Workshop on Complex Geometry

July 11 – 13, 2006

Room 517, Meng Wah Complex, HKU

Program and Abstracts

Institute of Mathematical Research

The University of Hong Kong

Speakers:

Jaehyun Hong	Seoul National U., Korea
Xiaojun Huang	Rutgers U., USA
Jun-Muk Hwang	KIAS, Korea
Andrei Iordan	Université Paris VI, France
Ngaiming Mok	HKU, Hong Kong
Tuen Wai Ng	HKU, Hong Kong
Dan Popovici	U. Warwick, UK
Yum-Tong Siu	Harvard U., USA
Luen Fai Tam	CUHK, Hong Kong
Wing Keung To	National U. Singapore, Singapore
Tom Yau Heng Wan	CUHK, Hong Kong
Xiaowei Wang	CUHK, Hong Kong
Richard Wentworth	Johns Hopkins U., USA
Siye Wu	HKU, Hong Kong
Sai-Kee Yeung	Purdue U., USA

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Program

July 11, 2006
Tuesday

9:50 – 10:00 *Opening Address*
Sun Kwok, Dean, Faculty of Science, HKU

10:00 – 11:00 **Yum-Tong Siu**, Harvard U., USA
Multiplier ideal sheaves and their applications

Tea Break

11:15 – 12:15 **Andrei Iordan**, Université Paris VI, France
Boundary regularity of the $\bar{\partial}$ operator in the complex projective space and applications

Lunch Break

14:00 – 15:00 **Xiaojun Huang**, Rutgers U., USA
On the complex structure of a Bishop surface near vanishing Bishop invariant

15:10 – 16:10 **Dan Popovici**, U. Warwick, UK
Singular Morse inequalities

Tea Break

16:30 – 17:30 **Siye Wu**, HKU, Hong Kong
Smooth and analytic Deligne cohomology groups

July 12, 2006
Wednesday

10:00 – 11:00 **Richard Wentworth**, Johns Hopkins U., USA
Harmonic maps to the Weil-Petersson completion of Teichmueller space

Tea Break

11:15 – 12:15 **Tom Yau Heng Wan**, CUHK, Hong Kong
Prescribing trees for holomorphic quadratic differentials and harmonic maps

Lunch Break

14:00 – 15:00 **Tuen Wai Ng**, HKU, Hong Kong
Solving non-linear differential equations by Nevanlinna Theory

15:10 – 16:10 **Luen Fai Tam**, CUHK, Hong Kong
Nonnegatively curved Kähler manifolds with quadratic curvature decay

Tea Break

16:30 – 17:30 **Ngaiming Mok**, HKU, Hong Kong
Analytic continuation of certain germs of holomorphic immersions between Fano manifolds of Picard number 1

July 13, 2006
Thursday

10:00 – 11:00 **Sai-Kee Yeung**, Purdue U., USA
Classification and construction of fake projective planes and fake projective spaces

Tea Break

11:15 – 12:15 **Jaehyun Hong**, Seoul National U., Korea
Rigidity of Schubert varieties

Lunch Break

14:00 – 15:00 **Wing Keung To**, National U. Singapore, Singapore
Singular potential functions on Hermitian symmetric manifolds and some applications

15:10 – 16:10 **Xiaowei Wang**, CUHK, Hong Kong
Moment map in Riemannian geometry

Tea Break

16:30 – 17:30 **Jun-Muk Hwang**, KIAS, Korea
Deformation of holomorphic maps to the blow-up of the plane

Abstracts

Jaehyun Hong, Seoul National U., Korea

Rigidity of Schubert varieties

In this talk we consider two kinds of rigidity for Schubert varieties on rational homogeneous varieties $S = G/P$. By Bruhat decomposition, S is the union of finitely many B-orbits $\mathcal{O}_w$, $w \in W^P$. We call the closure X_w of $\mathcal{O}_w$ a Schubert variety of type w.
Fix $w \in W^P$. Taking different Borel subgroups, we get the family of Schubert varieties of type w. We say that a Schubert variety of type w is *Schubert rigid* if any subvariety X of S, which is tangent to a Schubert variety of the same type (depending on x) at each smooth point x of X, is a Schubert variety of the same type. We prove that a smooth Schubert variety is Schubert rigid if it is an equivariantly embedded Hermitian symmetric space, with trivial exceptions. In particular, any smooth nonlinear Schubert variety on Hermitian symmetric spaces is Schubert rigid. Using this, we prove the Schubert rigidity of singular Schubert varieties of certain types.
We say that a Schubert variety X_w of type w is *Schur rigid* if any effective cycle of S with the homology class equal to $r[X_w]$, $r \in \mathbb{Z}^+$, is a sum of r Schubert varieties of type w. Its Schubert rigidity is a necessary condition. We discuss when it is a sufficient condition.

