Hidden Markov Model for the Detection of Machine Failure

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Abstract

Hidden Markov Models (HMMs) are widely used in applied sciences and engineering. The potential applications in manufacturing industries have not yet been fully explored. In this paper, we propose to apply HMM to the detection of machine failure in a process control problem. We propose models for both cases of indistinguishable production units and distinguishable production units. Numerical examples are given to illustrate the effectiveness of the proposed model.

Key Words: Machine Failure, Hidden Markov Model, Statistical Control Process, Transition Probability.

1 Introduction

Statistical process control is a powerful tool that is widely used in monitoring manufacturing processes. On a control chart, points are plotted as production proceeds, and the process is deemed to be out of control if a plotted point lies outside the control limits, or a group of points have a certain pre-defined abnormal pattern. Commonly used control charts, including the $\bar{x}$ -chart, p-chart, EWMA charts [12, 14]

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and the relatively new CCC- and CQC-charts [2, 3, 4], are designed for monitoring a single quality characteristic. In order to achieve high production efficiency, in many manufacturing process the equipment is design for parallel production of multiple items of the same product. An example is a soft-drink filling machine with 100 filling nozzles arranged in a circle, designed for filling 100 bottles simultaneously. In such a set-up, from a strict process control point of view, the operating condition of each nozzle should be monitored. In normal operation, however, bottles filled by the different nozzles are mixed when they come out from the filling machine, and it is not possible to tell which bottle is filled by which nozzle. If the products produced from the machine are inspected, the overall fraction of nonconforming products produced can be monitored using one p-chart or CCC-chart, but such a chart does not show the quality characteristic of individual filling nozzles. The same situation occurs in many other manufacturing processes. The objective of this paper is to introduce the hidden Markov model (HMM) for monitoring such processes.

In general, we consider a system of n independent production units each of which can be regarded as a stand-alone part in the system. In the above bottle-filling example, the system is the filling machine, and a unit is a filling nozzle. An item of product is a bottle filled by the filling machine, and each item produced can be classified as either conforming or nonconforming according to whether the amount of liquid filled is within a specified range or out of the range. Suppose that each production unit, when operating in the in control state, has probability $r_1 \in [0, 1)$ of producing conforming product, and when it is in the out of control state, it has probability r_2 of producing conforming product, where $r_2 \in [0, r_1)$. Assume that at any time during production, a production unit which is in the in control state will change to the out of control state with probability $p \in (0, 1)$, and if it is in the out of control state it will stay in this state until repair is carried out which will bring it back to the in control state. Suppose that the states of the individual units are unknown (unless production is stopped and each production unit is investigated). We assume that each of the n production units produces the same quantity of product within the same period of time. Assume further that the products produced by the n production units are mixed and inspected after production, and the overall number of nonconforming product items are counted. In what follows, we shall apply a HMM to indicate whether the process is out of control.

The rest of the paper is organized as follows. In Section 2, we will give a brief introduction to HMM. In Section 3, we will establish a HMM that can be used for statistical process control purpose, and perform a statistical analysis for the production problem. In Section 4, we will apply the model to a system of indistinguishable production units, while in Section 5 we will give an example of three distinguishable units. The paper is then concluded in Section 6.

2 Hidden Markov Model

Hidden Markov Models (HMMs) are widely used in applied sciences and engineering [6, 9, 7, 11], but its potential applications in manufacturing industries have not yet been fully explored. Readers may refer to [11] for a detailed discussion on HMM.

