THE UNIVERSITY OF HONG KONG

*Institute of Mathematical Research*
*Department of Mathematics*

# COLLOQUIUM

# On Polar Ovals in Cyclic Projective Planes

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### Abstract

A projective plane is Desarguesian if every pair of centrally perspective triangles are axially perspective. It is well known that a Desarguesian plane can be coordinatized by a division ring. Since any finite division ring is a Galois field, the order of a finite Desarguesian projective plane is a prime power. Although non-Desarguesian planes exist, all known cases have order a prime power. This leads to the following long-standing conjecture:

**Prime Power Conjecture** *Any finite projective plane has order a prime power*.

The smallest open case is order 12.

A cyclic projective plane is a finite projective plane which admits a cyclic Singer group of collineations. J. Singer (1938) has shown that any finite Desarguesian plane is a cyclic projective plane. The converse is another long-standing conjecture:

**Conjecture** *Any cyclic projective plane is Desarguesian*.

This conjecture implies the prime power conjecture for cyclic projective planes. A cyclic projective plane determines and is determined by a cyclic difference set. In this talk, we shall introduce a condition on the cyclic difference set $\mathcal{D}_q$ of a cyclic projective plane $\pi$ of odd order $q$, and construct a polarity on $\pi$ so that its set of absolute points is $2\mathcal{D}_q$. We shall then deduce that any such $\pi$ is Desarguesian, using a fundamental result of U. Ott (1975), which states that if a cyclic projective plane admits a different cyclic collineation group in addition to its Singer cyclic collineation group, then it must be Desarguesian.

This is a joint work with H.F. Law and P.P.W. Wong.

**Date:** **January 11, 2010 (Monday)**

**Time:** **3:00 – 4:00pm**

**Place:** **Room 210, Run Run Shaw Bldg., HKU**

*All are welcome*