The Malfatti Problem in nineteenth-century China

CHAN Yip-Cheung
Department of Curriculum and Instruction, The Chinese University of Hong Kong.

SIU Man-Keung
Department of Mathematics, University of Hong Kong.

Towards the latter part of the eighteenth century there was in England a movement in which humanitarianism and evangelism were closely related. A prime missionary society founded in 1795 was the London Missionary Society, which sent the first Protestant missionary, Robert Morrison (1782-1834), to China in 1807. Missionary work in China during the nineteenth century was invariably associated with colonialism, even if the foreign missionaries were "a decidedly mixed lot in every way --- in social and educational background, in temperament and disposition, in attitude to their calling, in approach to their objectives [7, Chapter 17]". In any case, "the missionary movement of the nineteenth century was a complex historical process forming an integral part of the wider world process of 'Europeanization' …. The European sense of superiority, fed by a growing industrial and technological pre-eminence, was to become dominant; attitude of compromise, conciliation or cultural sympathy were to become correspondingly more difficult to maintain. ... the new colonial-imperial age, more direct in the pursuit of its interests, concerned less with cultural reconciliation or adaptation than with command and control, less with the means than with the end. ... that sense of superiority --- cultural and religious as well as scientific and technical superiority --- which was to characterize the general European attitude to Asia in the nineteenth century [7, Chapter 17]". In this article we pay more attention to the more respectable and positive aspect of this movement amidst the understandably unfavourable political situation to religious mission at the time.

Along with spreading Christian faith some missionaries worked hard to propagate Western learning in old imperial China through various means, one of which was publishing periodicals. The periodical *Zhongxi Wenjian Lu* (Record of News in China and West) with English title *Peking Magazine* was founded in August of 1872 by the American missionary William Alexander Parsons Martin (1827-1916) and the English missionary Joseph Edkins (1823-1905), supported by *Guangxue Hui* (Society for the Diffusion of Useful Knowledge) in Beijing [Peking]. At the back of the title-page of the first issue (Figure 1a) the objective in publishing this periodical was clearly stated in a short note (Figure 1b), further elaborated in the ensuing preface. The short note says, "*Zhongxi Wenjian Lu* [*Peking Magazine*] adopts the practice and format of newspapers in the Western world in publishing international news and recent happenings in different countries, as well as essays on astronomy, geography and *gewu* [science, literally meaning "investigating things"]. The magazine will be published once every month. Any gentleman, Chinese or Westerner who gathers new information or has his own views to express, is invited to submit it to the editors at the *Shi Yiyuan* (Charity Hospital) of *Mishi* [*Street*] (Rice Market Street). The editors will select those items that are considered to be fit for print. In this way, new information will be attained through collective effort to benefit more people so as to enable them to become more and more knowledgeable." [15]

