Proper holomorphic mappings on flag domains of $SU(p,q)$-type on projective spaces

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1 Introduction

The objects of study in the present article are the domains in $\mathbb{P}^n$ defined by

$$\mathbb{D}_n^\ell = \{[z_0, \ldots, z_n] \in \mathbb{P}^n : \sum_{j=0}^{\ell} |z_j|^2 > \sum_{j=\ell+1}^{n} |z_j|^2\}$$

and the proper holomorphic mappings among them. They are examples of the so-called *flag domains* in $\mathbb{P}^n$ when the latter is regarded as a flag variety, i.e. they are open orbits of the real forms $SU(\ell+1, n-\ell)$ of the complex simple Lie group $SL(n+1, \mathbb{C})$ when both of which act on $\mathbb{P}^n$ as biholomorphisms.

The domain $\mathbb{D}_n^0$ is just the complex unit n-ball embedded in $\mathbb{P}^n$ and there has been an extensive literature in the study of their proper holomorphic mappings in the last couple of decades. For a survey, see [1]. In general, when the codimension is high, the set of proper holomorphic mappings between complex unit balls is large and difficult to determine. On the other hand, in the recent works of Baouendi-Huang [2] and Baouendi-Ebenfelt-Huang [3], the domains $\mathbb{D}_n^\ell$, $\ell \geq 1$ and the relevant holomorphic mappings are studied by methods in Cauchy-Riemann geometry. It appears that there is in general much more rigidity when $\ell \geq 1$. Indeed, there is one essential difference between the complex unit n-ball and the domains $\mathbb{D}_n^\ell$, $\ell \geq 1$, for the latter contain linear subspaces of $\mathbb{P}^n$. Motivated by this the author studied in [4] the domains $\mathbb{D}_n^\ell$, $\ell \geq 1$, and their generalizations in Grassmannians by exploiting the structure of the moduli spaces of compact complex analytic subvarieties. Rigidity results analogous to those of [2] are obtained in a more geometric way.

We will follow the terminologies in [2] and [3] to call ℓ the *signature* of the domain $\mathbb{D}_n^\ell$. As far as rigidity of holomorphic mappings is concerned, the determining factor should be the difference of signatures rather than the codimension. This is illustrated by, for instance, in [2] that when the domain and target are of the same signature, then any local proper holomorphic map is the restriction of a linear embedding between the ambient projective spaces. On the other hand, in [3], Baouendi-Ebenfelt-Huang studied the situations with a small signature difference. Together with other results, they proved that there is partial rigidity for local proper holomorphic mappings $h : U \subset \mathbb{D}_n^\ell \to \mathbb{D}_m^{\ell'}$ when $1 \leq \ell < n/2$, $1 \leq \ell' < m/2$ and $\ell' \leq 2\ell - 1$. Furthermore, simple examples can be constructed explicitly to demonstrate that

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their partial rigidity is best possible and in particular we cannot have full rigidity for local proper holomorphic mappings, i.e. there are local proper holomorphic maps which are not the restrictions of linear embeddings between projective spaces.

The main purpose of the current article is to prove the following theorem, which says that we do have full rigidity for global proper holomorphic mappings among $\mathbb{D}_n^\ell$ when the difference in signatures is small.

Theorem 1.1. *Let* $1 \leq \ell < n/2$, $1 \leq \ell' < m/2$ *and* $f : \mathbb{D}_n^\ell \to \mathbb{D}_m^{\ell'}$ *be a proper holomorphic map. If* $\ell' \leq 2\ell - 1$, *then* f *extends to a linear embedding of* $\mathbb{P}^n$ *into* $\mathbb{P}^m$.

Remarks. (1) In [3], it has been proved under the same assumptions that the image of f is contained in a projective linear subspace of dimension $n + (\ell' - \ell)$. Our proof is independent of this result. **(2)** For $\ell = 1$, which also implies $\ell' = 1$, the above result is already obtained in [2] and a more geometric proof is given in [4].

We now sketch the scheme of proof for Theorem 1.1. Our first observation is the following theorem of Feder [5].

