Jaehyun Hong, Seoul National U., Korea

Rigidity of Schubert varieties

In this talk we consider two kinds of rigidity for Schubert varieties on rational homogeneous varieties $S = G/P$. By Bruhat decomposition, S is the union of finitely many B-orbits $\mathcal{O}_w$, $w \in W^P$. We call the closure X_w of $\mathcal{O}_w$ a Schubert variety of type w.

Fix $w \in W^P$. Taking different Borel subgroups, we get the family of Schubert varieties of type w. We say that a Schubert variety of type w is *Schubert rigid* if any subvariety X of S, which is tangent to a Schubert variety of the same type (depending on x) at each smooth point x of X, is a Schubert variety of the same type. We prove that a smooth Schubert variety is Schubert rigid if it is an equivariantly embedded Hermitian symmetric space, with trivial exceptions. In particular, any smooth nonlinear Schubert variety on Hermitian symmetric spaces is Schubert rigid. Using this, we prove the Schubert rigidity of singular Schubert varieties of certain types.

We say that a Schubert variety X_w of type w is *Schur rigid* if any effective cycle of S with the homology class equal to $r[X_w]$, $r \in \mathbb{Z}^+$, is a sum of r Schubert varieties of type w. Its Schubert rigidity is a necessary condition. We discuss when it is a sufficient condition.