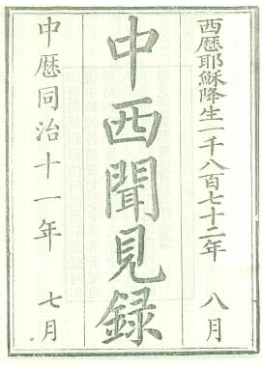


Figure 1a

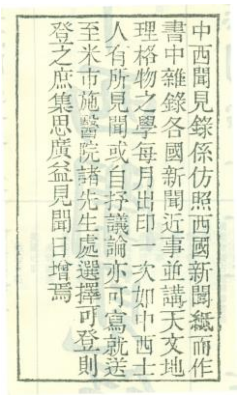


Figure 1b

Upon the closing down of the Society for the Diffusion of Useful Knowledge in August of 1875, the periodical was also terminated after thirty-six issues, to be revived in 1876 in the form of another periodical *Gezhi Huibian* (Compendium for Investigating Things and Extending Knowledge) with English title *The Chinese Scientific and Industrial Magazine*. The latter was founded by another English missionary John Fryer (1839-1928) and the Chinese scholar XU Shou (1818-1884) in Shanghai, and was modelled after the *Peking Magazine*. Of the some three-hundred-and-sixty essays published in the *Peking Magazine*, about forty-six percent dealt with science and technology, including mathematics. Among essays on mathematics, many were contributed by the famed Chinese mathematician LI Shan-lan (1810-1882), who was then heading the *Tianwen Suanxue Guan* (School of Astronomy and Mathematics) in *Tongwen Guan* (School of Combined Learning) [5, Chapter 8]. The establishment of *Tongwen Guan* was at first intended as a language school to train interpreters, but later developed into a college of Western learning. Further colleges of a similar nature also sprouted in other cities like Shanghai, Guangzhou, Fuzhou and Tianjin, along with the establishment of arsenals, shipyards and naval schools during the period known as "Self-strengthening Movement". This was a result of the fervent and urgent desire of the Chinese government to learn from the West, in order to resist the foreign exploitation that the country went through in the first and second Opium Wars [2, 3, 11].

The story we are going to tell, which by itself is only one page of the study of geometry, mirrors the attitude and enthusiasm of Chinese intellectuals of the time in learning fervently from the West when their country was facing a critical period of challenge, change and survival. The historian Immanuel Hsü describes this intent on modernizing China as a "search for a way to survival in the new world that had been forcibly thrust upon China by the West after the mid-nineteenth century". He continues to point out that the Chinese, "burdened by tradition and heritage, and as yet ignorant of the nature of the Western world, groped in the dark" and looked for a way to live with the great change [8, Chapter 1].

A detailed account of the curriculum in the School of Astronomy and Mathematics of *Tongwen Guan* can be found in many sources (for instance [13], or a summary in Section 5 of [3]). Usually one can see more of the curriculum by looking not just at examination papers but other works of the students as well, such as homework assignments, term papers or project reports. In the late nineteenth century such works were labeled as *keyi*, which were sometimes collected into book form by the respective schools, either *Tong Wen Guan* or other private academies. Some of these works were also published in periodicals founded by the foreign missionaries. Often the examination papers and homework in mathematics of students of Li's were included in the *Peking Magazine*. These articles appeared in the form of *nanti* (difficult problems) that invited solutions and responses from readers, which would then be published in

subsequent issues. The posing of problems in a scientific periodical is a rather commonplace practice nowadays, but it was a novel practice in those days in China.

In Issue No. 5 (December 1872) of the *Peking Magazine* there appeared a problem that says:

> "A plane triangle (acute, right or obtuse) contains thee circles of different radii that touch each other. We want to fix the centres of the three circles. What is the method?"

Mathematically speaking we are looking for three non-overlapping circles placed inside a given triangle, each touching two sides of the triangle as well as the other two circles (Figure 2). It is interesting to note (not an easy exercise) that the three radii are determined by the length of the three sides of the given triangle [4, Chapter 3]. In particular, the three kissing circles are not necessarily all mutually different as the problem demanded them to be.

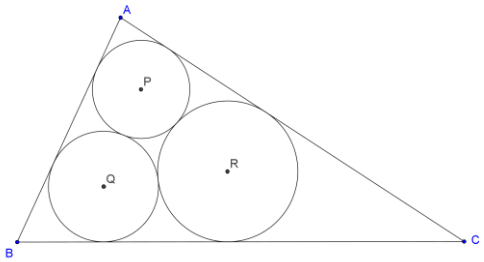
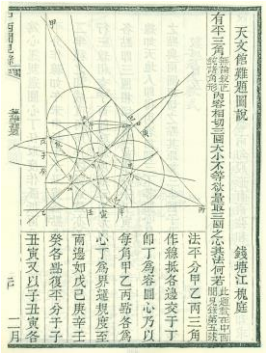


Figure 2

The problem was posed by the School of Astronomy and Mathematics of *Tongwen Guan*, for it was followed by the remark:

> "All students in *Tongwen Guan* retreated from trying this problem. Whoever can solve the problem should send the diagram [of the solution] to the School of Astronomy and Mathematics and would be rewarded with a copy of *Jihe Yuanben* [Chinese translation of Euclid's *Elements*]. The diagram [of the solution] would be published in this magazine so that the author would gain universal fame."

A solution submitted by one reader that was published in Issue No. 8 (March, 1873) was followed by a comment by another reader in Issue No. 12 (July, 1873) together with an acknowledgement of the error and a further comment by the School of Astronomy and Mathematics (Figure 3). This kind of fervent exchange of academic discussion carried on in public domain was a new phenomenon of the time in China [3, Section 5].