Theorem 1.2 (Feder)**.** *Let* $h : \mathbb{P}^\ell \to \mathbb{P}^{\ell'}$ *be an immersion. If* $\ell' \leq 2\ell - 1$, *then* h *is linear.*

In order to apply Feder's theorem, we have to show two things: (i) there exists some ℓ-plane $L \subset \mathbb{D}_n^\ell$ on which the restriction of f is an immersion; (ii) the image $f(L)$ is contained in some ℓ'-plane in $\mathbb{P}^m$. We prove (i) by first showing that f extends to a rational map from $\mathbb{P}^n$ to $\mathbb{P}^m$ and this is achieved by standard Hartog's extension techniques in several complex variables. From that we can deduce the finiteness of f on $\mathbb{D}_n^\ell$ and then by analyzing the kernel of the differential of f we get (i). For (ii), we basically follow the same approach as in [4]. We first prove that ℓ-planes in the boundary $\partial\mathbb{D}_n^\ell$ are mapped to ℓ'-planes in the target space due to the properness of f. Then by studying the space of ℓ-planes on $\mathbb{D}_n^\ell$ we prove that the boundary behaviour can be carried over to the interior and hence (ii).

After establishing (i) and (ii), Feder's theorem now says that the restriction of f on some ℓ-plane L is linear. In particular, we can find a line $\mathbb{P}_0^1$ on which f is linear or equivalently the restriction of f is a degree one rational map. This is a topological condition and therefore the same holds true for any line that can be deformed from $\mathbb{P}_0^1$. Thus, we have shown that f maps lines to lines. The main theorem now follows from a standard extension result in algebraic geometry (c.f. Lemma 3.5).

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2 Linear subspaces of $\mathbb{D}_n^\ell$ and foliations

In [4], the structure of the set of certain compact complex analytic subvarieties in $\mathbb{D}_n^\ell$ is studied and it is crucial to our study here also. To make the article more self-contained, we will briefly recall some relevant facts in this section.

For a point $[\mathbf{z}] = [z_0, \ldots, z_n] \in \mathbb{P}^n$, we will split its homogeneous coordinates as $[\mathbf{z}] = [\mathbf{z}', \mathbf{z}'']_\ell$, where $\mathbf{z}' = (z_0, \ldots, z_\ell)$ and $\mathbf{z}'' = (z_{\ell+1}, \ldots, z_n)$. We denote the closure of $\mathbb{D}_n^\ell$ in $\mathbb{P}^n$

by $\overline{\mathbb{D}}_n^\ell$. We first recall the definition of type-I irreducible bounded symmetric domains and its compact dual the complex Grassmannians.

Definition 2.1. *Let $M(p,q;\mathbb{C})$ be the set of $p\times q$ complex matrices. We identify $M(p,q;\mathbb{C})$ as $\mathbb{C}^{pq}$. The type-I irreducible bounded symmetric domain $\Omega_{p,q}$ is the domain in $\mathbb{C}^{pq}$ defined by $\Omega_{p,q}=\{A\in M(p,q;\mathbb{C}) : I-AA^H>0\}$, where A^H denotes the Hermitian transpose of A. As a Hermitian symmetric space, the compact dual of $\Omega_{p,q}$ is the complex Grassmannian of p-planes in $\mathbb{C}^{p+q}$ which we denote by $G_{p,q}$.*

Proposition 2.2. *$\mathbb{D}_n^\ell$ (resp. $\overline{\mathbb{D}}_n^\ell$) contains a family of ℓ-dimensional projective linear subspaces. They are maximal compact complex analytic subvarieties in $\mathbb{D}_n^\ell$ (resp. $\overline{\mathbb{D}}_n^\ell$). Moreover, the set of all such $\mathbb{P}^\ell$ is parametrized by the points in $\Omega_{\ell+1,n-\ell}$ (resp. $\overline{\Omega}_{\ell+1,n-\ell}$). Furthermore, if $\ell<n/2$, the boundary $\partial\mathbb{D}_n^\ell$ also contains a family of ℓ-dimensional projective linear subspaces and the Shilov boundary of $\Omega_{\ell+1,n-\ell}$ parametrizes precisely those contained in the boundary.*