Xiaojun Huang, Rutgers U., USA

On the complex structure of a Bishop surface near vanishing Bishop invariant

For a real analytic surface in a complex space of dimension two, the complex structure at a totally real point is trivial. However, at a point with a complex tangent, the structure would be quite rich. The first invariant in this regard was introduced by E. Bishop. In this talk, we will dicuss the local equivalence problem for such a surface near a point with vanishing invariant. This is a joint work with Wanke Yin from Wuhan University.

Jun-Muk Hwang, KIAS, Korea

Deformation of holomorphic maps to the blow-up of the plane

Let S be the blow-up of the projective plane at d distinct points and let $f : X \to S$ be any surjective holomorphic map from a compact complex manifold X. We will show that all deformations of f come from automorphisms of S if $d > 2$. The result is optimal in the sense that it is not true if $d = 1$ or 2.

Andrei Iordan, Université Paris VI, France

Boundary regularity of the $\overline{\partial}$ operator in the complex projective space and applications

Let Ω be a pseudoconvex domain with smooth boundary in a Hermitian manifold and $g \in C^{\infty}_{(p,q)}\left(\overline{\Omega}\right)$ a $\overline{\partial}$-closed form, $q > 0$. Under the hypothesis that there exists a strongly plurisubharmonic function in a neighborhood of $\partial\Omega$, by a classical theorem of J. J. Kohn there exists $f \in C^{\infty}_{(p,q-1)}\left(\overline{\Omega}\right)$ such that $\overline{\partial} f = g$. For pseudoconvex domains in the complex projective space the existence of a plurisubharmonic function in a neighborhood of $\partial\Omega$ is not true in general.

In this talk, we will discuss the boundary regularity of $\overline{\partial}$ on pseudoconcave domains in the complex projective space and its applications to a quantitative version of Martineau-Serre duality for the Bergmann space and to the non-existence of Levi-flat hypersurfaces.

Ngaiming Mok, HKU, Hong Kong

Analytic continuation of certain germs of holomorphic immersions between Fano manifolds of Picard number 1

Let X be a Fano manifold equipped with a choice of Chow component $\mathcal{K}$ of minimal rational curves. At a general point x of X the variety of minimalr rational tangents (VMRT) is the collection of projectivizations of vectors tangent to a minimal rational curve passing through x. We say that VMRTs are linear if at a general point x of X the VMRT is a union of linear subspaces (of the same dimension). This is in particular the case when the normal bundle of a general minimal rational curve is trivial.

In what follows we assume that X is of Picard number 1. With geometric applications in mind we consider the problem of analytic continuation of local holomorphic maps from X into a Fano manifold Y of Picard number 1. We say that X exhibits the Cartan-Fubini extension property if the following holds true. Let Y be any Fano manifold of Picard number 1 equipped with a Chow component of minimal rational curves, and $f : (X, x_0) \to (Y, y_0)$ be a germ of VMRT-preserving biholomorphic map, then f extends to a biholomorphism from X to Y. In a joint work with Jun-Muk Hwang, we established in 2001 that any Fano manifold X exhibits the Cartan-Fubini extension property provided that at a general point of X, the Gauss map is generically injective at the VMRT at x. In a more recent joint work in 2004, we generalized the method to show that the Cartan-Fubini extension property holds true provided that the VMRT at a general point is non-linear. As an application this shows that any generically finite surjective holomorphic mapping $g : X' \to X$ is rigid when the target manifold is fixed.

As far as the Cartan-Fubini extension property is concerned, it is natural to try to generalize to the non-equidimensional situation. In other words, we can consider the question of extension of a germ of holomorphic immersion $f : (X, x_0) \to (Y, y_0)$, where Y denotes a Fano manifold of Picard number 1 equipped with a Chow component of minimal rational curves, $x_0 \in X$ and $y_0 \in Y$ are general points, under the assumption that f transforms the VMRT $\mathcal{C}_x$ at a general point x of a neighborhood of x_0 into the VMRT $\mathcal{D}_{f(x)}$ at $f(x)$. Here we focus on the situation where the Gauss map at a general point

of $\mathcal{C}_x$ is generically injective. Assume that a general point of $[df](\mathcal{C}_x) \subset \mathcal{D}_{f(x)}$ is smooth. We can then show that f extends to a meromorphic mapping under an assumption on the the Gauss map of $\mathcal{D}_{f(x)}$ at a general point of $[df](\mathcal{C}_x)$. This condition is nontrivial even when all VMRTs are nonsingular.

