mathcal{A}_j : \mathcal{S} \cap \mathcal{T} \neq \emptyset} \gamma_{\mathcal{S}} \geq 1, \quad j \in [n], \\
& \gamma_{\mathcal{S}} \geq 0, \quad \mathcal{S} \subseteq [n], \\
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- Similar, but richer structure than clique covering bounds

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\end{aligned}$$

- Similar, but richer structure than clique covering bounds
- Decoding spanned over multiple subproblems (time slots)

More on composite coding

☺ Optimal up to $n = 5$ (9608 problems)

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- ☹ Recursive composite coding: difficult to evaluate

Concluding remarks

- Random coding is a powerful tool
  - Original network coding theorem (Ahlswede–Cai–Li–Yeung 2000)
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- Index coding
  - One of the most fundamental network information theory problems (cf. 2-DMBC)
  - Down the rabbit hole (full of exciting adventures)
  - Lower bounds (Sun–Jafar 2013)
  - Capacity region vs. optimal broadcast rate

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