In a HMM, there are two types of states, the hidden state and the observable state. The underlying process of both types of states is a Markov chain process. The hidden state q_t at time t cannot be observed, and may take any element in the set of

$$S = \{S_1, S_2, \ldots, S_N\}$$

of possible states, where $N \geq 1$ is a given integer. The observable state O_t at time t can be observed, and may be take any element in the collection

$$V = \{v_1, v_2, \ldots, v_M\}$$

of possible states, where $M \geq 1$ is a given integer. Suppose that $A_N = \{a_{ij}\}_{ij}$ is an $N \times N$ transition probability matrix for the hidden state at time $t+1$ given the hidden state at time t, and $B_{NM} = \{b_i(k)\}_{ik}$ is an $N \times M$ probability distribution matrix for the observable state at time t given the hidden state at time t. A HMM consists of the hidden states, the observable states, the transition probability matrix A_N and probability distribution matrix B_{NM} with the initial state probability distribution $\pi = \{\pi_i\}$ defined by

$$\begin{cases} a_{ij} & = \mathrm{Prob}(q_{t+1} = S_j \,|\, q_t = S_i), \quad (i, j = 1, \ldots, N), \\ b_i(k) & = \mathrm{Prob}(O_t = v_k \,|\, q_t = S_i), \quad (i = 1, \ldots, N;\ k = 1, \ldots, M), \\ \pi_i & = \mathrm{Prob}(q_1 = S_i), \quad (i = 1, \ldots, N). \end{cases}$$

Failures	0	1	2	3	4	5
A	1/8	1/8	1/4	1/4	1/8	1/8
B	7/10	1/10	1/10	1/30	1/30	1/30

Table 1

Here a_{ij} is the probability for the hidden state at time $t+1$ given the hidden state at time t, and $b_i(k)$ is the probability for the observable state at time t given the hidden state at time t. We also denote a_{ij} by $a_{q_t q_{t+1}}$ and $b_i(k)$ by $b_{q_t}(O_t)$ whenever appropriate.

2.1 An Illustration of a HMM with Two Machines

Suppose that an old machine A and a new machine B, are used in producing the same product. When a batch of five items of product is produced from either machine, the number of nonconforming items are observed. We assume that the probability distributions of the number of nonconforming product items produced by Machines A and B are given in Table 1. Here we further assume that the process of choosing a machine from Machines A and B is a hidden process, so that each time when a machine is to be chosen it is unknown (unobservable) whether A or B will be chosen. But it is known that A will be chosen with probability 0.3 and B will be chosen with probability 0.7. Suppose that during operation of the chosen machine, the number of nonconforming items produced can be observed and recorded.

The process can be described as a HMM with

$$N = 2, \quad M = 6, \quad S = \{A, B\}, \quad V = \{0, 1, 2, 3, 4, 5\} \quad \text{and} \quad \pi = (0.3, 0.7).$$

The transition probability matrix is the following 2×2 matrix

$$\begin{pmatrix} 0.3 & 0.7 \\ 0.3 & 0.7 \end{pmatrix}.$$

Also, we have $b_A(i) = \frac{1}{8}$ for $i = 0, 1, 4, 5$; $b_A(i) = \frac{1}{4}$ for $i = 2, 3$;
$b_B(i) = \frac{7}{10}$ for $i = 0$; $b_B(i) = \frac{1}{10}$ for $i = 1, 2$ and $b_B(i) = \frac{1}{30}$ for $i = 3, 4, 5$.

3 Statistical Analysis in a HMM

In this section, we present the numerical algorithms for simulation and computation in a HMM. Recall the symbols in Section 2, given the parameters in a HMM, a sequence of observed states $O_1 O_2 \ldots O_T$ ($O_i \in V$) can be simulated. In order to monitor a process using HMM, first of all it is necessary to estimate the relevant parameters in the HMM given an observed sequence $O_1 O_2 \ldots O_T$ from the process, and then to compute the most-likely sequence of the hidden states $q_1 q_2 \ldots q_T$. Computation of the most-likely sequence of the hidden states can be done by using the Viterbi algorithm [13], a dynamic programming approach, see for instance, [5, 8, 11]. Hence the model parameters can be estimated, see for instance a tutorial on the algorithms [11].

3.1 Simulation of HMM

We present the numerical algorithm for the simulation of a given HMM as follows.

