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You are living in a world of mathematics

by MAN-KEUNG SIU and NAM-KIU TSING

Department of Mathematics, University of Hong Kong, Hong Kong

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This article discusses the making of a slide show to promote the awareness of mathematics among the general public. After stressing the importance of popularization of mathematics, it describes the points the authors paid attention to while making the show. The script of the slide show is included.

1. Introduction

The following script is that of a slide show we prepared ourselves, with the aim of promoting the awareness of mathematics among the general public, and in particular among secondary school children.

All of us will probably agree that mathematics is essential to the advancement of science and technology; it finds useful applications in various disciplines such as physics, chemistry, biology, engineering, economics, management, etc. However, ironic as it may seem, most of the time people are so dazzled by its achievements that they forget about mathematics itself. Allan L. Hammond, of the magazine *Science*, refers to mathematics as 'our invisible culture'. Paul R. Halmos, a noted mathematician, goes even further to say that 'it saddens me that the educated people don't even know that my subject exists'. Certainly that does not mean people do not know that there is something called mathematics, but it does mean that the majority do not understand what mathematics is about. An educated person will know what sculpture, painting,... are without being an artist; will know what symphony, rhythm,... are without being a musician; will know what poems, novels,... are without being a writer; will know what nuclear energy, protein, planet,... are without being a scientist. But how many will know what a function, a postulational system, a commutative group, a manifold,... are, without being a mathematician? Why is it like that?

Probably the long history of mathematics has something to do with it. Even before other sciences began their modern forms, mathematics had already some two thousand years of illustrious accomplishment behind it; only a small portion of this is learnt in school. Even at the university level, while students in other sciences proceed from post-19th-century development to the latest achievement in the 20th century, their knowledge of mathematics stops (more or less) at the beginning of the 19th century! Thus mathematics gradually acquires a language of its own, which sounds foreign and inaccessible to people not in the field. Besides, mathematics is a subject with a cumulative nature; its past is forever assimilated in its present and future. By its nature, mathematics involves abstract thinking, and one must put in the requisite amount of effort and time in order to come to grips with it. Although it may be too much to ask us all to put in that amount of effort and time, it is possible and desirable to let each of us become aware of this human endeavour called mathematics, along with its social impact and its relevance to other human activities.

Here lies the objective of popularization of mathematics. It is desirable to popularize it for two reasons. Firstly, if we are convinced of the import of mathematics, then it follows as a natural consequence that literacy in mathematics of the majority should be elevated; but elevation is built upon popularization. Secondly, ignorance of mathematics breeds indifference to mathematical activities, which in turn has a detrimental effect on the healthy growth of the subject. This is especially marked since a segment of the general public will become policy-making people who exert great influence in running the community, the government, or even the nation. History has time and again informed us that mathematical development is not only closely connected with technological development, but even with progress of the civilization of mankind as a whole. Can we then afford to leave the majority oblivious of this vital human activity called mathematics? With this in mind we went ahead with the production of this slide show.

We decided that the slide show should address itself to a general audience, and not just to those with a fair amount of mathematical knowledge. It had to be comprehensible and appealing, even to laymen and school children. The aim was to show people that mathematics is all around us, affecting our daily lives either directly or indirectly. If possible, we also wanted to rectify certain common misconceptions people have concerning mathematics. Roughly speaking, such misconceptions fall into several dichotomies. Some people think that mathematics is a dry and recondite subject; others that it is merely a fascinating game for exercising the brain. Some think that mathematics is a subject totally unrelated to the real world; others that it is an almighty tool. Some think that mathematics without immediate applicability is but art for art's sake; others that only abstract pure theory can bear the beautiful name of mathematics. Certainly, even to scratch the surface of these problems one will unavoidably be led to ponder the nature of mathematics; that places a demand upon the mathematical maturity of the audience, a demand definitely too strict for the audience we had in mind. We finally decided that for the present project the first aim, to show the effect of mathematics on daily life was primary, while rectifying common misconceptions was only secondary. Since we would face a general audience, we had to make the content of the slide show interesting, the slides attractive, and the commentary appealing and easy to follow. We had to avoid technical explanation as far as possible, even the use of terms that sound technical.