Proof. The complete proof is given in [4] (Proposition 2.2, Proposition 2.3 and Lemma 2.4 therein). Here we just give the explicit parametrization of the linear subspaces. Let $A\in M(\ell+1,n-\ell;\mathbb{C})$. Consider the ℓ-dimensional linear subspace

$$\{[\mathbf{z}',\mathbf{z}'']_\ell\in\mathbb{P}^n : \mathbf{z}''=\mathbf{z}'A\}\cong\mathbb{P}^\ell\subset\mathbb{P}^n.$$

Then as $\mathbf{z}'\mathbf{z}'^H>\mathbf{z}'AA^H\mathbf{z}'^H$ for all $\mathbf{z}'$ if and only if $I-AA^H>0$, we see that such $\mathbb{P}^\ell$ is contained in $\mathbb{D}_n^\ell$ if and only if $A\in\Omega_{\ell+1,n-\ell}$. The parametrization extends to the respective closures in the natural way. Note that when $\ell<n/2$, the Shilov boundary of $\Omega_{\ell+1,n-\ell}$ is just the set of all matrices A such that $AA^H=I$. □

In the followings, by an ℓ-Grassmann bundle of a manifold M, denoted by $G_\ell TM$, we mean the bundle of Grassmannians of the ℓ-planes in each tangent space on M. We denote the Grassmannian of ℓ-planes in the tangent space at $p\in M$ by $G_\ell T_pM$.

Proposition 2.3. *Let $\pi : G_\ell T\mathbb{D}_n^\ell\to\mathbb{D}_n^\ell$ be the ℓ-Grassmann bundle of $\mathbb{D}_n^\ell$. There is an open set $V_n^\ell\subset G_\ell T\mathbb{D}_n^\ell$, $\pi(V_n^\ell)=\mathbb{D}_n^\ell$ such that V_n^ℓ is a trivial holomorphic $\mathbb{P}^\ell$-bundle over $\Omega_{\ell+1,n-\ell}$.*

Proof. Fix a point $p\in\mathbb{D}_n^\ell$. Since $\mathbb{P}^\ell$ is compact and $\mathbb{D}_n^\ell$ is a domain, we deduce that there is an open set $U_p\subset G_\ell T_p\mathbb{D}_n^\ell$ consisting of precisely all the tangent ℓ-planes which are tangent to some ℓ-dimensional projective linear subspace contained in $\mathbb{D}_n^\ell$. Since the tangent plane at a point uniquely determine the linear subspace, combining with Proposition 2.2, the statements in the proposition are immediate. □

The above $\mathbb{P}^\ell$ foliation of V_n^ℓ is just the universal family of ℓ-planes in $\mathbb{D}_n^\ell$ and we denote it as $\Pi : V_n^\ell\to\Omega_{\ell+1,n-\ell}$. Furthermore, it is simply the restriction of the standard universal family of ℓ-planes in $\mathbb{P}^n$ and we also denote it as $\Pi : G_\ell T\mathbb{P}^n\to G_{\ell+1,n-\ell}$.

Lemma 2.4. *If $\ell<n/2$, then any germ of complex submanifold in $\partial\mathbb{D}_n^\ell$ must lie in an ℓ-dimensional projective linear subspace contained in $\partial\mathbb{D}_n^\ell$.*

Proof. In fact, by [6], the ℓ-dimensional projective linear subspace contained in $\partial\mathbb{D}_n^\ell$ are the *holomorphic arc components* or *boundary components* of $\partial\mathbb{D}_n^\ell$, whose defining properties are precisely the statement in the lemma. For a more elementary proof of the lemma, see [4]. □

3 Proof of Theorem 1.1

Throughout this section, we let $f : \mathbb{D}^\ell_n \to \mathbb{D}^{\ell'}_m$ be a proper holomorphic map, $\ell \geq 1$.

We start with an elementary lemma in algebraic geometry which is used to prove that f is a finite map.

