

## Mathematics for Math Majors: Loss of Its Self-esteem

*FUNG Chun-Ip*  
*Department of Mathematics*  
*Northcote College of Education*  
*Hong Kong*

*SIU Man-Keung*  
*Department of Mathematics*  
*University of Hong Kong*  
*Hong Kong*

Paper given at the International Conference of Mathematics Education-7, Quebec, 1992

Many may think that mathematics for math majors presents no serious problem since math majors are supposed to form a highly motivated and capable group who can study mathematics almost on their own. In reality our teaching experience tells us otherwise. We see a wide spectrum of students who are labelled as math majors, at least so labelled for administrative reason because they take a certain number of units of courses in mathematics including such and such compulsory courses in the programme. At one end of the spectrum we are gratified to find a few very motivated students with high calibre and strong background in mathematics. At the other end we are depressed to find a small group of unmotivated and listless students whose sole aim is to obtain a university degree with (less than?) minimal effort. They are not necessarily weak, though they usually end up weak in mathematics. In between, we find a rather

---

**Mathematics is too vast and too old (yet forever new) a subject so that when school children learn of achievements of modern sciences in the present century, their mathematics lessons basically cover what had been done up to the sixteenth and seventeenth centuries.**

---

large group of students, some quite motivated and some not as motivated, some adequately prepared and some less prepared, but all reasonably willing to put in the effort. It is mainly this middle group that we wish to address here, but certainly what we say applies to the first group as well. There is little we can do about the remaining group, whose

attitude is moulded by priority and value judgement formed within a society of which they form a part. But if we do not do our job well, many in the middle group may be driven to the end of the spectrum. (In a small way we can still hope to influence the socio-political scenario of the time!) A student in the middle group may start as one who likes mathematics, at least not one who dislikes or is apprehensive of mathematics, but in the course of time may feel more and more confused, alienated and frustrated. When he was in secondary school he could compute and solve specific problems assigned by the teacher, and derive delight and satisfaction from doing that. Now, he is immersed in concepts which seems neither related to what he has learnt nor suggestive of where he is heading. Faced with this fragmentary picture he may resort to learning by rote. But that usually ends up in disaster. Even if he is lucky and passes the test which happens to ask for knowledge reproducible from rote learning, he is disillusioned. For that does not seem like what an intellectually exciting subject should be. In either case he no longer feels the same affinity to mathematics as he had felt before becoming a math major! (In this article "he", "his" are used in a generic sense. It is tiresome to write "he or she", "his or her" each time instead.) What goes wrong?

In a poster at ICME-5 of 1984, one of us wrote: "Although mathematics is universally recognized as a most basic, important, useful and encompassing discipline, it is also the least understood, the most misunderstood and the most neglected subject by the public. ... Mathematics is too vast and too old (yet forever new) a subject so that when school children learn of achievements of modern sciences in the present century, their mathematics lessons basically cover what had been done up to the sixteenth and seventeenth centuries. Even at