This, however, created another problem, *viz.*, how could we reveal the power of mathematics? If throughout the show no mathematical formula or mathematical symbol appears, the audience would be watching only slides which portray things seen in daily lives, beautiful scenes in nature, or achievements in science and technology; they would thus be distracted from the theme of the show. We solved this in two ways, firstly by adding graphics to certain slides. For example, we added some calculations on the slide about the market place, about banks; we added some mathematics exercises on the slide about a mathematics class; we outlined the triangle in bright colour on the slide about a bridge; we outlined the spiral on the slide about a nautilus shell; we drew hypothetical axes on the slide about radar antennas; we marked the angles on the slide about honeycomb. Secondly, at appropriate places we showed slides with a strong mathematical flavour, for example the cover of a mathematical treatise or a page of mathematical writing. Since we wished to emphasize the relationship between mathematics and our daily lives, we had to include slides depicting scenes familiar to the audience; scenes that they can knit into their daily experience. For instance, we included slides about the market place, about banks and about lotteries (this last also to dispel the myth prevalent among laymen that mathematics is invariably connected with gambling). We also included slides depicting local financial news, local buildings, subway stations, bus stops, and airports. To add to a feeling of intimacy, we actually went into the streets to take these photos, although we knew that better-looking ones on similar scenes could be found in the foreign magazines.

In the 18 months since its production, the slide show has been borrowed by over 40 local schools or educational organizations, including one college of education. We feel that our effort has not been spent in vain. We wish to thank CASTME (Commonwealth Association of Science, Technology and Mathematics Educators) for permission to publish this article, which is a slightly revised version of a longer paper that has won a 1981 CASTME Award.

We now go to the script.

2. 1, 2, 3, ... and beyond

(translated version of the script from Chinese)

From the time we learnt to speak we dealt with numbers. As little boys or girls we all counted with our little fingers, like this—1, 2, 3, ... As we grew up we learnt some more mathematics in schools—1, 2, 3, ... and beyond. But alas, some of us may have developed a distaste for the subject ever since. But wait, think about this—what does mathematics mean to you?

Is mathematics just good for finding out how much a catty of vegetables plus half a dozen of eggs will cost you? Or is it just for reckoning how much will be left of your pay-check after your monthly spending? Is mathematics just good for measuring land? Or is it just for building houses and fly-overs? Is mathematics just good for calculating your chance in winning the lottery? Or is it just for solving brain-twisters for fun? Or yet, is mathematics simply solving equations, computing integrals, or constructing geometical figures? Actually, mathematics embraces much more than all these. Has it ever occurred to you? You are living in a world of mathematics.

For instance, look at the Sun which provides us with light and warmth, the Moon which decorates the night sky, or the Earth on which we live. They are all more or less spherical in shape. Balloons going up in the air, soap-bubbles blown by children, the metal bead at the tip of a ballpoint-pen, and metal balls on a ball-bearing, are also spherical. The different kinds of ball we use in ball-games are spherical, and so are the different kinds of fruits and berries, and dew drops on leaves in the early morning. Or take the eyeballs with which you are watching this slide show now, aren't they spherical as well? And the sphere is but one among thousands and thousands of mathematical objects of investigation.

What else have you noticed? Chemical crystals are in the form of polyhedra with geometric symmetries. The nautilus shell grows into a logarithmic spiral. The sea-star is in the shape of a pentagonal star. The Morning-glory crawls up the hedge in a helix. Nuts of screws are usually made hexagonal. Radar antennas are made parabolic. Collapsable gates consist of many rhombi. Joints in bridges are triangular frames. Most decorative patterns consist of interlocking equilateral triangles, squares or regular hexagons. A reflected beam of light makes an equal angle with the reflecting surface as that of the incident beam. Planets revolve around the sun in elliptical paths. The molecular structure of DNA, which carries the genetic code, is a double helix. A honeycomb is made up of many hexagonal cells. Speaking of honeycomb, we may as well take a closer look at it. Each cell is not a hexagonal prism,

but has a base composing of three congruent rhombi, each with angles equal to 109°28′ and 70°32′. Mathematical computation reveals a fascinating fact; this configuration is most economical—it uses the least material possible. Can you imagine it? The honeybees are such ingenious architects!