Lemma 3.1. *Let $h : \mathbb{P}^n \to \mathbb{P}^m$ be a rational map. If $S \subset \mathbb{P}^n$ is a compact complex analytic subvariety in the domain of h and h is constant on S, then S is a finite set of points.*

Proof. By composing h with a linear transformation, we may assume that $h(S) = [1, 0, \dots, 0]$. Let $h = [h_0, \dots, h_m]$, where all h_j are polynomials of the same degree. By assumption, for $1 \leq j \leq m$, we have $h_j|_S \equiv 0$. If S is of positive dimension, then the zero set of h_0 must intersect S and hence S intersects the set of indeterminacy of h and this violates our initial assumption. Thus, S is finite set of points. □

Proposition 3.2. *f extends to a rational map from $\mathbb{P}^n$ to $\mathbb{P}^m$. Furthermore, f is a finite map.*

Proof. For each $j \in \{0, \dots, n\}$, let $U_j \subset \mathbb{P}^n$ be the open set defined by $z_j \neq 0$. Note that the complement $\mathbb{P}^n \setminus \mathbb{D}^\ell_n$ is the domain defined by $\sum_{j=0}^{\ell} |z_j|^2 < \sum_{j=\ell+1}^{n} |z_j|^2$. In particular, we have $\mathbb{P}^n \setminus \mathbb{D}^\ell_n \subset \bigcup_{j=\ell+1}^{n} U_j$.

Hence, it suffices to establish the meromorphic extension of the component functions of f (as meromorphic functions) on U_j, for each $j \in \{\ell+1, \dots, n\}$. Now fix $j \in \{\ell+1, , \dots, n\}$, then in terms of the standard inhomogeneous coordinates $(w_1, \dots, w_n)$ on U_j, the domain $\mathbb{D}^\ell_n \cap U_j$ is defined by the equation

$$\sum_{k=1}^{\ell+1} |w_k|^2 > \sum_{k=\ell+2}^{n} |w_k|^2.$$

If we decompose $U_j \cong \mathbb{C}^n = \mathbb{C}^{\ell+1} \times \mathbb{C}^{n-\ell-1}$, then for every relatively compact open set $V \Subset \mathbb{C}^{n-\ell-1}$ containing the origin, the component functions of f extend meromorphically over $\mathbb{C}^{\ell+1} \times V \subset \mathbb{C}^n$ by Hartog's extension [7] since $\ell + 1 \geq 2$. In other words, f extends to a meromorphic map from U_j to $\mathbb{P}^m$. We have thereby established the meromorphic extension of f on each U_j, $j \in \{\ell+1, \dots, n\}$ and hence f extends to a rational map from $\mathbb{P}^n$ to $\mathbb{P}^m$.

Now since $f : \mathbb{D}^\ell_n \to \mathbb{D}^{\ell'}_m$ is proper and holomorphic, for every $p \in \mathbb{D}^{\ell'}_m$, the preimage $f^{-1}(p) \subset \mathbb{D}^\ell_n$ is a compact complex analytic subvariety in $\mathbb{P}^n$ and hence is a finite set by Lemma 3.1. □

Proposition 3.3. *If $\ell < n/2$, $\ell' < m/2$ and $\ell' \leq 2\ell - 1$, there exists an ℓ-dimensional projective subspace of $\mathbb{P}^n$ on which the restriction of f is an immersion.*

Proof. Let $df : T\mathbb{D}^\ell_n \to T\mathbb{D}^{\ell'}_m \subset T\mathbb{P}^m$ be the differential of f. Since f is finite, in particular not totally degenerate, df induces naturally a meromorphic map $[df]$ from the ℓ-Grassmann bundle of $\mathbb{D}^\ell_n$ to the ℓ-Grassmann bundle of $\mathbb{P}^m$. Let $Z \subset G_\ell T\mathbb{D}^\ell_n$ be set of the indeterminacy of $[df]$. We are going to show that all the irreducible component(s) of the complex analytic subvariety Z are of dimension less than $(\ell+1)(n-\ell)$. Assume this dimension estimate for the moment. Now let $\Pi : V_n^\ell \subset G_\ell T\mathbb{D}_n^\ell \to \Omega_{\ell+1,n-\ell}$ be the universal family of ℓ-planes in $\mathbb{D}_n^\ell$ (c.f. Proposition 2.3). Note that Π is proper and hence $\Pi(Z\cap V_n^\ell) \subset \Omega_{\ell+1,n-\ell}$ is a complex analytic subvariety. But $\dim(\Omega_{\ell+1,n-\ell}) = (\ell+1)(n-\ell)$ and therefore by our dimension estimate in the previous paragraph we see that $\Pi(Z)$ is a proper subvariety. Thus, there exists a point $q \in \Omega_{\ell+1,n-\ell}$ such that $[df]$ is well defined on $\Pi^{-1}(q) \cong \mathbb{P}^\ell$. It is equivalent to saying that the restriction of f on the ℓ-plane corresponding to $\Pi^{-1}(q)$ is an immersion and the proof is complete.

