Workshop on Complex and Algebraic Geometry

July 12 – 14, 2005

Room 517, Meng Wah Complex

Program and Abstracts

Institute of Mathematical Research

The University of Hong Kong

Speakers:

➢ Avery Ching	HKUST, Hong Kong
➢ Lawrence Ein	U. Illinois, USA
➢ Gordon Heier	Harvard U., USA
➢ Jun-Muk Hwang	KIAS, Korea
➢ Naichung Leung	CUHK, Hong Kong
➢ Xiaonan Ma	Ecole Polytechnique, France
➢ Ngaiming Mok	HKU, Hong Kong
➢ Tuen Wai Ng	HKU, Hong Kong
➢ Yum-Tong Siu	Harvard U., USA
➢ Xiaotao Sun	Academia Sinica, China
➢ Sheng-Li Tan	East China Normal U., China
➢ Wing Keung To	National U. Singapore, Singapore
➢ Tom Wan	CUHK, Hong Kong
➢ Yihu Yang	Tongji U., China
➢ Sai-Kee Yeung	Purdue U., USA

Sponsor: Hong Kong Mathematical Society

Email: imr@maths.hku.hk

THE UNIVERSITY OF HONG KONG

Institute of Mathematical Research

Workshop on Complex and Algebraic Geometry

July 12 – 14, 2005

Room 517, Meng Wah Complex

Time / Date	July 12 (Tue)	July 13 (Wed)	July 14 (Thur)
10:00 – 10:50	Siu	Ein	Hwang
10:50 – 11:10	*Tea Break*		
11:10 – 12:00	Heier	Tan	Sun
12:00 – 14:00	*Lunch Break*		
14:00 – 14:50	To	Leung	Wan
15:00 – 15:50	Ma	Ching	Ng
15:50 – 16:10	*Tea Break*		
16:10 – 17:00	Mok	Yang	Yeung

June 27, 2005

Program

July 12, 2005
Tuesday

10:00 – 10:50 **Yum-Tong Siu**, Harvard U., USA
Algebraic geometric techniques for estimates of partial differential equations

Tea Break

11:10 – 12:00 **Gordon Heier**, Harvard U., USA
Algebraic methods in the theory of finite type domains

Lunch Break

14:00 – 14:50 **Wing-Keung To**, National U. Singapore, Singapore
Effective isometric embeddings and Lojasiewicz inequality

15:00 – 15:50 **Xiaonan Ma**, Ecole Polytechnique, France
Heat kernel, Bergman kernel and constant scalar curvature

Tea Break

16:10 – 17:00 **Ngaiming Mok**, HKU, Hong Kong
On isometric embeddings of the Poincaré disk

July 13, 2005
Wednesday

10:00 – 10:50 **Lawrence Ein**, U. Illinois, USA
Contact loci in arc spaces

Tea Break

11:10 – 12:00 **Sheng-Li Tan**, East China Normal U., China
Trigonal algebraic surfaces and applications

Lunch Break

14:00 – 14:50 **Naichung Leung**, CUHK, Hong Kong
Counting curves on K3 surfaces

15:00 – 15:50 **Avery Ching**, HKUST, Hong Kong
A proposed filtration of Chow groups

Tea Break

16:10 – 17:00 **Yihu Yang**, Tongji U., China
Cohomologies of harmonic bundles over quasi-compact Kähler manifolds

July 14, 2005
Thursday

10:00 – 10:50 **Jun-Muk Hwang**, KIAS, Korea
Complex singularity exponents of divisors on Grassmannians

Tea Break

11:10 – 12:00 **Xiaotao Sun**, Academia Sinica, China
Remarks on Gieseker's degeneration

Lunch Break

14:00 – 14:50 **Tom Wan**, CUHK, Hong Kong
Mean curvature flows in hyperkähler manifolds

15:00 – 15:50 **Tuen Wai Ng**, HKU, Hong Kong
On Briot-Bouquet differential equations

Tea Break

16:10 – 17:00 **Sai-Kee Yeung**, Purdue U., USA
Geometric results in Teichmüller and moduli space of curves

Abstracts

Avery Ching, HKUST, Hong Kong

A proposed filtration of Chow groups

The Chow groups of varieties are studied by the cycle class map and the Abel Jacobi map. But certain parts of Chow groups are much more complicated and they cannot be studied by these two maps. They are supposed to be given by the existence of a conjectural filtration. In this talk, we will describe a proposed filtration given by P. Griffiths and discuss the shortcoming for such a construction. We will also see some problems solved by this method and some open problems which are potentially done in this way.