(Step S1) Choose an initial hidden state $q_1 = S_i \in S$ with the initial distribution π.

(Step S2) Set $t = 1$.

(Step S3) Generate $O_t = v_k \in V$ according to the probability $b_i(k)$.

(Step S4) Make a transition of hidden state from $q_t = S_i$ to $q_{t+1} = S_j$ according to the transition probability distribution a_{ij}.

(Step S5) Set $t = t + 1$ and return to Step 3 if $t < T$; otherwise terminate the procedure.

4 A System of n Indistinguishable Production Units

In this section, we consider a production system consisting of n independent, indistinguishable production units, each of which can produce items of the same product. A production unit is either in the normal state w_1 or in the subnormal state w_2. We assume the followings.

(i) During the duration in which a production unit is producing an item, the state of the production unit does not change.

(ii) If a production unit is in state w_1, immediately after an item is produced it will deteriorate to state w_2 with probability p or remain in state w_1 with probability $1 - p$.

(iii) If a production unit is in state w_2, it will remain in this state until a perfect maintenance action is carried out which will bring the unit back to state w_1.

(iv) An item produced can be classified as either conforming or nonconforming.

(v) When a production unit is in state w_i, it will produce a conforming item with probability r_i or produce a nonconforming item with probability $1 - r_i$ $(i = 1, 2)$, where $r_1 > r_2$.

We assume that during production, the states of the production units are unobservable. The states of the production units are known only if production is stopped and a full inspection on the system is carried out. Furthermore, we assume that all product items produced are inspected, and inspection is perfect in the sense that it will correctly indicate whether the item inspected is conforming or nonconforming.

In what follows, for any integers k and m, we denote by C_k^m the number of combinations of k objects from m objects if m and k are integers and $0 \leq k \leq m$, and $C_m^k = 0$ otherwise. The system is said to be in state i if i production units are in state w_2 and the other $(n - i)$ units are in state w_1. The true states of the production units during production are unobservable. As production proceeds their states form a HMM with $N = n + 1$ states. The transition probability matrix of this HMM is an $(n + 1) \times (n + 1)$ matrix $A_{n+1} = \{a_{ij}\}$, where

$$a_{ij} = C_{j-i}^{n-i+1} (1 - p)^{n-j+1} p^{j-i} \quad (i, j = 1, \ldots, n + 1).$$

For example, when $n = 2$, the transition matrix A_N is given by

$$\begin{array}{cc} \text{state} & \begin{array}{ccc} \quad 0 \quad & \quad 1 \quad & \quad 2 \end{array} \\ A_3 = \begin{array}{c} 0 \\ 1 \\ 2 \end{array} & \left(\begin{array}{ccc} (1-p)^2 & 2p(1-p) & p^2 \\ 0 & (1-p) & p \\ 0 & 0 & 1 \end{array} \right) . \end{array}$$

Suppose that $0 \le k \le n$. When the production system is in state i, the probability for $(n-i)$ production units in state w_1 to produce $(k-l)$ nonconforming items, and i production units in state w_2 to produce l nonconforming items, one from each production unit, is

$$p(k-l,l) = C_{k-l}^{n-i} r_1^{(n-i)-(k-l)}(1-r_1)^{k-l} \times C_l^i r_2^{i-l}(1-r_2)^l,$$

which can be easily seen from Figure 1.

$$\overbrace{\text{x x x x x x x x x}}^{k-l}\;\overbrace{\text{x x x x x x}}^{l}$$

$$\underbrace{-\;-\;-\;-\;-\;-\;-\;-\;-\;-\;-\;-\;-\;-\;-}_{\substack{n-i \text{ items produced}\\ \text{by production units}\\ \text{in state } w_1}}\;\underbrace{-\;-\;-\;-\;-\;-\;-\;-\;-}_{\substack{i \text{ items produced}\\ \text{by production units}\\ \text{in state } w_2}}$$

Figure 1. Production of a total of k nonconforming items.

