THE UNIVERSITY OF HONG KONG

Institute of Mathematical Research
Department of Mathematics

GEOMETRY SEMINAR

Malmquist-type theorems for second-order ODEs

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Abstract

At the beginning of the twentieth century, Painlevé and Gambier analyzed the possible forms of f when the equation $y''(z) = f(z; y(z); y'(z))$, with $f(z; \zeta; \xi)$ rational in ζ and ξ and with z dependent coefficients, has *all* of its solutions to be single-valued around their movable singularities. They found fifty possible classes for f so that the equations possess the required property. Forty four of the f are known to be solvable in terms of known functions, and the remaining six give raise to what is known to be the six Painlevé equations. On the other hand, Malmquist showed in 1931 that if the equation $y'(z) = R(z; y(z))$, where R is rational in y and with polynomial coefficients, admits a meromorphic solution, then $R(z; y) = a_0(z) + a_1(z)y + a_2(z)y^2$, that is, the equation must be reduced to a Riccati equation. We extend Malmquist's and others' results that when f in $y''(z) = f(z; y; y')$ is suitably restricted, then we can recover some of the six Painlevé equations by *only assuming* the equation to admit meromorphic solution. The method of approach is based on a combination of Painlevé analysis and Nevanlinna's theory of value distribution of meromorphic functions. In fact, the Nevanlinna theory even allows us to consider coefficients of f to be transcendental meromorphic functions which have small Nevanlinna order when compared to that of the solution y. This is a joint project with R. G. Halburd and E. Lingham.

Date:	**December 11, 2008 (Thursday)**
Time:	**4:30 – 5:30pm**
Place:	**Room 517, Meng Wah Complex, HKU**

All are welcome