Workshop on Complex Geometry

July 28 - 30, 2014

Room 210, Run Run Shaw Bldg., HKU

Program and Abstracts

Institute of Mathematical Research
Department of Mathematics

Speakers:

Rong Du	HKU, Hong Kong
Roman Dwilewicz	Missouri U. Sci. & Tech., USA
Lawrence Ein	U. Illinois, Chicago, USA
James Fullwood	HKU, Hong Kong
Peter Greiner	Toronto U., Canada
Zhiguo Liu	East China Normal U., Shanghai
Ngaiming Mok	HKU, Hong Kong
Sui-Chung Ng	East China Normal U., Shanghai
Tuen-Wai Ng	HKU, Hong Kong
Yum-Tong Siu	Harvard U., USA
Toshiyuki Sugawa	Tohoku U., Japan
Wing-Keung To	National U. Singapore, Singapore
Chiu-Yin Tsang	HKUST, Hong Kong
Sai-Kee Yeung	Purdue U., USA

Organizers: Tuen-Wai Ng, Ngaiming Mok

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Time / Date	July 28 (Mon)	July 29 (Tue)	July 30 (Wed)
9:30 – 10:30	Siu	Sugawa	Ein
10:30 – 10:50	*Tea Break*		
10:50 – 11:50	NgSC	NgTW	Fullwood
Lunch Break			
14:00 – 15:00	Greiner	Liu	Du
15:10 – 16:10	Dwilewicz	Tsang	Yeung
16:10 – 16:30	*Tea Break*		
16:30 – 17:30	Mok	To	

Program

July 28, 2014
Monday

9:30 – 10:30 **Yum-Tong Siu**, Harvard U., USA
Strong rigidity and Hodge filtration in homology setting

Tea Break

10:50 – 11:50 **Sui-Chung Ng**, East China Normal U., Shanghai
Holomorphic splitting tangent sequence in Hermitian locally symmetric spaces

Lunch Break

14:00 – 15:00 **Peter Greiner**, Toronto U., Canada
A CR-heat kernel on complex spheres

15:10 – 16:10 **Roman Dwilewicz**, Missouri U. Sci. & Tech., USA
Hartogs-type extension for unbounded domains in C^n

Tea Break

16:30 – 17:30 **Ngaiming Mok**, HKU, Hong Kong
Geometric structures and substructures: from uniruled projective manifolds to bounded symmetric domains

July 29, 2014
Tuesday

9:30 – 10:30 **Toshiyuki Sugawa**, Tohoku University, Japan
Hyperbolic metric with conic singularities on Riemann surfaces and its applications

Tea Break

10:50 – 11:50 **Tuen-Wai Ng**, HKU, Hong Kong
Critical points of random polynomials and random finite Blaschke products

Lunch Break

14:00 – 15:00 **Zhiguo Liu**, East China Normal U., Shanghai
A q-partial differential equation, complex analysis in several variables and q-series

15:10 – 16:10 **Chiu-Yin Tsang**, HKUST, Hong Kong
Symmetry of the Darboux equation

Tea Break

16:30 – 17:30 **Wing-Keung To**, National U. Singapore, Singapore
Hyperbolicity and Weil-Petersson geometry

July 30, 2014
Wednesday

9:30 – 10:30 **Lawrence Ein**, U. Illinois, Chicago, USA
Gonality conjecture and asymptotic syzygies

Tea Break

10:50 – 11:50 **James Fullwood**, HKU, Hong Kong
On motivic characteristic classes and universal tadpole relations

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Lunch Break

14:00 – 15:00 **Rong Du**, HKU, Hong Kong
Hypergeometric differential equations and ball quotient surfaces

15:10 – 16:10 **Sai-Kee Yeung**, Purdue U., USA
Geometry of a special arithmetic complex two ball quotient

Abstracts

Rong Du, HKU, Hong Kong

Hypergeometric differential equations and ball quotient surfaces

I will introduce the Gauss-Schwarz Theory for hypergeometric differential equations. By using this theory, I will describe the complex geodesics in ball quotient surfaces which are branched covers over the projective plane with special line arrangements as branch loci.

Roman Dwilewicz, Missouri U. Sci. & Tech., USA

Hartogs-type extension for unbounded domains in C^n

The classical Hartogs theorem solves the extension problem for bounded domains in C^n and clearly shows the difference between one and several-variables cases. Surprisingly, not many results are in the unbounded case. In the talk there will be presented two results: one for generalized tubular domains in C^n and another for tube-like domains in C^2.

Lawrence Ein, U. Illinois, Chicago, USA

Gonality conjecture and asymptotic syzygies

We discussed joint work with Rob Lazarsfeld. We'll describe how to use Hilbert schemes of points to study syzygies. In particular we are able to solve a conjecture of Green and Lazarsfeld which says that one detects the gonality of an algebraic curve from the Betti numbers of a minimal resolution of the coordinate ring of a sufficiently ample line bundle on the curve. We also discuss the generalization of the theorem to higher dimensions.

James Fullwood, HKU, Hong Kong

On motivic characteristic classes and universal tadpole relations

Motivic characteristic classes are class generalizations of motivic measures, which in turn may be viewed as higher analogues of counting. We first review the basic theory of such objects, and then discuss how certain tadpole relations arising in physics are merely reflecting certain identities between such classes.

Peter Greiner, Toronto U., Canada

A CR-heat kernel on complex spheres

I shall give a geometrically invariant derivation of the heat kernel for the CR-subLaplacian on complex spheres. There appears the possibility of the CR-curvature modifying the Hamilton-Jacobi equation.