universities, most students study mathematics that was done up to the beginning of the nineteenth century; only a few math majors may go beyond that. Mathematics gradually acquires a language of its own, which can sound quite obscure to one without that training. It must also be admitted that mathematics demands abstract thinking so that one must put in the requisite amount of time and effort to really understand it. .... Thus, in schools, mathematics teaching tends to emphasize the technical content with the advantage that a reasonable amount of knowledge can be transmitted in the time allotted so that students can learn their language and skill in a reasonably short time. However, in so doing, the cultural aspect is bound to be neglected. Students may be totally unaware that mathematics has its life, that it has a past as well as a future, that it is not just a mess of neatly packed but lifeless formulae and theorems." (M.K. Siu, History of Mathematics Teachers, Mathematical Tall Timbers, Mu Alpha Theta, No. 11 (March 1985), French translation in Bull. de l'Association des Prof. de Math., no. 354 (1985), 309319.) He also wrote in the same article: "What do we mean by a learned teacher? A learned teacher should possess the following qualities: (i) ability, (ii) knowledge, (iii) wisdom. They differ from one another but are closely related, each complementing the other two. An eighteenth century Chinese scholar, Yuan Mei, once said (but in a literary context), 'Knowledge is like the bow, ability like the arrow; but it is wisdom which directs the arrow to the bull's eye.' .... A well-balanced mathematics curriculum should address three aims (i) training of the mind, (ii) transmission of technical knowledge, (iii) awareness of the cultural aspect. If the reader is willing to bear with a looser usage of vaguer terms, we shall characterize the three aims as: (i) ability, (ii) knowledge, (iii) wisdom, which brings us back to our starting point." This viewpoint was further elaborated: "in schools we tend to emphasize the technical content of mathematics and teach it as a skill and a tool. By so doing we attend to 'knowledge' or perhaps even some 'ability', but unfortunately not 'wisdom' as well. ... This may seem effective, at least it seems so from examination performance. In Hong Kong, the passing percentage and average score in mathematics in the Secondary School Leaving Certificate Examination are usually higher than that in other subjects. The recent report of the 2nd International Evaluation of Educational Achievement reveals that Hong Kong students

score pretty high marks in mathematics among some twenty countries/districts. But does this really signify success in our mathematics education? Are we complacent about this? I am doubtful of that. I even feel confused, because among the students I have taught only a few show interest in mathematics or perform well, the remaining majority, if not downright abhorrent of mathematics, are indifferent to mathematics. Is it because, in demanding technical ability, we sacrifice other aspects and pay a price for those qualities which cannot be assessed by short-term standard tests? .... From a broad viewpoint of education, this is a big failure. A.N. Whitehead once said in *The Aims of Education* (1929): 'Culture is activity of thought, and receptiveness to beauty and human feeling. Scraps of information have nothing to do with it. A merely well-informed man is the most useless bore on God's earth. What we should aim at producing is men who possess both culture and expert knowledge in some special direction.'" (M.K. Siu, Who needs history of mathematics (in Chinese), Shuxue Tongbao (Mathematics Bulletin), 4 (1987), 42-44.)

In short, students lack a global view of the subject of mathematics and rarely appreciate or enjoy it from a cultural aspect, and too little emphasis is put

---

**'Culture is activity of thought, and receptiveness to beauty and human feeling. Scraps of information have nothing to do with it. A merely well-informed man is the most useless bore on God's earth. What we should aim at producing is men who possess both culture and expert knowledge in some special direction.'**

---

on the subject's own worth. This is rather at variance with the view expressed by great mathematicians since antiquity. Let us just quote that of a great mathematician in our century, Hermann Weyl, who said; "We do not claim for mathematics the prerogative of a Queen of Science, there are other fields which are of the same or even higher importance in education. But mathematics sets the standard of objective truth for all

intellectual endeavors; science and technology bear witness to its practical usefulness. Besides language and music, it is one of the primary manifestations of the free creative power of the human mind, and it is the universal organ for world-understanding through theoretical construction. Mathematics must therefore remain an essential element of the knowledge and abilities which we have to teach, of the culture we have to transmit, to the next generation" The unsatisfactory situation of mathematics teaching in schools seems to persist at the university level, which is further aggravated by the fact that the subject material becomes more abstract and diversified at that level. Students tend to have a feeling that they are on a "sight-seeing" tour of museum exhibits, only they are not particularly interested as museum-goers! Throughout the undergraduate years for the majority of students, if they do form a good opinion of mathematics, it would be that mathematics is a useful tool. But then, for those who do not need to use that tool that much, what worth is there left of the subject? We share the education philosophy that views "the critical and the evaluative function of education as its central contribution to the intellectual and spiritual development of the student" (Abe Shenitzer, "Some thoughts on the teaching of mathematics," *Math. Intelligencer*, 8 (1986), 21-24). Abe Shenitzer goes on to say: "Uninterpreted or underinterpreted technical material .... destroys mind and soul, and teaches cynicism and contempt for teacher and subject alike." (A recent article of Abe Shenitzer, titled "An unorthodox "test" which appears in vol. 99 (January, 1992) of *Amer. Math. Monthly* will give us much food for thought.)