In Figure 1, a horizontal bar "–" denotes an item produced, and a cross "x" above the horizontal bar indicates that the item is nonconforming. Figure 1 shows that out of the k nonconforming units, $(k-l)$ are produced by $(n-i)$ production units in state w_1, and l are produced by i production units in state w_2. Let the probability for this to happen be $b_i(k)$. Hence the probability distribution matrix for the observable states of the n items produced from the production system, one from each of the n production units, is an $(n+1)\times(n+1)$ matrix $B_{n+1} = \{b_i(k)\}$.

Figure 1 shows that when $k \le i$ and $k \le n-i$, we have $0 \le l \le k$, and in other cases, l takes different ranges. From such results, we have

$$
b_i(k) = \begin{cases}
\sum_{l=0}^{k} p(k-l,l), & k \le i,\ k \le n-i, \\
\sum_{l=0}^{i} p(k-l,l), & i \le n-i,\ k > i,\ k \le n-i, \\
\sum_{l=k-(n-i)}^{k} p(k-l,l), & i > n-i,\ k \le i,\ k > n-i, \\
\sum_{l=k-(n-i)}^{i} p(k-l,l), & k > i,\ k > n-i.
\end{cases}
$$

When $n = 2$, for example, we have

$$
B_3 = \begin{array}{c} \text{state} \\ 0 \\ 1 \\ 2 \end{array}
\begin{array}{c} \begin{array}{ccc} 0 & \qquad 1 & \qquad 2 \end{array} \\
\begin{pmatrix}
r_1^2 & 2r_1(1-r_1) & (1-r_1)^2 \\
r_1 r_2 & (1-r_1)r_2 + (1-r_2)r_1 & (1-r_1)(1-r_2) \\
r_2^2 & 2r_2(1-r_2) & (1-r_2)^2
\end{pmatrix} \end{array} .
$$

4.1 A Numerical Example

Suppose that $n = 2$, so that the states of the production units are given by

$$
S_1 = \{w_1, w_1\}, \quad S_2 = \{w_1, w_2\}, \quad S_3 = \{w_2, w_2\},
$$

and the transition probability matrix for the states of the production unit is A_3. If the two production units are in state S_i, immediately after each of them has produced an item they will change to state S_j with probability

$$
a_{ij} \quad (i, j = 1, 2, 3).
$$

Any item produced is defined as in state u_1 or state u_2 according to whether it is conforming or nonconforming. Therefore the three possible states of the two items are given by

$$
v_1 = \{u_1, u_1\}, \quad v_2 = \{u_1, u_2\}, \quad v_3 = \{u_2, u_2\},
$$

and the transition probability matrix for the states of two items is given by B_3.

0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 1 0 0 0 1 0 0 0 0 1 1 0 0 0 0 0 1 1 1 1 0
1 0 0 0 1 2 0 0 0 1 1 0 0 0 0 0 0 0 0 1 1 0 1 0 1 1 0 0 1 2 2 0 0 2 1 0 1 2 0 1
1 2 1 2 1 2 1 0 2 2 2 0 1 1 1 0 1 2 1 0 1 1 1 1 0 1 1 2 0 1 1 1 1 1 1 1 0 2 1 2
1 0 1 1 1 2 1 2 2 2 1 1 1 0 1 1 0 2 0 0 1 1 2 1 2 1 1 2 2 0 1 1 2 2 1 2 2 0 2 1
1 0 0 1 2 2 1 1 1 0 0 2 1 2 1 1 1 0 1 1 1 2 1 1 0 1 0 1 1 1 2 1 2 1 2 1 0 0 1 0

Table 2

In the following numerical example, we let $r_1 = 0.95$, $r_2 = 0.5$ and $p = 0.02$ and we use A_3 to generate a time series of hidden states of length 200 for the states of the production units. The hidden sequence $\{q_t, t = 1, 2, \ldots, 200\}$ is given as follows:

$$q_t = \begin{cases} \{w_1, w_1\}, & t = 1, 2, \ldots, 27; \\ \{w_1, w_2\}, & t = 28, 29, \ldots, 69; \\ \{w_2, w_2\}, & t = 70, 71, \ldots 200. \end{cases}$$

¿From the generated hidden sequence, we simulate the number of nonconforming product items and obtained a sequence of length 200 in Table 2, $\{O_t, t = 1, 2, \ldots, 200\}$ by using the procedure in described in Section 3.

