Hong Kong Mathematics Education: A Time for Change?
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Session on Appraisal of the Hong Kong Mathematics Curriculum

Core curriculum = Second class curriculum?
Core curriculum = Core syllabus?

SIU Man Keung
Department of Mathematics
University of Hong Kong

1. Having been given only three-days' notice to speak on the topic of core curriculum in this session, I cannot possibly write up a paper to elucidate my views at length. To be frank, I would not be able to do so even if I had been given more time, for I happened to bump into this topic only recently, and have given it some thought only in the past month. In the text below I will try to piece together relevant parts from my files, if just to help me organize my thoughts. Please bear with the rambling and disconnected discourse in this hastily prepared manuscript. This matter on core curriculum may have such far-reaching consequence (for better or for worse) that it warrants any sort of discourse, no matter how rambling and disconnected it may be.

Questoins: Should we regard a core curriculum as a "watered-down" substitute for the academically low achievers? Can a core curriculum be established by a simple pruning of the existing syllabus?

- Some references that may be helpful:
  孫淑南,中學數學課程改革:從第五組別學生談起
  蕭文強,少者多也:普及教育中的大學數學教育
  陳鳳潔,黃毅英,蕭文強,教(學)無止境:數學「學養教師」的成長
  (All three articles appear in the following book: 蕭文強主編,《香港數學教育的回顧與前膽:梁鑑添博士榮休文集》,香港大學出版社,1995.)
  F.K. Siu, M.K. Siu, N.Y. Wong, The changing times in mathematics education: The need of a scholar teacher, in C.C. Lam, H.W. Wong, Y.W. Fung (Ed), "Proceedings of the International Symposium on Curriculum Changes for Chinese Communities in Southeast Asia: Challenges of 21st Century", The Chinese University of Hong Kong, 1993, 223-226.

2. Even when we are facing practical reality, we must not lose sight of the ideal aims of mathematics education. Vision usually sounds utopian, but the world belongs to dreamers! In the long run, it is an ideal that brings about progress. Below I will extract some passages from my file of notes on this issue.

- (From a paper published in September 1992: 蕭文強,數學史和數學教育,《數學傳播》16卷3期(1992年9月), 23-29頁.)

這兩種反應貌似不同,實則反映了同一件事: 在數學教育中, 我們往往只強調實用知識這一個目標.不同時代不同地區的數課程綱要,內容和使用字眼或許不相同, 但籠統扼要地說,它們的目標都可以分爲三方面,即是:(1)思維訓練、(2)實用知識、(3)文化素養. 但往往我們只注重(2), 把數學單單作爲一種技能、一種工具去講授.這樣做的話,縱使傳授了知識,亦必掩蓋了數學作爲文化活動的面目.學生不易瞭解數學有它的生命和發展、有它的過去和未來;學生容易把數學看成是一堆現成的公式和定理,雖然完美無誤但也是僵硬不變而且刻板枯燥;學生見到的儘是技巧堆砌和邏輯遊戲,予人閉門造車的印象.難怪只有極少數學生被數學吸引了,也有少數一些學生爲了日後需要使用這種工具姑且把它捱過去 ,其餘絕大部份學生都與數學疏離,或者厭惡害怕它,或者對它持冷漠態度.很多學生中學畢業了,卻像完全沒有學過數學這科,只當它是一場惡夢!

數學教學有"狹義"和"廣義"兩方面:前者是指傳授數學知識,後者較難界定, 籠統地說它是指"數學觀"的體現.什麼是"數學觀"呢?有些人以爲那是抽象的哲學問題,其實它並不抽象,你的數學觀就是你對數學的看法、你對數學本質和意義的見解.每個人總有自己對事物的看法,因此每個人一定有自己的數學觀.(如果你認爲毋須理會數學的本質和意義,那也是一種數學觀!)每個社會的成員的數學觀匯集起來,其主流即形成該社會數學觀.千萬不要小看這一點,千萬不要以爲數學觀與數學教學無干.就個人而言,不論你自覺也好, 不自覺也好,你的數學觀必定流露反映於你的教學中,從而影響了你的學生.就整個社會而言,證諸歷史,數學和數學教育的內容及發展,決定於當時當地的數學觀.