Zhiguo Liu, East China Normal U., Shanghai

A q-partial differential equation, complex analysis in several variables and q-series

The concept of q-partial differential equations is first introduced and then we discuss a specific q-partial differential equation. Using the theory of analytic functions in several variables, we prove that any analytic solution of this q-partial differential equation can be expressed in terms of the Rogers-Szego polynomials. This fact allows us to develop a general method of deriving q-hypergeometric identities. Using this method, we can not only give new derivations of many classic q-series identities, but also find new q-formulas.

Ngaiming Mok, HKU, Hong Kong

Geometric structures and substructures: from uniruled projective manifolds to bounded symmetric domains

With Jun-Muk Hwang the speaker has developed a geometric theory of uniruled projective manifolds X, which by Miyaoka-Mori include Fano manifolds, modeled on varieties of minimal rational tangents $\mathcal{C}_x(X) \subset \mathbb{P}T_x(X)$, alias VMRTs, defining geometric structures called VMRT structures. Our theory applies especially to rational homogeneous manifolds $X = G/P$. In collaboration with Jaehyun Hong the speaker has considered pairs (X_0, X) of uniruled projective manifolds, and characterized certain standard embeddings $i : G_0/P_0 \hookrightarrow G/P$ between rational homogeneous spaces of Picard number 1 basing on a non-equidimensional Cartan-Fubini extension theorem generalizing results of Hwang-Mok in the equidimensional case. Our study leads to geometric substructures called VMRT substructures. VMRT structures and substructures are also defined on irreducible bounded symmetric domains D and their quotient manifolds when D is identified as an open subset of its compact dual manifold S by the Borel embedding. We define the Recognition Problems for VMRT structures and substructures defined by VMRTs, and discuss their applications to Algebraic Geometry, Several Complex Variables, Complex Differential Geometry and Arithmetic Geometry.

Sui-Chung Ng, East China Normal U., Shanghai

Holomorphic splitting tangent sequence in Hermitian locally symmetric spaces

A complex submanifold is said have its tangent sequence split holomorphically if the pull-back holomorphic tangent bundle from the ambient space is a holomorphic direct sum of its holomorphic tangent bundle and normal bundle. A theorem of Van de Ven states that a complex submanifold of a projective space is linear if and only if its tangent sequence splits holomorphically. In this talk, we are going to discuss what happens when the ambient space is a compact Hermitian locally symmetric space of non-compact type.

Tuen-Wai Ng, HKU, Hong Kong

Critical points of random polynomials and random finite Blaschke products

The study of zero distribution of random polynomials has a long history and is currently a very active research area. Traditionally, the randomness in these polynomials comes from the probability distribution followed by their coefficients. One can introduce randomness in the zeros (instead of the coefficients) of polynomials, and then investigate the locations of their critical points (relative to these zeros). Such a study was initiated by Rivin and the late Schramm in 2001, but only until 2011, Pemantle and Rivin proposed a precise probabilistic framework of it which will first be explained in this talk. Following this framework, we will consider the problem of finding the zero distributions of the derivatives of random polynomials and random finite Blaschke products with i.i.d. zeros following a common distribution supported on a subset of the complex plane. This is a joint work with Pak-Leong Cheung, Jonathan Tsai and Phillip Yam.

Yum-Tong Siu, Harvard U., USA

Strong rigidity and Hodge filtration in homology setting

For a compact Kähler manifold the Hodge filtration filters cohomology classes by representability by closed forms of type (p, q). In this talk we use the method of strong rigidity to discuss Hodge filtration in the homology setting by considering the representability of homology classes by CR manifolds.

Toshiyuki Sugawa, Tohoku University, Japan

Hyperbolic metric with conic singularities on Riemann surfaces and its applications

We first review fundamentals of hyperbolic metric on Riemann surfaces which admits conic singularities. In this talk, we will be mainly concerned with the simplest case when the Riemann surface is the complex projective line (Riemann sphere) and when the number of singularities is three. In this case, the density function of the metric can be explicitly described in terms of hypergeometric functions. As an application, we will give some refinements of Schottky and Landau theorems. This talk is based on the joint work with Daniela Kraus and Oliver Roth (University of Wuerzburg).

Wing-Keung To, National U. Singapore, Singapore

Hyperbolicity and Weil-Petersson geometry

In this talk I will talk about some joint works with Sai-Kee Yeung studying hyperbolicity properties on moduli spaces of certain polarised manifolds using Weil-Petersson metrics.

Chiu-Yin Tsang , HKUST, Hong Kong

Symmetry of the Darboux equation

The Darboux equation (1882) was a generalization of both Picard's and Hermite's equations. All these equations are generalizations of the well-known Lamé equation (1837). The equation was rediscovered by Treibich and Verdier in the 1980s concerning it having finite-gap property in an algebraic geometric characterization. The equation is a (doubly periodic) torus version of the Heun equation which lives on the Riemann sphere. It turns out that the Darboux equation has a better symmetry structure compared to that of the Heun equation. In this talk, we will describe the symmetry of the Darboux equation via the study the transformations which induce the automorphisms of the Darboux equation. We show how to apply the automorphisms to generate the 192 local solutions of the Darboux equation. This is a joint work with Yik-Man Chiang and Avery Ching.

Sai-Kee Yeung, Purdue U., USA

Geometry of a special arithmetic complex two ball quotient

The purpose of the talk is to explain a joint work with Vincent Koziarz and Donald Cartwright on the geometry of a complex two ball quotient constructed earlier by Cartwright and Steger. The surface has the smallest possible Euler number 3 among surfaces of general type but is a not a fake projective plane. Basic geometric properties of the surface had not been understood, such as the genus of the Albanese fibration. We would answer some questions in this direction and use the example to study several problems in the geometry of algebraic surfaces and complex ball quotients.