For $t = 1$ to 200, based on the above observable sequence and the algorithms mentioned in Section 3, it can be estimated that the most likely hidden sequence $\{\hat{q}_t\}_{t=1}^{200}$ is:

$$\hat{q}_t = \begin{cases} \{w_1, w_1\}, & t = 1, 2, \ldots, 30; \\ \{w_1, w_2\}, & t = 31, 32, \ldots, 71; \\ \{w_2, w_2\}, & t = 72, 73, \ldots 200. \end{cases}$$

In this numerical example, comparison of $\{q_t\}$ and $\{\hat{q}_t\}$ shows that 3 more observations are required to identify the change of hidden state from $\{w_1, w_1\}$ to $\{w_1, w_2\}$ and 2 more observations are required to identify the change of hidden state from $\{w_1, w_2\}$ to $\{w_2, w_2\}$.

The same set of parameters $r_1 = 0.95$, $r_2 = 0.5$ and $p = 0.02$ is used to simulate the above process for 50 times. In a particular simulation, the number of observations that indicates a failure may occur before or after the real failure. ¿From the simulation results, the average number of extra observations required to identify the change of the system from hidden state $\{w_1, w_1\}$ to hidden state $\{w_1, w_2\}$ is calculated to be 0.041667 with standard deviation 4.8508, and the average number of extra observations required to identify the change from hidden state $\{w_1, w_2\}$ to hidden state $\{w_2, w_2\}$ is -0.43478 with standard deviation 9.6209.

5 A System of Three Distinguishable Production Units

In this section, we consider a production system consisting of independent but distinguishable production units. For simplicity of discussion, we only consider a system of three machines, namely M_1, M_2 and M_3. Each of these production units can produce the same product item by item. A production unit is either in the normal state w_1 or in the subnormal state w_2. We assume the followings.

(i) During the production period a production unit is producing an item, the state of the production unit does not change.

(ii) If the production unit M_i is in state w_1, immediately after an item is produced it will deteriorate to state w_2 with probability p_i or remain in state w_1 with probability $p'_i = 1 - p_i$.

(iii) If a production unit is in state w_2, it will remain in this state until a perfect maintenance action is carried out which will bring the unit back to state w_1.

(iv) An item produced can be classified as either conforming or nonconforming.

(v) When the production unit M_i is in state w_j, it will produce a conforming item with probability r_{ij} or produce a nonconforming item with probability

$$r'_{ij} = 1 - r_{ij} \ (j = 1, 2), \quad \text{where} \quad r_{i1} > r_{i2}.$$

The transition probability matrix of this HMM is an 8×8 matrix A:

$$
\begin{array}{c}
(w_1, w_1, w_1) \\ (w_2, w_1, w_1) \\ (w_1, w_2, w_1) \\ (w_1, w_1, w_2) \\ (w_2, w_2, w_1) \\ (w_2, w_1, w_2) \\ (w_1, w_2, w_2) \\ (w_2, w_2, w_2)
\end{array}
\begin{pmatrix}
p_1'p_2'p_3' & p_1p_2'p_3' & p_1'p_2p_3' & p_1'p_2'p_3 & p_1p_2p_3' & p_1p_2'p_3 & p_1'p_2p_3 & p_1p_2p_3 \\
0 & p_2'p_3' & 0 & 0 & p_2p_3' & p_2'p_3 & 0 & p_2p_3 \\
0 & 0 & p_1'p_3' & 0 & p_1p_3' & 0 & p_1'p_3 & p_1p_3 \\
0 & 0 & 0 & p_1'p_2' & 0 & p_1p_2' & p_1'p_2 & p_1p_2 \\
0 & 0 & 0 & 0 & p_3' & 0 & 0 & p_3 \\
0 & 0 & 0 & 0 & 0 & p_2' & 0 & p_2 \\
0 & 0 & 0 & 0 & 0 & 0 & p_1' & p_1 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 1
\end{pmatrix}.
$$

