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*The Use of History in the Teaching of Mathematics: Theory, Practice, and Evaluation of Effectiveness**

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The use of history of mathematics in mathematics teaching is widely recognized. However, to date, its effectiveness has not been systematically evaluated. An experiment was conducted with the topic on Pythagorean Theorem at Secondary 2 under the framework of "intended–implemented–attained curricula." The duration was three weeks and the effectiveness of the class under experimentation was compared with a parallel one. A teaching capsule was designed to remedy the shortcomings of current practice of teaching the particular topic as found in a pilot research. Pre-assessment and post-assessment of students' attitude towards mathematics, enjoyment of learning in mathematics, learning motivation and mathematics self-concept were conducted at the start and at the end of the course. Their test scores were also collected. In the experimental group, these questionnaires were supplemented by a list of open-ended questions on how well they received the new way of teaching and how well they received the capsule (teaching material). Semi-structured interviews were also conducted on the teacher and six students from the experimental group on their perception of these three weeks of teaching and learning experience.

* Part of the paper reports on the work of the first author in his M.Phil. study at The Chinese University of Hong Kong under the supervision of the third author.

The Expanding Goal of Mathematics Education

To go for "mathematics-for-all," mathematics should no longer be taught in school just as a tool, but should be taught as a subject which possesses several very different goals that reflect the diverse roles mathematics plays in the society. As it is stated in *Reshaping School Mathematics* (National Research Council, 1990), mathematics education plays an expanding role comprising a practical goal, a civic goal, a professional goal and a cultural goal. This can only be achieved if the learners see the relevance of what is learned and engage in either the process of constructing mathematical knowledge by themselves or the study of how others, including our ancestors, construct their mathematics (Fung & Wong, 1998). The process of how mathematics is learned is as important as what mathematical product is learned. Mathematics becomes a part of the learner's asset if we can let the learner appreciate "mathematics-in-its-making" and not just "mathematics-as-an-end-product" (Siu & Siu, 1979).

With such an expanding goal, we should let students shape a broader "Anschauung" of mathematics (Siu, 1995), realize that mathematics is not confined to "calculables" (Wong, Lam, & Wong, 1998) and regard mathematics as an academic discipline as well as a cultural activity. These "cultural goals" are clearly stated in the recently released Hong Kong 2001 school mathematics curriculum, namely, students should be made aware "that mathematics is a dynamic field with its roots in many cultures; of the aesthetic nature of mathematics; the precise and aesthetic nature aspect of mathematics and the role of mathematics in human affairs" (Curriculum Development Council, 1998; Wong, 1997). To gain such a broad perspective of the subject mathematics as a whole, we feel that an acquaintance with the history of mathematics is indispensable. As pointed out in Swetz (1994), history of mathematics can give students "an awareness of tradition, a feeling of belonging, and a sense of participation.... By incorporating some history into teaching mathematics, teaching can lessen its stultifying mystique. Mathematics isn't something magic and forbiddingly alien" (p. 31). A number of reasons and ways in using history of mathematics in the classroom was given in Fauvel (1991). For enhancing the study of the subject itself we can "look for insight and motivation in the illustrious examples from history, thereby gaining an enlightened interpretation" (Siu, 1985). A recent ICMI (International Commission on Mathematical Instruction) Study has as its theme *The Role of the History of Mathematics in the Teaching and Learning of Mathematics*. A book on this study, published in 2000, contains many useful references on all aspects of history of mathematics and mathematics education (Fauvel & van Maanen, 2000).

Effectiveness of Using History of Mathematics in Education

Although there is no dearth of books and papers on the integration of history of mathematics in the teaching of mathematics, not much research seems to have been done on the actual experimentation and evaluation of effectiveness of teaching mathematics with the use of the history of mathematics. McBride and Rollins (1977) reported on such an experiment on college algebra students and found an improvement in the attitude towards mathematics of the students who were exposed to items from the history of mathematics. In this paper we will report on an experiment on Secondary 2 students and discuss the underlying theoretical framework of the experiment.

Furthermore, a pilot study carried out on 360 school teachers in 41 secondary schools in Hong Kong (conducted by the first author) revealed that although the use of history in mathematics teaching is highly regarded by school teachers, they lack the initiative of actually using it. The tight teaching schedule, lack of knowledge and supporting material as well as examination-orientation of the Hong Kong school culture were some of the reasons stated by the teachers for not using it (Lit & Siu, 1998; Lit & Wong, 1999).

