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Basics of Crystal Bases and Geometrical Crystals

The theory of crystal base has been initiated by Masaki Kashiwara. Nowadays, it is widely known and applied to many areas in mathematics and physics. In the former half of this lecture, we will introduce the theory of crystal bases from the beginning, which includes the following topics: representation theory of quantum groups; Kashiwara operators; definiton of crystal base; crystal graphs; existence theorem; tensor products of crystal bases; tableaux realizations; crystal base of the negative part of the quantum group; polyhedral realizations.

In the latter half of the lectures, we shall introduce theory of geometric crystals, which has been invented by A.Berenstein and D.Kazhdan. It is a kind of geometric analogue of crystal base theory, indeed, which is defined on some algebraic variety and each Kashiwara operator will be realized as some rational action of the 1-dimensional multiplicative group. Geometric crystal is not a simple and superficial analogue of crystal base, but it holds certain functorial correspondence to crystal base, which is called "tropicalization". In the lecture, the following topics will be treated: definition of geometric crystal; product structure of geometric crystals; positive structure of geometric crystals; tropicalization of geometric crystals; monomial realization of "crystal bases"; geometric crystal on Schubert variety; decorated geometric crystals.

If time permits, we also present some relations between crystal bases and cluster algebras.