The transition matrix for the observable states of the three items produced from the production system, one from each of the production unit, is an 8×4 matrix B:

$$
\begin{array}{c}
(w_1, w_1, w_1) \\ (w_2, w_1, w_1) \\ (w_1, w_2, w_1) \\ (w_1, w_1, w_2) \\ (w_2, w_2, w_1) \\ (w_2, w_1, w_2) \\ (w_1, w_2, w_2) \\ (w_2, w_2, w_2)
\end{array}
\begin{pmatrix}
r_{11}r_{21}r_{31} & r_{11}'r_{21}r_{31} + r_{11}r_{21}'r_{31} + r_{11}r_{21}r_{31}' & r_{11}'r_{21}'r_{31} + r_{11}'r_{21}r_{31}' + r_{11}r_{21}'r_{31}' & r_{11}'r_{21}'r_{31}' \\
r_{12}r_{21}r_{31} & r_{12}'r_{21}r_{31} + r_{12}r_{21}'r_{31} + r_{12}r_{21}r_{31}' & r_{12}'r_{21}'r_{31} + r_{12}'r_{21}r_{31}' + r_{12}r_{21}'r_{31}' & r_{12}'r_{21}'r_{31}' \\
r_{11}r_{22}r_{31} & r_{11}'r_{22}r_{31} + r_{11}r_{22}'r_{31} + r_{11}r_{22}r_{31}' & r_{11}'r_{22}'r_{31} + r_{11}'r_{22}r_{31}' + r_{11}r_{22}'r_{31}' & r_{11}'r_{22}'r_{31}' \\
r_{11}r_{21}r_{32} & r_{11}'r_{21}r_{32} + r_{11}r_{21}'r_{32} + r_{11}r_{21}r_{32}' & r_{11}'r_{21}'r_{32} + r_{11}'r_{21}r_{32}' + r_{11}r_{21}'r_{32}' & r_{11}'r_{21}'r_{32}' \\
r_{12}r_{22}r_{31} & r_{12}'r_{22}r_{31} + r_{12}r_{22}'r_{31} + r_{12}r_{22}r_{31}' & r_{12}'r_{22}'r_{31} + r_{12}'r_{22}r_{31}' + r_{12}r_{22}'r_{31}' & r_{12}'r_{22}'r_{31}' \\
r_{12}r_{21}r_{32} & r_{12}'r_{21}r_{32} + r_{12}r_{21}'r_{32} + r_{12}r_{21}r_{32}' & r_{12}'r_{21}'r_{32} + r_{12}'r_{21}r_{32}' + r_{12}r_{21}'r_{32}' & r_{12}'r_{21}'r_{32}' \\
r_{11}r_{22}r_{32} & r_{11}'r_{22}r_{32} + r_{11}r_{22}'r_{32} + r_{11}r_{22}r_{32}' & r_{11}'r_{22}'r_{32} + r_{11}'r_{22}r_{32}' + r_{11}r_{22}'r_{32}' & r_{11}'r_{22}'r_{32}' \\
r_{12}r_{22}r_{32} & r_{12}'r_{22}r_{32} + r_{12}r_{22}'r_{32} + r_{12}r_{22}r_{32}' & r_{12}'r_{22}'r_{32} + r_{12}'r_{22}r_{32}' + r_{12}r_{22}'r_{32}' & r_{12}'r_{22}'r_{32}'
\end{pmatrix}.
$$