We now prove the dimension estimate.

For $r \in \{1, \ldots, n\}$, let $I_r \subset \mathbb{D}_n^\ell$ be the set of points where the kernel of df (as a linear map at each individual point) is of dimension at least r. As f is finite, I_r is a complex analytic subvariety of dimension at most $n-r$ and $I_n \subset \cdots \subset I_1 = \pi(Z)$, where $\pi : G_\ell T\mathbb{D}_n^\ell \to \mathbb{D}_n^\ell$ is the canonical projection. Now let $p \in \mathbb{D}_n^\ell$ at which the kernel of df is of dimension k. The fibre of Z over p, i.e. $Z \cap G_\ell T_p\mathbb{D}_n^\ell$, is the set of ℓ-planes in $T_p\mathbb{D}_n^\ell$ that intersect the kernel of df at p. Therefore it is simply a Schubert variety in $G_\ell T_p\mathbb{D}_n^\ell$ and by standard calculation we obtain readily that its dimension is $(k-1)+(\ell-1)(n-\ell)$ if $k \le n-\ell$ and $\ell(n-\ell)$ if $k > n-\ell$.

Let $Z_r = \pi^{-1}(I_r) \subset Z$, where $1 \le r \le n$. It is clear that each Z_r is also a complex analytic subvariety of $G_\ell T\mathbb{D}_n^\ell$ and $Z_n \subset Z_{n-1} \subset \cdots \subset Z_1 = Z$. We start from Z_n. The dimension of Z_n is $\dim(Z \cap G_\ell T_p\mathbb{D}_n^\ell) + \dim(I_n)$, for any $p \in I_n$. Thus,

$$\dim(Z_n) \le \ell(n-\ell) + 0 < (\ell+1)(n-\ell)$$

by the previous paragraph. Next, $Z_{n-1} \setminus Z_n$ is a locally closed complex analytic subvariety and its dimension, by the previous paragraph again, is at most equal to

$$\ell(n-\ell)+1 \le \ell(n-\ell)+\ell < \ell(n-\ell)+(n-\ell) = (\ell+1)(n-\ell)$$

since $\ell < n/2$. Hence Z_{n-1} is also of dimension less than $(\ell+1)(n-\ell)$. We continue the same argument to conclude that

$$\dim(Z_{n-\ell+1}) \le \ell(n-\ell)+(\ell-1) < \ell(n-\ell)+(n-\ell) = (\ell+1)(n-\ell).$$

Now for $Z_{n-\ell} \setminus Z_{n-\ell+1}$, it is a locally closed complex analytic subvariety of dimension at most

$$[(n-\ell-1)+(\ell-1)(n-\ell)] + [n-(n-\ell)] = (n-1)+(\ell-1)(n-\ell) < (\ell+1)(n-\ell).$$

The last inequality is again due to the assumption $\ell < n/2$. Repeat the same argument and we see that for $1 \le r \le n-\ell$, the dimension of Z_r is also at most $(n-1)+(\ell-1)(n-\ell)$ and hence we have shown that Z is of dimension less than $(\ell+1)(n-\ell)$. □

Proposition 3.4. *If $\ell < n/2$ and $\ell' < m/2$, then f maps ℓ-planes into ℓ'-planes. More precisely, for each ℓ-dimensional projective linear subspace $L \subset \mathbb{D}_n^\ell$ (as described in Proposition 2.2), we have $f(L) \subset L'$, where L' is some ℓ'-dimensional linear subspace in the target $\mathbb{P}^m$.*

