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Cohomologies of harmonic bundles over quasi-compact Kähler manifolds

Let $\overline{M}$ be a compact Kähler manifold, $\rho \to GL(m, C)$ a semisimple linear representation. Canonically one has a local system L_ρ over $\overline{M}$ and a harmonic metric h on L_ρ; on the other hand, by the Siu-Bochner technique, L_ρ has a Higgs bundle structure $(E, D'' = \overline{\partial} + \theta)$. Consequently, one can define various cohomologies: Cech cohomology and de Rham cohomology with coefficients in L_ρ, and Holomorphic Dolbeault cohomology and Higgs cohomology with coeffients in E, and canonically identify with them. In this Talk, we try to generalize the above consideration to the case of quasi-compact Kähler manifolds $M = \overline{M} \setminus D$, here D being a normal crossing divisor. In particular, I will consider a special case, namely M being a noncompact curve and ρ being unipotent at the divisor; in such a case, we will show that the corresponding cohomologies can be identified. In the case of variations of Hodge structures, this was proved by S. Zucker.