5.1 A Numerical Example

In this subsection, we demonstrate the efficiency of the method by a numerical example. We let

$$
p = \begin{pmatrix} 0.008 \\ 0.015 \\ 0.030 \end{pmatrix} \quad \text{and} \quad r = \begin{pmatrix} 0.65 & 0.3 \\ 0.80 & 0.4 \\ 0.95 & 0.5 \end{pmatrix}.
$$

We use A to generate a time series of hidden states of length 200 and obtain the observable sequence in Table 3. The hidden sequence $\{q_t, t = 1, 2, \ldots, 200\}$ is:

$$
q_t = \begin{cases} (w_1, w_1, w_1), & t = 1, 2, \ldots, 23; \\ (w_2, w_1, w_1), & t = 24, \ldots, 27; \\ (w_2, w_1, w_2), & t = 28, \ldots, 97; \\ (w_2, w_2, w_2), & t = 98, \ldots, 200. \end{cases}
$$

0	2	1	1	0	0	0	1	1	0	0	1	0	0	1	0	0	1	1	1	1	1	1	0	0	1	0	1	1	1	2	2	2	2	2	2	2	2	2	2
1	2	2	0	1	1	2	2	2	2	2	0	2	0	3	0	1	2	1	1	2	2	1	2	1	1	0	0	1	2	2	2	2	1	1	3	1	1	2	3
0	1	1	0	0	2	1	2	2	0	1	2	2	1	1	1	2	2	2	2	2	1	1	3	2	2	2	3	1	2	1	3	3	1	0	1	1	2	2	2
1	2	2	2	2	2	1	1	2	3	1	2	3	2	3	2	2	3	1	3	2	1	1	3	0	2	2	3	2	3	2	3	2	2	1	0	2	1	3	3
2	2	1	2	2	0	0	2	1	3	2	2	2	2	1	3	3	2	2	1	1	3	1	0	2	3	2	2	2	2	2	2	3	2	2	1	2	2	2	3

Table 3

Using the algorithm described in Section 3, one can find the most likely hidden sequence $\{\hat{q}_t\}_{t=1}^{200}$ as follows:

$$\hat{q}_t = \begin{cases} (w_1, w_1, w_1), & t = 1, 2, \ldots, 17; \\ (w_2, w_1, w_1), & t = 18, \ldots, 30; \\ (w_2, w_1, w_2), & t = 31, \ldots, 96; \\ (w_2, w_2, w_2), & t = 97, \ldots, 200. \end{cases}$$

We use the set of parameters $r_1 = 0.95$, $r_2 = 0.5$ and $p = 0.02$ to simulate the above process for 200 times. In this simulation, the algorithm is able to identify the correct hidden sequence 74 times. In fact, there are 16 possible scenarios for the breakdown of machines in this system. They are (i) no machine breaks down; (ii) one machine breaks down (M_1, M_2, or M_3); (iii) two machines break down ($M_1M_2, M_2M_1, M_1M_3, M_3M_1, M_2M_3, M_3M_2$); and (iv) all machines break down ($M_1M_2M_3, M_1M_3M_2, M_2M_1M_3, M_2M_3M_1, M_3M_1M_2, M_3M_2M_1$). Thus with the HMM, better prediction can be obtained. Among those correct identifications the average extra numbers of observations required to identify the first, second and third machine failures, calculated from the simulation results, are -1.5676, 0.028986 and -0.28571, with standard deviations $7.3094, 12.726$ and 17.618, respectively.

6 Summary

In this paper, we discuss hidden Markov models for the detection of machine failures. We present two models for detecting machine failures of different production systems, one is for a system of indistinguishable production units, and the other is for a system of distinguishable production units. Numerical examples are given to demonstrate the effectiveness of the models. Simulation results show that the HMM is efficient in detecting machine failures in both cases.

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