以前我曾在一篇題爲"數學. 數學史.數學教師"的文章裡談到數學上"才、學、識"(刊於《抖擻》雙月刊第53期(1983年7月),67-72頁),這個提法是源於清代文學家袁枚的話:"學如弓弩,才如箭鏃,識以領之、方能中鵠."於數學而言,才是指計算能力、推理能力、分析和綜合能力、洞察力、直觀思維能力、獨立創作力、...;學是指各種公式、定理、算法、理論、...; 識是指分析鑒別知識再經融會貫通後獲致個人見解的能力.如果把這三點套用於上述的兩方面,"學"便對應於狹義數學教學,而"才、學、識"三者合起來才對應於廣義數學教學.至於這兩方面的功能,大別之或者可以這樣說: 狹義數學教學達致的社會功能,就短線而言乃日常計算或專業需要,就長線而言乃數學研究及科技進展,總而言之,數學是一種工具.廣義數學教學達致的還有教育功能,這包括數學思維伸延至一般思維,培養正確的學習方法和態度、良好學風和品德修養,數學欣賞帶來的學習愉悅以至對知識的尊重.

- (Notes written on February 19, 1995)

For the overall aim in education the drafted statement issued by the Education Commission (School Education in Hong Kong: A Statement of Aims, October 1992) already captures it quite well:

> "the fundamental aim of the school education service
> is to develop the potential of every individual child, so

that our students become independent-minded and socially aware adults, equipped with the knowledge, skills and attitudes which will enable them to lead a full life and play a positive role in the social and economic development of the community."

In the subject of mathematics the aims should follow this same spirit in general, but at the same time be implemented and elaborated according to the nature of the specific subject.

In broad strokes we wish our students to be brought up in such a (mathematics) classroom culture and environment that they can:

(1) acquire active and effective learning habits so that they are able to **read** and know how to access knowledge; able to **write** and to **speak** clearly in order to express their views and to communicate with others; able to make **sense** out of mathematics; willing to **think**, to query, to challenge and to probe;

(2) have first-hand mathematical experience so that they realize the **dual natures** of mathematics as an exact science as well as an imaginative endeavour, as an abstract intellectual pursuit as well as a concrete subject with real-life applications; appreciate the beauty, the import, the power as well as the limitation of mathematics.

In the course of achieving these aims the subject content must be introduced in such a way that a student will learn basic mathematical concepts and skills, and learn how to apply them to solve problems in everyday life or in a future career, be it academic or vocational. In this way, we hope students will regard mathematics not merely as a technical tool, which it certainly is, but more importantly as an intellectual endeavour and a mode of thinking. This will help students to form their own conception of the discipline, and convince them that mathematics is an intellectually rewarding discipline which plays a central role in human culture in a more general context.

Although the aims stated above should permeate through the mathematics curriculum, at different stages in school the emphasis and the subject material are bound to vary. It will be helpful to set down more specific goals and to devise some main themes so that the syllabus can be planned based on these goals and themes.

- (Proposed themes, jointly worked out with N.Y. Wong in April 1995)

PRIMARY

numbers
shapes
measurements
(mostly inductive reasoning and heuristics)

JUNIOR SECONDARY

operations, patterns, functions & their graphs
algebraic concepts
geometric concepts
statistical concepts
(deductive reasoning)

SENIOR SECONDARY

inverse operations, functions
3-dimensional spatial sense
probabilistic concepts
(generalization and abstraction)

NOTE: Themes are not confined to a single level. The level to which each theme is attached just indicates that it can be introduced and be emphasized from that level on.

- There are four dimensions to the shaping and development of a mathematics curriculum. Vertically speaking these are the primary, the secondary, the tertiary and the teachers' education. Horizontally speaking these are the students, the teachers, the mathematicians and the mathematics educators. A successful curriculum relies on the contribution from all of them, summed up in the slogan (coined by Dr. K.T. Leung)"數學教育四維共濟 ".

3. In recent years, there is a move towards tailoring the mathematics syllabus so as to cater for students of a wide range of abilities, interests and needs. Here the idea of a core curriculum comes up. The intention is to identify a core which satisfies the following five criteria:

(a) it is a minimum body of learning for every student,

(b) it forms the foundation on which future study of mathematics at a higher level could be built,

(c) it contains different components that constitute a coherent curriculum,

(d) it covers a wide range of cognitive ability,

(e) it emphasizes important knowledge, concepts and skills but not the details.

At one point I was asked to examine the pruned syllabus and to offer comments. Below I will reproduce the notes I wrote down on May 8, 1995. (Refer to the Mathematics Syllabus recommended by the Hong Kong Curriculum Development Committee, 1985.)

