THE UNIVERSITY OF HONG KONG

Institute of Mathematical Research

Department of Mathematics

GEOMETRY SEMINAR

Projective-algebraicity of minimal compactifications of non-arithmetic quotients of the complex unit ball

Professor Ngaiming MOK
The University of Hong Kong

Abstract

Let Ω be a bounded symmetric domain and $\Gamma \subset \mathrm{Aut}(\Omega)$ be a torsion-free discrete group of automorphisms such that $X := \Omega/\Gamma$ is of finite volume with respect to the canonical Kähler-Einstein metric. If $\Gamma \subset \mathrm{Aut}(\Omega)$ is arithmetic, it is well-known that the minimal compactification $\overline{X}_{\min}$ of Satake-Baily-Borel is projective-algebraic.

When X is globally irreducible according to Margulis' arithmeticity theorem Γ is necessarily arithmetic unless Ω is the complex unit ball, which is of strictly negative sectional curvature bounded between two negative constants. For a complete Kähler manifold (Z, h) of finite volume and of strictly negative curvature it was proven already in 1982 by Siu-Yau that Z can be compactified by adding a finite number of normal isolated singularities to give the minimal compactification $\overline{Z}_{\min}$. By the method of L^2-estimates of $\overline{\partial}$ and a theorem due to Andreotti-Tomassini it was proven that Z is biholomorphic to a quasi-projective manifold. It was however left open the question whether the minimal compactification is itself projective-algebraic.

In this talk we give a proof of projective-algebraicity of the minimal compactification $\overline{X}_{\min}$ for not necessarily arithmetic quotients $X = \Omega/\Gamma$ of the complex unit ball. For this purpose we first generalize the Mumford compactification $\overline{X}_M$ to the non-arithmetic case. This generalization relies on the geometric description of the structure of ends of Siu-Yau. We then prove by means of L^2-estimates of $\overline{\partial}$ that the q-th power of the logarithmic canonical line bundle of $\overline{X}_M$ with respect to the compactifying divisor D (which is a disjoint union of Abelian varieties) is base point-free whenever $q \geq 2$, which is the essential fact allowing the compactifying divisors to be blown down to normal isolated singularites. The general question whether the minimal compactification $\overline{Z}_{\min}$ (as in the above) is projective-algebraic remains open.

In a recent article Koziarz-Mok has studied the Submersion Problem, viz., submersions $f: X \to X'$ between finite volume quotients of the complex unit ball. We prove that any such holomorphic submersion must be a biholomorphism provided that either X is compact or it is noncompact but of complex dimension at least 2. One motivation of the proof of projective-algebraicity of $\overline{X}_{\min}$ was to give an alternative deduction of the finite-volume case from the cohomological arguments of the compact case by slicing by hyperplane sections.

Date:	**March 11, 2009 (Wednesday)**
Time:	**4:00 – 5:00pm**
Place:	**Room 517, Meng Wah Complex, HKU**

All are welcome