Lawrence Ein, U. Illinois, USA

Contact loci in arc spaces

Let X be a complex variety. We give a one to one correspondence between divisorial valuations with center in X and the contact loci in the space of arcs of X. We describe applications to inversion of adjunction and geometry of rational singularities.

Gordon Heier, Harvard U., USA

Algebraic methods in the theory of finite type domains

We will present a way to effectively compute the type of an ideal of holomorphic functions vanishing only at the origin in $\mathbb{C}^n$ (math.CV/0506557, joint with R. Lazarsfeld). As time permits, we will also give an introduction to J. J. Kohn's algorithm for subelliptic multiplier ideal sheaves and point out its significance as an effective Frobenius theorem for Artinian subschemes.

Jun-Muk Hwang, KIAS, Korea

Complex singularity exponents of divisors on Grassmannians

Given a complex manifold X and an effective divisor $D \subset X$, the complex singularity exponent of D at a point $x \in X$ is the real number

$$\sup\{c > 0 : \ |f(z)|^{-c} \text{ is locally } L^2 \text{ near } x\}$$

where $f(z)$ is a local defining function of D near x. The complex singularity exponent is an important local invariant of the divisor D, but is usually not easy to compute or estimate. We will discuss an optimal lower bound for the complex singularity exponents of divisors on Grassmannians.

Naichung Leung, CUHK, Hong Kong

Counting curves on K3 surfaces

We will discuss several methods in computing the number of curves in K3 surfaces. The generating function is given by a modular form as predicted by the Yau-Zaslow formula.

Xiaonan Ma, Ecole Polytechnique, France

Heat kernel, Bergman kernel and constant scalar curvature

By using the asymptotic expansion of the Bergman kernel, Donaldson found that the existence of Kähler metrics with constant scalar curvature is shown to be closely related to Chow-Mumford stability.

In this talk, we establish the asymptotic expansion of the Bergman kernel in the general context of symplectic manifolds and orbifolds. We use some techniques from the local index theory (or the heat kernel). One motivation of our work is to extend Donaldson's work to orbifolds and to understand the relationship between heat kernel, index formula and stability.

Ngaiming Mok, HKU, Hong Kong

On isometric embeddings of the Poincaré disk

Let $f : (D, 0) \to (D', 0)$ be a germ of holomorphic isometry up to a normalizing constant between two bounded symmetric domains equipped with the Bergman metric. We pose the question of characterizing such maps and of finding conditions which force such maps to be totally geodesic. The special case of the problem where D is the unit disk, D' is a polydisk, and f satisfies some supplementary conditions, was studied by Clozel-Ullmo in connection to an arithmetic problem. There first of all they proved that f extends algebraically by making use of real algebraic functional identities arising from Kähler potentials.

By the standard procedure of polarizing real-analytic identities one obtains an infinite number of holomorphic identities. They define subvarieties of $D \times D'$ reminiscent of Segre varieties. We show that, in the event that there are nontrivial deformations of solutions of the holomorphic identities, the germ of holomorphic isometry must take values on intersections of hyperplane sections of the embedding of the domain into the infinite-dimensional projective space $\mathbb{P}^\infty$ defined by an orthonormal basis of $H^2(D')$. Such hyperplane sections correspond to zero sets of extremal holomorphic functions, which can be read off from the Bergman kernel $K(z, \overline{w})$. As a consequece, we show that $f : (D, 0) \to (D', 0)$ extends algebraically.