As an application we consider local holomorphic mappings between rational homogeneous manifolds of Picard number 1. This type of situation occurs in connection with rigidity problems on proper holomorphic mappings between bounded symmetric domains. It was proven by Tsai (1993) that when domain and target bounded symmetric domains are of the same rank ≥ 2, then any proper holomorphic mapping f is totally geodesic. A bounded symmetric domain can be realized as an open subset of its compact dual by means of the Borel embedding, and the properness and rank conditions imply that f sends VMRTs into VMRTs. Here we examine the general question whether such local holomorphic maps between rational homogeneous manifolds of Picard number 1 are necessarily totally geodesic.

Tuen Wai Ng, HKU, Hong Kong

Solving non-linear differential equations by Nevanlinna Theory

In this talk, I shall report some recent progresses on applying Nevanlinna Theory to find exact solutions of certain non-linear differential equations. In particular, I shall show that the all meromorphic solutions of the differential equation, $y^{(4)} = y^5$ are of the form $\frac{a}{z-b}$. This solves an old problem of Alexandre Eremenko. This is a joint work with L.W. Liao.

Dan Popovici, U. Warwick, UK

Singular Morse inequalities

We prove singular Morse inequalities estimating the asymptotic growth of the cohomology groups of high tensor powers of singular Hermitian holomorphic line bundles twisted by the corresponding multiplier ideal sheaves over a compact complex manifold. The main step in the proof is the construction of a new regularisation of closed almost positive $(1,1)$-currents with controlled Monge-Ampère masses. To this end, we prove two results describing the asymptotic growth of multiplier ideal sheaves associated with increasingly singular metrics: almost linear growth and an effective version of their coherence property. The asymptotics of Bergman kernels associated with singular metrics will play an important part.

Yum-Tong Siu, Harvard U., USA

Multiplier ideal sheaves and their applications

Will discuss multiplier ideal sheaves and their various applications, culminatinng in their application to the finite generation of canonical rings.

Luen Fai Tam, CUHK, Hong Kong

Nonnegatively curved Kähler manifolds with quadratic curvature decay

We will report some recent joint works with Albert Chau on the complex structure of a noncompact complete Kähler manifold M with nonnegative holomorphic bisectional curvature. We will discuss the proof of the following result: If M has maximal volume growth, then M is biholomorphic to $\mathbb{C}^n$. Note that by a result of Ni, such a manifold must have quadratic curvature decay. The proof will use this fact in an essential way.

Wing Keung To, National U. Singapore, Singapore

Singular potential functions on Hermitian symmetric manifolds and some applications

In this tak, I will discuss some joint works with J.-M. Hwang on the construction of certain potential functions with isolated singular points on Hermitian symmetric manifolds. In particular, I will discuss some applications of such constructions such as bounding Seshardri constants and volume of complex analytic subvarieties of Hermitian symmetric manifolds as well as some recent result on level structures of abelian varieties over a complex function field.

Tom Yau Heng Wan, CUHK, Hong Kong

Prescribing trees for holomorphic quadratic differentials and harmonic maps

A sufficient condition for the existence of holomorphic quadratic differential on a non-compact simply-connected Riemann surface with prescribed horizontal and vertical trees is obtained. In particular, for any pair of complete $\mathbb{R}$-trees of finite vertices with $n+2$ infinite edges, there exists a polynomial quadratic differential on $\mathbb{C}$ of degree n such that the associated vertical and horizontal trees are isometric to the given pair. Relationship to a problem in harmonic maps will also be discussed.

Xiaowei Wang, CUHK, Hong Kong

Moment map in Riemannian geometry

In this talk, we will discuss a notion of moment map in the Riemannian geometric setting. We will show some example and finally we will give an application to Riemannian geometry.

Richard Wentworth, Johns Hopkins U., USA

Harmonic maps to the Weil-Petersson completion of Teichmueller space

The talk will review the structure of the metric completion of Teichmueller space with respect to the Weil-Petersson metric as a nonpositively curved Alexandrov space. Harmonic maps with Teichmueller

space as a target may be used to study representations of fundamental groups into mapping class groups. A regularity result for the case of two dimensional domains will be discussed.

Siye Wu, HKU, Hong Kong

Smooth and analytic Deligne cohomology groups

In this talk, we compare the smooth and analytic Deligne cohomology groups. In particular, we study the cohomological product, fiber integration, and the relation with differential character.

Sai-Kee Yeung, Purdue U., USA

Classification and construction of fake projective planes and fake projective spaces

A fake projective plane is a complex surface different from but has the same Betti numbers as the complex projective plane. The first example was constructed by Mumford. Later on two more examples were found by Ishida and Kato. Very recently a fourth possible one was proposed by Keum. In this talk we present a joint work with Gopal Prasad on the classification of fake projective planes and their higher dimensional analogues. Furthermore, new examples are constructed both in dimension two and higher dimensions, one corresponding to each class of our table of classification.