> 1. The five criteria set down on the first page of this document sound reasonable, the last three particularly well-spoken. Indeed, these criteria can well be those for planning a comprehensive secondary mathematics curriculum rather than for tailoring the existing syllabus! But so far as the tailoring purpose is

concerned, criterion (b) will no doubt lead to a syllabus which covers more than that which satisfies only criterion (a). We are thus being drawn back to the old question: can a student, who follows only the core syllabus, continue to study mathematics profitably and effectively after S5?

2. Criterion (c) on the importance of coherence can be supplemented by those of relevance and unity. It is usually comforting and motivating for a student to see things learnt previously pop up in other parts of mathematics, or better yet, in subjects other than mathematics. It would be a pity if we do not try to strengthen such links, and worse yet if we try to play them down, thinking that bringing in knowledge of other subjects can make mathematics more difficult.

3. There seems to be a dilemma in deleting certain topics or examples which are interesting, challenging, enlivening and relevant to "daily experience", on the ground that these are "difficult" for weak students. It is debatable whether easy (routine?) topics are conducive to positive learning, and whether "difficult" topics have the opposite effect. Some such examples, which are labelled as "difficult", are counting (completely absent in the syllabus now), arithmetic and geometric progressions, tessellation. Some topics are instructive and interesting as exercises but not suitable when labelled as separate items by themselves, e.g. sum of interior angles of a polygon. Under the demand of cutting-for-cutting's sake, all these easily become the first batch to go!

4. Certain topics can be deleted without regret, and not just from the core syllabus. They are of two types: (1) topics which have become obsolete, e.g. square root tables, trigonometric tables, logarithm tables, "assumed mean"; (2) topics which are not suitable for school mathematics, e.g. open sentences, explanation of operation on negative numbers through basic axioms, numerical analysis (bisection method), summation notation.

5. Certain topics are "frills" and can be deleted from the core syllabus (and in some cases even from the full syllabus as well), e.g. binary numbers, polar coordinates, trigonometric ratios of angles other than acute angles, trigonometric identities, relations between roots of a quadratic equation, rationalization of surds, simultaneous equations in which one is linear and one is quadratic, manipulation of ratio and proportion, linear programming.

6. Certain topics can be deleted from the core syllabus, or deferred to a later stage when they can be studied in a richer context with related development rather than as an isolated, and hence unmotivating, technique, e.g. algebraic inequalities, radian measure.

7. Certain topics have been accorded excessive attention, even to the point of being overdone, either with fragmented repetition or with lots of secondary technicalities, e.g. significant figures and rounding off, percentages (stretched from S1 to S3), different "standard forms" of the equation of a straight line.

8. Certain (mathematically) closely related topics are severed with one part retained and the other part dropped, for reasons I fail to understand, e.g. midpoint theorem/intercept theorem, remainder theorem/factor theorem. Certain topics which can enhance understanding are deleted, for reasons I fail to appreciate, e.g. graphical interpretation of ratio/proportion, graphical interpretation of algebraic inequalities, completing the square (without it, the quadratic formula by itself is so awkward and dry!)

9. The notion of locus is a basic concept, not just in geometry but in mathematics as a whole. It is helpful in illuminating other parts of mathematics.

Even in the syllabus now it is not accorded a prominent role. It would be a pity if it were left out altogether.

10. One should separate the notion of logarithm from the use of logarithm table. Although the latter is obsolete and can be deleted altogether, the former is a basic notion. Maybe it can be deleted (or deferred) from the core syllabus, but its omission will create difficulty, if not impossibility, in the further pursuit of mathematics built on the core syllabus.

11. The perennial (and no doubt difficult) problem about the teaching of geometry is not solved, nor even addressed, in the tailoring process. Should we stay at the intuitive level all the way, or should we let students have a sense of the more abstract geometric conceps at some point? Should we confine ourselves to numerical calculations, or should we introduce students to deductive reasoning at some point? Should we let school geometry remain a practical subject, or should we use it as a means to let students see the synthetic mode of thinking at work? For instance, the inclusion or not, and at which level, of the topic on geometric construction (e.g. angle and line segment bisection) is a moot point affected by an answer to this difficult problem.

12. In general, a simple cut-and-paste job can easily end up with problems mentioned in the points raised above. That is why Mr. N.Y. Wong suggests that the planning of the syllabus should be based on certain themes. For instance, almost all of the content now appearing in Strand A plus part of that appearing in Strand B are actually implementations of the themes on operations/patterns/functions and their graphs, and algebraic concepts.

4. From reading the excerpts above you can well guess that my answers to the two questions raised at the beginning are both "no". It falls upon the shoulders of all of us, as mathematics teachers, to see how we can bring closer reality and Utopia!