Some may think that as long as students acquire the technical content, it is rather secondary whether students see the cultural value of mathematics or not. We disagree. Looking at the history of our subject we believe that the development of mathematics education and even of mathematics itself is, to a large extent, dictated by the general prevalent "Mathematics Anschauung" of the community at the time and the place. (As an illustration, see a poster display at this Congress: M.K. Siu, *Mathematics Education in Ancient China: What Lesson Do We Learn From It?*) Emphasizing only the usefulness of mathematics at the expense of its cultural aspect leads to a loss of self-esteem for the subject (and for those math majors who study the subject). As a result, the supply of good teachers and good researchers is

dwindling. Furthermore this problem has a snowballing effect, for quite a number of math majors are to become teachers later, in schools and universities. If they lack a general outlook of the subject and fail to realize how the subject is woven into the full tapestry of a general education, then they will produce crops of students who will be exposed to such an outlook and in turn some of these students will become teachers, and so on.

Our teaching aim in mathematics is to let students acquire good study habits and the ability to stand on their own, to teach them to think both logically and heuristically, to give them the room to think, to let them appreciate the beauty and import of mathematics, and more generally to instill in them a

---

**In short, students lack a global view of the subject of mathematics and rarely appreciate or enjoy it from a cultural aspect, and too little emphasis is put on the subject's own worth.**

---

regard for learning. This seems far more important than to make sure they know a hundred theorems. (But if our aim is achieved, they will know those hundred theorems as a by-product!) Actually, the same objective has been stressed in other reports, for instance Standard 6 of developing perspectives in "A Call For Change: Recommendations For the Mathematical Preparation of Teachers of Mathematics MAA Report", Mathematical Association of America (editor, J.R.C. Leitzel), 1991 and the five goals in "Curriculum and Evaluation Standards for School Mathematics", National Council of Teachers of Mathematics, 1988. What we wish to add is that these goals are equally important, if not more, for those math majors who go on to become research mathematicians.

What can be done? We believe attention to a general attitude is more important than the subject-content itself, which is generally agreed upon except for details. We believe the curriculum for math majors, for both prospective research mathematicians or mathematics teacher (these two should not be exclusive of each other) should be the same; for as Klein put it in his 1872 Inaugural

Lecture at Erlanger on mathematics education (not to be confused with his more famous "Erlanger Program" which was on geometry and which was written before his appointment; see D.E. Rowe, Felix Klein's Erlanger Antrittsrede, *Hist. Math.*, 12 (1984), 123-141): "We want the future teacher to stand above his subject, that he have a conception of the present state of knowledge in his field, and that he generally be capable of following its further development. ... Although hardly one in ten will later for himself take up scientific research, anyone who completes such a study even once takes with him an altogether different type of certainty in judgement and liveliness of conception." We call this a "mathematical taste", which corresponds to the item "wisdom" mentioned before. In this respect, the "interface" between school mathematics and university mathematics should be paid attention to in all undergraduate courses whenever possible, in the spirit of Klein's "elementary mathematics from an advanced standpoint". Although skills in mathematics are to be acquired, understanding and a global view should be of primary importance. In

certain instances such awareness cannot be tested easily, but likewise many time-honoured aims in education cannot be tested either; they show up in the growth of personality. We should encourage the doing, the reading, the writing and the talking of mathematics among students. We should pay attention to maintaining a "continuity" in the courses, i.e. what was learnt in a previous course may pop up in subsequent courses, as part of the content, as examples, or even just as remarks in passing. We should incorporate a "sense of history" into all courses to let students appreciate the long process of evolution and of the formation of key ideas and concepts. (We need not stress further the usefulness of mathematics, not because it is not but because enough has been said about that. Ironically, its usefulness, in a sense, attributes to its neglect as an intellectual discipline!) To regain the self-esteem of mathematics (and mathematics majors) we must convince the audience that mathematics is an intellectually rewarding subject which plays a central role in human culture.