Effectiveness: Perspective from Curriculum Evaluation

We are going to propose that a simple experimental-control-groups design by comparing their respective "post-test vs. pre-test differences" is not sophisticated enough to evaluate the effectiveness of using history of mathematics in mathematics instruction (Wong, 1998). There is a need to have a theoretical framework, for which we borrow the intended–implemented–attained curriculum framework used in the IEA Mathematics Studies (Travers & Westbury, 1989). At the intended level, the aims and objectives of incorporating components from the history of mathematics are first identified (what educational issues can be addressed with the use of history) and we evaluate whether these aims are met in the design of the corresponding teaching capsule. At the implemented level, we do not only check whether the intended curriculum (teaching capsule) is implemented according to the original ideas of design, we also evaluate the feasibility of the design. At the attained level, both the process and the outcome will be assessed by means of formative and summative evaluative methods. Cognitive and affective outcomes are to be considered. Both quantitative (grades in conventional tests, questionnaires) and qualitative methods (students' feedback gathered from interviews) are utilized to tap the cognitive and affective changes among the students and the change in their conceptions of mathematics.

Research Report of an Experiment

Situational Appraisal

Large class size (around 40) and an examination-driven curriculum relying on lecturing, drilling and memorization are not uncommon features found in the Hong Kong classroom (Llewellyn, Hancock, Kirst, & Roeloffs, 1982; Morris, 1985, 1988). Prior to the design of the teaching capsule a situational appraisal was performed by interviewing 11 secondary school students. The result revealed that students perceive the "lecture-exercise-lecture-exercise" stereotype as boring. A tight schedule leaves them with no time to think. Cited factors which are conducive to learning include: being able to understand, previous experience of success, a teacher who can make full use of activities and teaching aids to create a lively classroom atmosphere, a teacher who is patient and knows the need of the students, a teacher who makes abstract concepts tangible and gives clear explanation. The result found is quite consistent with that from previous research (Leung, 1995; Wong, 1993, 1996).

The Intended Curriculum

The Pythagorean Theorem for Secondary 2 students was chosen as the topic of experimentation. The existing curriculum material (textbook) was analyzed and an attempt at remedying the shortcomings found were made by using components from history of mathematics. The design was pre-tested in two Secondary 2 classes in October 1996 before the actual implementation. The capsule was revised according to the feedback obtained in the pre-testing. In brief, *worksheets, problems taken from ancient classics*, activities, manipulatives and proofs of the Pythagorean Theorem with various cultural origins as well as relevant stories from history of mathematics were incorporated into the teaching capsule

(samples of them are listed in Appendices 1 to 3). The details of the final design are shown in Table 1.

Experiment Design

The capsule lasting three weeks of class time was experimented in November 1997. Two parallel Secondary 2 classes in the same school were chosen as the experimental group and the control group respectively. The teaching in the experimental group and the control group occupied 14 and 9 lessons of 40 minutes respectively. They were conducted by the same teacher who had a mathematics degree, possessing a professional qualification and having two years of teaching experience. Pre-assessment and post-assessment of students' attitude towards mathematics, enjoyment of learning in mathematics, learning motivation and mathematics self-concept were conducted at the start and at the end of the course. Their test

Table 1. Design of the Capsule

Shortcomings of the existing curriculum material	Remedy taken in the capsule
Inadequate material depicting the cultural aspect of mathematics	1. Introduce the Chinese, Egyptian, Greek and Babylonian origins of the Pythagorean Theorem. 2. Introduce the Chinese "water weed problem" and the Indian "lotus problem." 3. Passages from original classics given to the students for reference.
Rely too much on drilling	Multiple perspectives to a single problem by introducing methods from various cultures.
Rely too much on algebraic treatment	Diagrammatic proofs from ancient Chinese mathematics introduced.
No connection between the Pythagorean Theorem and irrational numbers	A story on the discovery of incommensurable magnitude leading to a crisis in mathematics introduced.
Lack readily available activities	Activities adopted from historical stories introduced, such as making a right-angled triangle from a string with 11 knots (dividing the string in 12 = 3+4+5 equal parts) and deciphering the Babylonian tablet known as Plimpton 322.
Rare use of manipulatives	Let students make a right-angled triangle with the above string and let them prove the Pythagorean Theorem themselves by geometrical dissection.

scores were also collected. In the experimental group, these questionnaires were supplemented by a list of open-ended questions on how well they received the new way of teaching and how well they received the capsule (teaching material). Semi-structured interviews were also conducted on the teacher and six students from the experimental group on their perception of these three weeks of teaching and learning experience. So the mathematics performance (test score), affective factors (questionnaire) and their reactions on the experiment (open-ended questions and interview) were collected via quantitative as well as qualitative methodologies.