However, we human beings are certainly more ingenious than honeybees. We know more mathematics, and we apply it to solve more problems. Our ancestors were already doing just that four or five thousand years ago. Look at this famous 'Wonder of the Ancient World'—the great pyramid, tomb of ancient Pharaohs of Egypt. The largest one was built almost 4800 years ago, and has a base perimeter of nearly 1 km and a height of 146·5 m. It was the highest building in the whole world before the Eiffel Tower of Paris was erected in 1889! Building such a pyramid would have meant the mobilization of tens of thousands of workers, the moving of hundreds of thousands of blocks of stones, and the investment of several decades of time. How much mathematics had to be used in designing, surveying, computing and organizing throughout such an immense project? We can only guess. But luckily we do know quite a lot about ancient Egyptian mathematics, because the Egyptians had recorded their mathematical knowledge on papyri, some of which had been preserved unto this day. This is the famous 'Rhind Papyrus' which now resides in the British Museum in London.

The mathematics of other ancient civilizations are equally fascinating and worthy of our admiration. For instance, this is an ancient mathematical tablet of the Babylonians, this is the famous Chinese mathematical classic 'Jiu Zhang Suan Shu', this is the famous Greek mathematical treatise 'Elements', and this is an ancient Indian mathematical manuscript. People in those days already involved themselves in various mathematical activities such as astronomical observation, calendar making, tax calculation, land surveying, and map drawing, in which they applied the sort of mathematics we now learn in school. They were also aware that some physical phenomena could be understood through mathematics. For instance, Greek mathematicians of the 6th century B.C. discovered the following interesting fact: if strings with lengths in ratio of integral multiples were plucked, they emitted harmonious tones. This problem about a vibrating string once more emerged in the limelight almost 2000 years later, and attracted the attention of eminent mathematicians of the 18th and 19th centuries. By that time, owing to further development in mathematics, the phenomenon was understood much better. The study of this kind of problem calls for mathematics that can deal with dynamic situations. You may have heard of the name 'calculus' in your secondary school days. Calculus is a relatively young subject; it came of age in the mid 17th century—only about 300 years old! With the advent of calculus, we can solve more problems since it can be used to study dynamic situations. For instance, we can calculate the trajectory of cannon-balls. We can calculate the path of a planet, and from thence even predict the presence of planets yet undiscovered. Neptune was discovered with pen and paper in this way in 1845! We can estimate the growth in population. We can find out how to make a cylinder with the largest volume using a fixed amount of material. We can study the differential equations which describe the relationship between electricity and magnetism. As a matter of fact, this is a magnificent triumph of mathematics and physics in the 19th century—the theoretical deduction of the existence of electromagnetic wave, which urged scientists to search for it in the laboratories. After 27 years, scientists found it, and have since applied it to realize the dream of telecomunication. There are many other such examples.

Today, as the civilization of mankind progresses, its reliance on mathematics increases with it. Every day the news programme will include a section on financial news, in which you must have frequently heard terms such as share prices, market indices, and lending rate. If you open the pages of a news magazine, you will frequently come across terms such as gross national product, production curve, and trade balance. Or you may have come across tables and graphs which display some statistics. Although not every one of us will understand thoroughly the meaning behind these terms or figures, we all know that they have a lot to do with our daily lives.