Proof. By Proposition 3.2, f extends as a rational map and in particular, f extends holomorphically in an open neighborhood of $\overline{\mathbb{D}}_n^\ell$. Thus, the meromorphic map $[df] : G_\ell T\mathbb{D}_n^\ell \to G_\ell T\mathbb{P}^m$ (in the proof of Proposition 3.3) also extends to an open neighborhood of $G_\ell T\mathbb{D}_n^\ell$ and in particular to an open neighborhood $W \supset \overline{V}_n^\ell$, where $\Pi : W \to U$, $U \supset \overline{\Omega}_{\ell+1,n-\ell}$, is an open neighborhood of the universal family $\Pi : V_n^\ell \to \Omega_{\ell+1,n-\ell}$. Now we consider the composition $f^\sharp := \pi \circ [df]$, where $\pi : G_\ell T\mathbb{P}^m \to \mathbb{P}^m$ is the canonical projection. Take a general point b in the Shilov boundary of $\Omega_{\ell+1,n-\ell}$ so that $[df]$ and hence $f^\sharp$ is defined on the ℓ-plane over the point b (i.e. $\Pi^{-1}(b)$). By the properness of f and Lemma 2.4, we have $f^\sharp(\Pi^{-1}(b)) \subset \partial\mathbb{D}_m^\ell$ and hence $f^\sharp(\Pi^{-1}(b)) \subset L'_b$ for some ℓ'-plane $L'_b \subset \mathbb{P}^m$. In other words, in the holomorphic $\mathbb{P}^\ell$-bundle $W \to U \supset \Omega_{\ell+1,n-\ell}$, the map $f^\sharp$ maps the general fibres over the Shilov boundary of $\Omega_{\ell+1,n-\ell}$ to ℓ'-planes in $\mathbb{P}^m$. Note that the latter is an analytic condition, i.e. it can be expressed in terms of the vanishment of a set of holomorphic functions in local coordinates (e.g. some degeneracy conditions of a set of vertical derivatives). Now we have a set of holomorphic functions vanishing on the intersection of an open set and the Shilov boundary of $\Omega_{\ell+1,n-\ell}$ and therefore they must vanish on the whole open set. (For a proof of this, see [4], Lemma 2.9 therein.) Hence, we conclude that the same property holds for the general fibres in the interior, i.e. $f^\sharp$ maps any general ℓ-plane into an ℓ'-plane. This precisely means that f map a general and hence every ℓ-plane into an ℓ'-plane. $\square$

We are almost ready to prove Theorem 1.1. We will need to use the following standard extension result in algebraic geometry. For a proof, see [8].

Lemma 3.5. *Let $U \subset \mathbb{P}^n$ be a connected open set. Let $h : U \to \mathbb{P}^m$ be a holomorphic embedding such that for every line L in $\mathbb{P}^n$ with $L \cap U \neq \emptyset$, we have $f(U \cap L)$ being an open subset of a line in $\mathbb{P}^m$. Then h extends to a linear embedding of $\mathbb{P}^n$ into $\mathbb{P}^m$.*

Proof of Theorem 1.1. By Proposition 3.3, there exist an ℓ-plane $L_0 \subset \mathbb{D}_n^\ell$ on which the restriction of f is an immersion. However, by Proposition 3.4, $f(L_0)$ is contained in some ℓ'-plane $L'_0 \subset \mathbb{P}^m$ and since $\ell' \leq 2\ell - 1$, we have by Feder's Theorem 1.2 that the restriction of f on L_0 is linear. Fix a line $\mathbb{P}_0^1 \subset L_0$, the restriction of f on $\mathbb{P}_0^1$ is again linear and it is equivalent to $\deg(f|_{\mathbb{P}_0^1}) = 1$ for a suitable choice of the generators in $H_2(\mathbb{P}_0^1, \mathbb{Z})$ and $H_2(\mathbb{P}^m, \mathbb{Z})$. Since the degree is a topological invariant and any other line $\mathbb{P}^1 \subset \mathbb{D}_n^\ell$ can be continuously deformed to $\mathbb{P}_0^1$, we deduce that the restriction of f on any line in $\mathbb{D}_n^\ell$ is also linear. Finally, as linearity is an analytic condition we readily see that the hypothesis in Lemma 3.5 is satisfied and hence f extends to a linear embedding of $\mathbb{P}^n$ to $\mathbb{P}^m$. $\square$

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