The case of isometric embeddings of the Poincaré disk Δ is of special interest, since there is an ample supply of geodesic disks on a bounded symmetric domain. We show that any $f : (\Delta, 0) \to (D', 0)$, which extends algebraically, must be asymptotically totally geodesic at a good boundary point. However, there are examples of holomorphic isometries of the Poincaré disk into the polydisk where singularities develop in the algebraic extension. In the case of holomorphic isometries into the polydisk, it can be shown that f is totally geodesic if and only if there are no singularities of the algebraic extension on the boundary circle.

Tuen Wai Ng, HKU, Hong Kong

On Briot-Bouquet differential equations

Many nonlinear partial differential equations encountered in physics are autonomous, i.e. do not depend explicitly on the independent variables x (space) and t (time). In such a case, they admit a reduction, called traveling wave reduction, to an autonomous nonlinear ordinary differential equation (ODE), defined in the simplest case by $Y(x,t) = y(\xi)$, $\xi = x - ct$, with c a constant speed.
It turns out that many ODEs obtained from the traveling wave reduction is of the form

$$P(y^{(k)}, y) = 0 \ ,$$

where $P(u, v)$ is an irreducible polynomial of two variables. This is the so-called Briot-Bouquet differential equations. It was proved by Briot and Bouquet in 1856 (when $k = 1$) and Picard in 1880 (when $k = 2$) that any solution y which is meromorphic in the complex plane must belong to the class W, which consists by definition of the following functions: (i) rational functions; (ii) rational functions of $\exp(az)$, $a \in \mathbb{C}$; (iii) elliptic functions. In 1982, Eremenko proved that for every k, if the genus of the algebraic curve defined by $P(u, v) = 0$ is one, then all meromorphic solutions must be elliptic functions. He also proved that when k is odd and a solution y is meromorphic in the plane and has at least one pole, then y must belong to the class W. He then conjectured that any meromorphic solution of a Briot-Bouquet differential equation must be one of the above three types. In this talk, we shall report some recent progress on this conjecture.

Yum-Tong Siu, Harvard U., USA

Algebraic geometric techniques for estimates of partial differential equations

Will discuss the motivation, techniques, results, and problems in the application of the algebraic geometric methods of multiplier ideal sheaves to estimates of partial differential equations.

Xiaotao Sun, Academia Sinica, China

Remarks on Gieseker's degeneration

In 1984, in proving of a conjecture of Newstead-Ramanan, Gieseker introduced a degeneration of moduli spaces of stable rank two vector bundles on curves when the smooth curves degenerate into an irreducible curve X with one node. It is a GIT quotient with only normal crossing singularities. In 1999, Nagaraj and Seshadri constructed a generalized Gieseker's degeneration of moduli spaces of arbitrary rank stable bundles, which is also a GIT quotient with only normal crossing singularities. Recently, Young-Hoon Kiem and Jun Li constructed the normal crossing degeneration as an algebraic space whose normalization is a smooth algebraic space. Then they constructed a sequence of moduli spaces (as algebraic spaces) M^α $(0 < \alpha < 1)$ so that when α closed to 1, M^α is the normalization of the degeneration, and when α closed to 0, M^α is a fiber bundle over the moduli space $\mathcal{U}_{\widetilde{X}}(r, d)$ of stable bundles on $\widetilde{X}$, where $\widetilde{X}$ is the normalization of X. In this talk, we remark firstly that Kiem-Li's degeneration coincides with Nagaraj-Seshadri's degeneration. Thus it is also a GIT quotient. Secondly, the M^α are GIT-quotients for a linearization depending on α.