Actually, the use of mathematics in economics is not confined to a few terms like that. A subject called econometrics deals with problems in economics by means of mathematics, thereby helping the planning of the economy. The Nobel Prizes in Economics, awarded in 1973 and 1980, went to scholars who applied mathematics to study economic systems. Living in a city like ours, we all know full well that the local economy is closely tied up with real estate. In our city we can see different kinds of tall buildings on the ground, railway stations underground, and tunnels across the harbour. These structures are erected with the help of mathematics. In other parts of the world, many famous buildings—the World Trade Centre in New York, Yangtze Bridge in Nanking, Space Needle in Seattle, Geodesic Dome in Los Angeles, Finlandia Hall in Helsinki, National Gallery in Washington, D.C.—are built for different uses (and hence of different design), but they are all backed by mathematics. What about the streets that surround such buildings, the highways that connect one district with the next? They also involve mathematics. One must estimate the flow of vehicle traffic and pedestrian traffic, and plan accordingly where to locate interchanges, where to construct fly-overs, how many lanes to set on a highway, where to erect traffic lights, how frequently the traffic lights should change signal. In transportation we also use mathematics to plan the schedules and routes of public vehicles. Or take the airport as an example. During the busiest period, planes come and go at a rate of nearly one flight per half minute. How should the flights be scheduled? How should the delays be estimated? How should the waiting time for take-off be minimized? All these are not just for efficiency, but also for the safety of passengers concerned. The use of mathematics in the overall organization of a huge system is one of the main accomplishments of applied mathematics in recent decades. The subject is called 'Operations Research', 'operations' no doubt reminding you of military affairs. Operations research is indeed a product of the Second World War; when England was under heavy air attack by the Germans, the English had to think of some means to utilize their resources fully to defend their country. A group of mathematicians was asked to apply their expertise to analyse and to plan, so as to make optimal effective use of what the country could provide. Modern military affairs not only call for bravery and wisdom, but technology as well. For instance, weather conditions should be taken into account. In the old days, commanders used to emphasize the importance of 'heaven, earth, and man', but then their knowledge about 'heaven' could hardly match that of ours today. On 6 June 1944, the Allies landed at Normandy, thereby opening up the road to their final victory. It was said that the decision to attack was made after the highest command knew of a short period of relative calm amidst bad weather. The Allies seized upon this opportunity to effect that unprecedented military operation thanks to the fine work of meteorologists in the army. Thirty years after that, weather forecasting has made great progress. Today, besides using instruments for collecting relevant data,

it also invokes mathematics, either to investigate the atmospheric conditions by differential equations, or to analyse the cumulative data by statistics. In both weather forecasting and operations research, the amount of data involved is enormous, and the computation lengthy. Were it not for the invention of electronic computers, the whole thing would become empty talk!

Electronic computers are popularly known as 'electronic brains'. They operate and calculate according to programmed instructions, appearing to think for themselves. Modern computers are capable not only of carrying out instructions, but also of learning, of simple reasoning. Research on this aspect, known as 'artificial intelligence', is going on, making 'electronic brains' a more and more appropriate name. However, there is always someone behind it all. The computers are very clever because we human beings are very clever. Once more it is mathematics that enables computers to work. The advent of computers has a hand in the successful attempts at space exploration. Do you remember the date: 20 July 1969? On that day the human race set foot on the surface of the moon for the first time, marking an important milestone in the technological advance of the present century. Space exploration, besides being a probe into the unknown, besides having the meaning of pioneering effort, also brings along many new products, such as communication satellites, medical technology, coding theory, new alloys, etc. Have you ever thought about it? It requires a lot of mathematics to launch even a tiny satellite. For instance, we must calculate how to put it in its orbit. We must know how to control it during its flight. Remote control at such a far distance requires the mathematical theory of control and communication. The messages it sends back get distorted as it undergoes various kinds of disturbance. To recover the original message is the central topic of coding theory, which relies on different branches of mathematics. Other than that, the control centre on ground is a huge working unit, or perhaps several working units closely co-ordinated. Mathematics is needed in the co-ordination, organization and communication between them.

So you see, mathematics is like that. It is present in our surroundings, affecting our daily lives directly or indirectly. Its past, present, or, we can even predict, its future, will always be fascinating and exciting. From the days when men gazed at the starlit sky tens of thousands of years ago, to this day when our horizon of the universe has expanded so much, mathematics has always done its part in our understanding of the laws of Nature. No wonder a mathematician once said, 'Mathematics is one of the oldest of the sciences; it is also one of the most active, for its strength is the vigour of perpetual youth'.