Instrument

Minato's *Mathematics Semantic Differential* was used in the questionnaire to tap students' attitude towards mathematics (Minato, 1983). It consists of 14 bipolar statements put in a 6-point response scale. Sample items include "School mathematics is (simple–complicated)," "School mathematics is (beautiful–ugly)." The scale had been translated into Chinese and used in Hong Kong several times, all with reliabilities (Cronbach's alpha) greater than .90. The scale was supplemented by the two subscales of "enjoyment" and "motivation" of Aiken's *Mathematics Attitude Scale* (Aiken, 1974). Each subscale consists of 6 items put in a 5-point scale (strongly disagree, disagree, fairly agree, agree, strongly agree). Sample items of the scales include "I have usually enjoyed studying mathematics in school" and "I want to develop my mathematical skills and study this subject more" for "enjoyment" and "motivation" respectively. Finally, the mathematics self-concept subscale from *Self Description Questionnaire I* (Marsh, 1992), which consists of 8 items tapping student ratings of their ability and their enjoyment/interest in mathematics put in the same 5-point scale, was administered. Sample items include "I enjoy doing work in mathematics."

Result

Responses to the Inventories

Satisfactory reliability indices (Cronbach's alphas) ranging from .71 to .96 were obtained for all the inventories used. The t-test was used to investigate the change of attitude in both the experimental and control groups. The result is listed in Table 2.

Results revealed that the enjoyment among students in the control

Table 2. Differences in Scores of Affective Factors of Experimental and Control Groups

	Pre-assessment			Post-assessment		
	Experimental group	Control group	t-value	Experimental group	Control group	t-value
Attitude	3.24	3.60	-1.82	3.24	3.32	-0.29
Enjoyment	3.00	3.06	-0.38	3.04	2.70	2.02*
Motivation	3.26	3.43	-0.98	3.19	3.15	0.17
Self-concept	2.57	2.80	-1.24	2.88	2.59	1.48

* $p < .05$

group dropped during the instruction of the Pythagorean Theorem whereas that in the experimental group rose slightly. The difference between the two scores in these groups was significant statistically.

Test Scores

The conventional test scores in both groups were also compared and the results are listed in Table 3. Results revealed that the test scores of the experimental group were generally lower than that of the control group, both before and after the treatment. The differences were statistically significant in tests 2 and 3. While the mean score of the control group increased by 3.22 marks, that of the experimental group dropped by 4.56 marks after the treatment.

Open-ended Questions

Open-ended questions administered in the experimental group revealed

Table 3. Differences in Test Scores of Experimental and Control Groups

	Mean		
	Experimental group	Control group	t-value
Test 1: Angles in triangles and polygons	50.24	54.70	75.67
Test 2: Rate, ratio and proportion	38.95	55.73	3.80*
	Treatment		
Test 3: Pythagorean Theorem	34.39	58.95	5.75*

* $p < .001$

that most (26 out of 38) of the students like the new mode of instruction. The main reason given was that the lesson became more lively and enjoyable. One of the reasons given by those who did not like it is that the new way of teaching was even more boring as too many words (in the historical stories) were involved.

Student Interviews

Responses to the student interviews revealed that most of the students judged the new teaching method by whether it could increase the interest of learning and whether it could enhance understanding of the content. So when they reflected on their learning experience they like the new mode of teaching. "Interesting," "enjoyable," "pleasurable," "easier to understand," and "not so boring" were the reasons they offered. In contrast, "boring," "could not understand what the teacher tried to deliver," and "troublesome [clumsy]" were the reasons offered by those who did not like the new teaching method.

For those who did not welcome the capsule, the main objection is the extra reading, which is a sheet of handout of some historical stories. One student reflected that "I simply don't like reading, ... I don't like words." She continued to say that "When everyone concentrated on reading the passage in class, the atmosphere became so boring (as everyone is silent, reading)." Though there was one who loved the passages very much and reflected that the stories provided rich information about the content.

The manipulatives and activities used were most welcomed by the students. Proofs originated from different cultures were also well received. When the students were asked how they felt about the history of mathematics in general, some students just focused their attention on how one could apply the formula to work on problems. This group of students did not see the significance of history of mathematics in their learning though there were some who agreed that it adds liveliness to the learning of mathematics.

Teacher Interviews

The teacher who implemented the curriculum was very supportive to the new mode of instruction. He felt that the capsule provided him with rich information and added liveliness to his teaching. He thought that the students should find interest in the activities, though he suspected that some students could find difficulty in understanding the proofs. Nevertheless, he reflected that the proofs with various cultural origins are more interesting than the standard proof found in the textbook. The teacher showed interest in using a problem (the "water weed problem") as the center of instruction, even in other topics, but he said that we should avoid too many words in introducing such problems. What the teacher found most successful was the handout of historical stories. He said that the reading of such stories kept the class silent and attentive. In general, the teacher showed high regard for the use of history. His sole concern is the provision of good teaching material for teachers to use in the classroom.