天文館難題圖說

錢塘江規庭

Figure 3

This problem became well-known in the Western world after it appeared as a proposed problem in the inaugural volume of *Annales mathématiques pures et appliquées (de Gergonne)* in 1810. In the next volume a reader, "Mr. BIDONE, professor at the Academy of Turin" [Giorgio Bidone (1781-1839)], wrote a letter to the editor to point out that the problem was raised and solved by the Italian mathematician Gian Francesco Malfatti (1731-1807) in 1803 [14]. This problem, originally posed as a problem on optimal area and solved only in the early 1990s, has a very interesting history [1]. Malfatti presumed that the three kissing circles would be those that yield optimal area, not knowing that this is far from being the case. The situation was clarified many years afterwards and the final solution was obtained by V.A. Zalgaller and G.A. Los as late as in 1992. In this article we refer to the Malfatti Problem as that of the three kissing circles.

The first geometric construction by straight-edge and compasses was proposed by the famed Swiss geometer Jakob Steiner (1796-1863) in 1826 without proof, later supplied by the Irish mathematician Andrew Hart in 1856. Readers are invited to read an interesting and perceptive discussion on the dispersion of this famous problem in the Western world in a recent paper by Jemma Lorenat which also provides a detailed bibliography [9]. We cannot ascertain when the Malfatti Problem was first introduced into China. Apparently this problem was introduced by Westerners into China only two to three decades after the problem became well-known in the West, at a time when the Chinese were just beginning to familiarize themselves with Euclidean geometry, which was not part of their traditional mathematics [10]. It is worth noting, from the active discussion generated around this problem, how enthusiastic the Chinese were in learning mathematics from the Westerners in the late nineteenth century

In a book on *keyi* by students of *Longcheng Shuyuan* (Academy of the Dragon City), a private academy famous for its mathematics curriculum, there appeared two articles in 1897 that were on this same three-circle-in-triangle problem, with two different solutions and accompanying remarks by the professor (Figure 4a, 4b) [12, Volume 12].

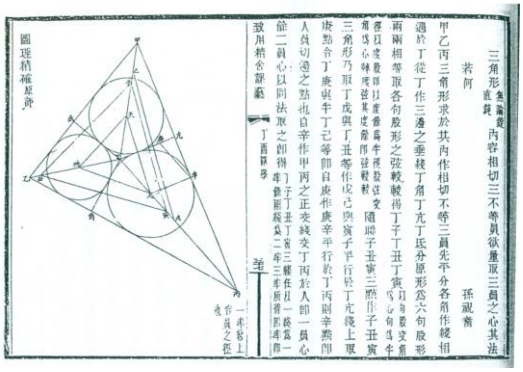


Figure 4a

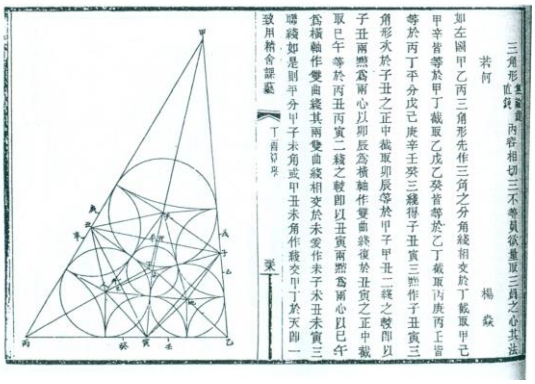


Figure 4b

One solution (Figure 4b) is particularly interesting because it made use of a hyperbola, which is a mathematical object that was totally foreign to Chinese traditional mathematics and was newly introduced in a systematic way only by the mid-nineteenth century. Knowledge on conic sections, particularly on the ellipse, was transmitted into China along with astronomy in the seventeenth century. A more systematic treatment came with the introduction of calculus in the mid-nineteenth century. LI Shan-lan translated a book titled *Yuanzhui Quxian Shuo* (Explanation on Conic Sections) in collaboration with the English missionary Joseph Edkins in 1859. It would be interesting to investigate whether the student independently discovered such a solution on his

own or learnt of it from some other source. We lack the documentary evidence to tell more on this point. [3, Section 5].