Sheng-Li Tan, East China Normal U., China

Trigonal algebraic surfaces and applications

Gonality of a complex projective curve C is defined as the minimal degree d of the finite covers of C over $\mathbb{P}^1$. This number d divides algebraic curves into some classes, $\mathbb{P}^1$, *hyperelliptic* $(d = 2)$, *trigonal* $(d = 3)$, and *d-gonal*. For example, $d = 1$ if $g(C) = 0$; $d = 2$ if $g = 1$ or 2; $d = 2$ or 3 if $g = 3$ or 4.
Similarly, the gonality d of an algebraic surface X can be defined as the minimal degree of generically finite covers of X over some *ruled surfaces* (not just $\mathbb{P}^2$). So algebraic surfaces are divided into classes: *ruled, hyperelliptic, trigonal*, and *d-gonal*. Because double cover is well understood now, we have a good method to classify hyperelliptic surfaces, which include surfaces with a genus 2 fibration, surfaces whose canonical maps are of degree 2, and surfaces whose invariants satisfy $c_1^2 < 3p_g - 7$.
We will talk about our triple cover method and its applications in the study of trigonal surfaces. Precisely, we will give upper bounds on the slope of trigonal fibrations, and describe by using rank two vector bundles the surfaces whose canonical maps are of degree 3 (joint work with Zhijie Chen). We will also present a cubic defining equation for each rational triple singularity of dimension two (joint work with Zhijie Chen, Rong Du and Fei Yu). Finally, we will show the relationship between rank two vector bundles on an algebraic surface and the discriminant curves of binary cubic forms.

Wing Keung To, National U. Singapore, Singapore

Effective isometric embeddings and Lojasiewicz inequality

In this talk, I will discuss some joint work with Sai-Kee Yeung on effective estimates on certain modifications needed to turn positive bihomogeneous polynomials on complex Euclidean spaces into squares of norms of vector-valued holomorphic polynomials. Such results can be interpreted as effective estimates on powers of associated Hermitian holomorphic line bundles needed for isometric projective embedding. We will also discuss generalizations to the case of bihomogeneous polynomials positive on a homogeneous affine hypersurfaces. In the higher codimensional case of arithmetically defined homogeneous affine varieties, we also obtain an effective Lojasiewicz inequality for such varieties, which is needed for the generalization.

Tom Wan, CUHK, Hong Kong

Mean curvature flows in hyperkähler manifolds

We will discuss a class of middle dimensional submanifolds in hyperkähler manifolds which is preserved under mean curvature flow. Submanifolds in this class also has a natural map to the real projective plane which should be regarded as an analog of the phase map of lagrangian submanifolds in Calabi-Yau manifold. We will see that there is a setting parallel to the case of lagrangian in Calabi-Yau.

Yihu Yang, Tongji U., China

Cohomologies of harmonic bundles over quasi-compact Kähler manifolds

Let $\overline{M}$ be a compact Kähler manifold, $\rho \to GL(m, C)$ a semisimple linear representation. Canonically one has a local system L_ρ over $\overline{M}$ and a harmonic metric h on L_ρ; on the other hand, by the Siu-Bochner technique, L_ρ has a Higgs bundle structure $(E, D'' = \overline{\partial} + \theta)$. Consequently, one can define various cohomologies: Cech cohomology and de Rham cohomology with coefficients in L_ρ, and Holomorphic Dolbeault cohomology and Higgs cohomology with coeffients in E, and canonically identify with them. In this Talk, we try to generalize the above consideration to the case of quasi-compact Kähler manifolds $M = \overline{M} \setminus D$, here D being a normal crossing divisor. In particular, I will consider a special case, namely M being a noncompact curve and ρ being unipotent at the divisor; in such a case, we will show that the corresponding cohomologies can be identified. In the case of variations of Hodge structures, this was proved by S. Zucker.

Sai-Kee Yeung, Purdue U., USA

Geometric results in Teichmüller and moduli space of curves

We would like to discuss a few results on Teichmuller and moduli spaces of curves that are obtained using methods of complex geometry. These include a problem of Yau about quasi-isometry of invariant metrics, a question of Gromov about Kähler hyperbolicity of the Bergman metric, and an analogue of Zucker's conjecture on the relation between L^2 cohomology and singular cohomology. Some geometric consequences would also be discussed.