Discussion

The present study forms one of the first systematic experimentation and evaluation of the use of history of mathematics in mathematics instruction in the intended-implemented-attained curriculum framework. Though the result is not as promising as one anticipates, it gives us information on how students learn mathematics and how they prefer to do in learning mathematics. It sheds light on how one should design a curriculum (teaching capsule) geared more to the classroom culture and preferences of the students.

At the level of the intended curriculum, a teaching capsule was designed not just to incorporate in it historical components, but designed with the purpose of incorporating such components to address certain shortcomings of the existing curriculum material. At the level of the implemented curriculum, teachers' feedback revealed that such a practice (of inserting historical components) is feasible and manageable. Their reception of the capsule was generally positive. At the level of the attained curriculum, comparison of affective factors revealed that students in the experimental group felt the learning process more enjoyable, though the cognitive outcome of that group did not show improvement. This indicates the possibility of a gap between what is taught and what is assessed. When conventional testing relies heavily on manipulations and paper-and-pencil exercises, drilling and practices remain the most "effective" way of pushing up test scores. Academic performance is definitely not the sole measure of the effectiveness of the use of history in the teaching of mathematics. As pointed out in Fraser, Williamson & Tobin (1987), standard achievement criteria often have been overemphasized at the expense of equally important process criteria. However, it does not necessarily mean that those possessing a lower score are less mathematically competent than their counterparts, especially if we take into account conception formation and authentic problem solving abilities.

Responses to open-ended questions and student interviews revealed more in-depth issues of the question. Apparently, students' responses to the introduction of history of mathematics were dichotomized. Some felt it increased the interest to learning and some said that it made learning even more boring. Student interviews reflected that the sole concern of the students is interest. In other words, students would judge a mode of teaching attractive or not by whether it makes learning stimulating and interesting. Along this line, those students who are scared of too many words would find historical stories boring rather than the other way round. One way to lessen this "language anxiety" is to think of introducing the historical stories orally (by narration) instead. Written texts can be distributed as optional reading material for students to read after class at leisure. This is in line with previous research that person-environment fit (congruence between actual and preferred classroom environment) is the crux of making classroom environments conducive to learning (Fraser, 1986; Wong & Watkins, 1996).

A mismatch between the teacher's and students' responses was also noted. Interest and liveliness were the major concerns of the students. But for the teacher, order and attentiveness come first. This is consistent with previous research. It was found that Hong Kong students preferred a classroom environment which is not boring with classmates engaged in learning, yet quiet with order observed (Wong, 1993, 1996). These anticipations are not conflicting but it requires high teaching skill to achieve that.

Test scores of the students in the experimental group were lower than those in the control group, even though the former group generally found the new mode of teaching more enjoyable. This can be easily explained. Besides the use of conventional testing methods as mentioned above, it could be a long way from cultivation of interest to better performance in solving mathematics problems. Interest is the first step and student needs to spend more time on mathematical task to consolidate their skills in mathematics. However, we believe that once the students are motivated, the rest would come about gradually.

Thus, how successful the introduction of history of mathematics is depends largely on how it is introduced. History of mathematics is not a panacea. We believe that no single teaching method can solve all the

problems in classroom learning, and each of these can only address a part of them. By identifying the strengths and limitations of these teaching methods, we can employ them to complement and supplement each other.

Concluding Remarks

With the implementation of universal education, mathematics education assumes an expanding goal. We should let students actively involve themselves in the experience of doing mathematics and constructing knowledge for themselves. The use of history of mathematics in the teaching of mathematics is a natural solution. Though most teachers value the use of history, it is a problem that teachers do not find enough readily available teaching material. The present paper reports not only the design of such a capsule but the process of its experimentation. Though the time elapsed in the experimental group was longer than that in the control group, the process of development should shed light on how a capsule could be gradually constructed based on situational appraisal of learning difficulties. It also serves as an example of later designs and students' feedbacks provide rich information for improvements. Eventually, the teacher would not only depend on ready-made teaching material but gradually develop the capability of designing curriculum material that serves the purpose of learning best. The teacher should become a thinker, an evaluator as well as a curriculum designer (Siu, Siu, & Wong, 1993; Wong & Su, 1995). In such a case, the development of curriculum concepts through history should be of great help to the teacher in designing the teaching schedule.

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