Another question of interest has to do with an earlier instance of the discussion of the Malfatti Problem, even before Malfatti did. The problem was in fact posed a bit earlier than Malfatti did by the Japanese mathematician Ajima Chokuyen (also known as Ajima Naonobu, 1732-1798). A related problem that asked for the radius of the inscribed circle of the triangle in terms of the radii of the three kissing circles was proposed by another Japanese mathematician Takatada Shichi on a *sangaku* (mathematical tablet) of the Meiseirinji Temple in Ogaki City, and solved by Kazuhide Omura (1824-1891) in his book *Sanpo Tenzan Tebikigusa* (Algebraic Methods in Geometry) of 1841 [6, Chapters 3, 6, 8]. A question of historical interest would be to study how familiar Chinese mathematicians were with Japanese mathematics at the time, or would they pay no attention at all to *Wasan* (Japanese mathematics) of the Edo period, thinking that *Wasan* was but a "tributary" of Chinese traditional mathematics [3, Section 5]?

REFERENCES

1. Andreatta, M., Bezdek, A, Boroński, J.P. (2010) The problem of Malfatti: Two centuries of debate, *Mathematical Intelligencer*, **33(1)**, 72-76.
2. Biggerstaff, K. (1961) *The Earliest Modern Government School in China,* Cornell University Press, Ithaca.
3. Chan,Y.C., Siu, M.K. (2012) Facing the change and meeting the challenge: mathematics curriculum of *Tongwen Guan* in China in the second half of the nineteenth century, *Zentralblatt für Didaktik der Mathematik – The International Journal of Mathematics Education*, **44(4)**, 461-472.
4. Coolidge, J.L. (1916) *A Treatise on the Circle and the Sphere*, Clarendon Press, Oxford.
5. Elman, B. A. (2005) *On Their Own Terms :Science in China 1550-1900*, Harvard University Press, Cambridge.
6. Fukagawa, H., Rothman, T. (2008) *Sacred Mathematics: Japanese Temple Geometry*, Princeton University Press, Princeton.
7. Harrison, B. (1979) *Waiting For China: The Anglo-Chinese College at Malaca, 1813-1843, and Early Nineteenth-Century Missions*, Hong Kong University Press, Hong Kong.
8. Hsü, C.Y. (1970/1995) *The Rise of Modern China*, 5th edition. Oxford, Oxford University Press.
9. Lorenat, J. (2012) Not set in stone: nineteenth-century geometrical constructions and the Malfatti Problem, *British Society for History of Mathematics Bulletin*, **27(3)** ,169-180.
10. Siu, M.K. (2011) 1607, a year of (some) significance: translation of the first European text in mathematics – *Elements* – into Chinese, in *History and Epistemology in Mathematics Education: Proceedings of the 6th European Summer University*, edited by E. Barbin, M. Kronfellner, C. Tzanakis, Verlag Holzhausen, Vienna, 573-589.
11. Xiong, Y.Z. (1994) *Xixue Dongjian Yu Wanqing Shehui* (The Dissemination of Western Learning and the Late Qing Society) [in Chinese], Shanghai People's Press, Shanghai.
12. *Longcheng Shuyuan Keyi* [*Homework Assignments of the Academy of the Dragon City*] (1901) [in Chinese], edited by R.X Hua, Q.S, Miu, Longcheng Shuyuan, Changzhou; also in: Zhao, S.S., Xue, Z.X. (eds.) (1995) *Zhongguo Lidai Shuyuan Zhi* (Compendium of Historical Records, Documents and Publications of Private Academies in China in the Past Ages) [in Chinese], Jiangsu Educational Press, Nanjing.
13. Zhu, Y.Z. (ed.) (1983) *Zhongguo Jindai Xuezhi Shiliao* (Historical Material on the Education System in Modern China) [in Chinese], East China Normal University Press, Shanghai.

14. *Annales mathématiques pures et appliquées (de Gergonne)*, available on-line in http://www.numdam.org.
15. *Zhongxi Wenjian Lu* [*Peking Magazine*] (1872-1875) [in Chinese], Beijing; thirty-six issues reprinted and collected in four volumes by the Nanjing Antique Bookshop, Nanjing, published in 1992.