THE UNIVERSITY OF HONG KONG

Institute of Mathematical Research
Department of Mathematics

SPECIAL SESSION IN ALGEBRAIC GEOMETRY

December 13, 2002 (Friday)

Room 517, Meng Wah Complex, HKU

3:00 – 4:00pm **Xiaotao Sun**
HKU
Degeneration of moduli spaces of SL(n)-bundles on curves

Tea Break

4:20 – 5:20pm **Fu Lei**
Nankai University and IMS, CUHK
l-adic Fourier Transformations and the Thom-Sebastiani Theorem

All are welcome

Special Session in Algebraic Geometry

December 13, 2002 (Friday)

Room 517, Meng Wah Complex, HKU

Xiaotao Sun
HKU

Degeneration of moduli spaces of SL(n)-bundles on curves

Abstract

When a smooth curve C degenerates to a stable curve C_0, the moduli space $\mathcal{U}_C$ of semistable vector bundles on C will degenerate to the moduli space $\mathcal{U}_{C_0}$ of semistable torsion free sheaves on C_0. The moduli space $\mathcal{S}U_C(L)$ of semistable bundles with fixed determinant L is a subvariety of $\mathcal{U}_C$. A natural question is to understand the degeneration of $\mathcal{S}U_C(L)$ in $\mathcal{U}_{C_0}$. Namely, one expects a definition of $SL(n)$-torsion free sheaves on C_0 such that the moduli functor of semistable $SL(n)$-torsion free sheaves is universally corepresented by a closed subscheme $\mathcal{S}U_{C_0}(L_0)$ of $\mathcal{U}_{C_0}$. The scheme $\mathcal{S}U_{C_0}(L_0)$ should satisfy at least the following conditions: (1) $\mathcal{S}U_{C_0}(L_0)$ should be the "limit" of $\mathcal{S}U_C(L)$ when b tends to 0 and L degenerates to L_0, (2) when L_0 is a line bundle, $\mathcal{S}U_{C_0}(L_0)$ should be reduced and contain a dense open set of locally free sheaves. When C_0 is irreducible with only one node, D.S. Nagaraj and C.S. Seshadri defined $\mathcal{S}U_{C_0}(L_0)$ and conjectured (1) and (2). In this talk, we will give a proof of (2) and thus (1) when L degenerates to a line bundle.

Fu Lei
Nankai University and IMS, CUHK

l-adic Fourier Transformations and theThom-Sebastiani Theorem

Abstract

Let $f : \mathbf{C}^m \to \mathbf{C}$ and $g : \mathbf{C}^n \to \mathbf{C}$ be two holomorphic germs with isolated singularities at the origin and let $h = f \oplus g : \mathbf{C}^{m+n} \to \mathbf{C}$ be the germ defined by $h(z, w) = f(z) + g(w)$. One can define the so called vanishing cycle cohomology groups H and monodromy operators T associated to these germs. A classical theorem of Thom and Sebastiani says that

$$(H,T)_h \cong (H,T)_f \otimes (H,T)_g .$$

In this talk, I will formulate a Thom-Sebastiani theorem in the characteristic p case and prove it using l-adic Fourier transformations and the stationary phase principle. It turns that result is not tensor product, but a convolution:

$$(H,T)_h \cong (H,T)_f * (H,T)_g .$$