The good, the bad and the pleasure (not pressure!) of mathematics competitions

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Introduction

A mathematics competition, as but one among many different kinds of extracurricular activity, should enhance the teaching and learning of mathematics in a positive way rather than present a controversy in a negative way. But why does it sometimes become a controversial issue for some people as to its negative effect? What are the pros and cons of this extracurricular activity known as a mathematics competition?

I cannot claim myself to be actively involved in mathematics competitions, but will attempt to share some of my views of this activity gleaned from the limited experience gained in the past years. The intention is to invite more discussion of the topic from those who are much more experienced and know much more about mathematics competitions[1].

To begin with I like to state clearly which aspect of this activity I will **not** touch on. In the recent decade mathematics competitions have mushroomed into an industry, some of which are connected with profit-making or fame-gaining intention. Even if these activities may have indirect benefit to the learning of mathematics, which I seriously doubt, an academic discussion of the phenomenon is, mathematically speaking, irrelevant. Rather, it is more a topic of discussion for its social and cultural aspect, namely, what makes parents push their children to these competitions and to training centres which are set up to prepare the children for these competitions, sometimes even against the liking of the children? With such a disclaimer let me get back to issues on mathematics competitions that have to do with mathematics and mathematics education.

The "good" of mathematics competitions

The only experience I had of an international mathematics competition occurred in 1988 and in 1994. I helped as a coach when Hong Kong first entered the 29th IMO (International Mathematical Olympiad) in 1988 held in Canberra and I worked as a coordinator to grade the answer scripts of contestants in the 35th IMO in 1994 held in Hong Kong. Through working in these two instances I began to see how the IMO can exert good influence on the educational side, which I overlooked before. I wrote up my reflections on the IMO in an article, from which I now extract the three points on "the good" of mathematics competitions [4, pp.74-76].

[1] This paper is an extended text of an invited talk given at a mathematics education forum held in conjunction with the International Mathematics Competition scheduled on July 24-27, 2012 in Taipei. I thank the organizers, particularly SUN Wen-Hsien of the Chiu Chang Mathematics Education Foundation, for inviting me to give a talk so that I can take the opportunity to organize my thoughts and share them with those who are interested in mathematics competitions.

"(1) All contestants know that clear and logical presentation is a necessary condition for a high score. When I read their answers I had a markedly different feeling from that I have in reading the answer scripts of many of my students. I felt cozy. Even for incomplete or incorrect answers I could still see where the writing is leading me. But in the case of many of my students, despite advice, coaxing, plea, protest (anything short of threat!) on my part, they write down anything that comes to their minds, disconnected, disorganized and perhaps irrelevant pieces. One possible reason for this bad habit is the examination strategy they have adopted since their school days --- write down everything you can remember, for you will score certain marks for certain key points (even if these key points are not necessarily presented in a correct logical order!) and the kind-hearted examiner will take the trouble to sift the wheat from the chaff! Many undergraduates still follow this strategy. Correct or incorrect answer aside, the least we can ask of our students should be clear communication in mathematics (but sadly we cannot).

(2) All contestants know that one can afford to spend up to one and half hour on each problem on the average, and hence nobody expects to solve a problem in a matter of minutes. As a result, most contestants possess the tenacity and the assiduity required of in problem solving. They will not give up easily, but will try all ways and means to probe the problem, to view it from different angles, and to explore through particular examples or experimental data. On the contrary, many of my students, again too much conditioned by examination techniques since their school days, would abandon a problem once they discover that it cannot be disposed of readily by routine means. In an examination when one races against time, this technique may have its excuse. Unfortunately, many students bring the same habit into their daily study. Any problem that cannot be disposed of in 3 minutes is a difficult problem and is beyond one's capacity, hence no time should be wasted in thinking about it! This kind of 'instant learning' is detrimental to the acquirement of true understanding and it kills curiosity, thence along with it the pleasure of study.

(3) Some contestants have the commendable habit of writing down not only the mathematics, but remark on their progress as well. Some would write down that they could not go from that point on, or what they did so far seemed to lead nowhere, or that they decided to try a new approach. I really appreciate the manifestation of this kind of 'academic sincerity'. (It is ironic to note that some leaders or deputy leaders tried to argue that those contestants were almost near to the solution and should therefore be credited with a higher score. They might be, but they did not, and the good intention of the leaders or deputy leaders is like filling in between the lines for the contestants.) On the contrary, some of my students behave in the opposite. They write down the given in the first line (amounting to copying the first part of the question), and write down the conclusion at the end (amounting to copying the final part of the question), then fill in between with disconnected pieces of information which may be relevant or irrelevant, ending with an unfounded assertion "hence we conclude …"! I am deeply disappointed at this kind of insincerity, passing off gibberish as an answer. I would have felt less disappointed if the student does not know the answer at all."

I should add one more point about the "good" of mathematics competitions. A young friend of mine and a member of the Hong Kong 2012 IMO team, Andy Loo, by recounting his own experience since primary school days with mathematics competitions, highlights the essential "good" of mathematics competitions as lying in arousing a passion in the youngster and piquing his or her interest in the subject. The experience of participating in a mathematics competition can exert strong influence on the future career of a youngster, whether he or she chooses to become a research mathematician or not. For those who finally do not benefit from this experience for one reason or another, perhaps it is just an indication that they lack a genuine and sustained passion for the subject of mathematics itself.

The "bad" of mathematics competitions

Despite what has been said in the previous section I have one worry about mathematics competitions which has to do with the way of studying mathematics and doing mathematics, even more so for those who are doing well in mathematics competitions. Those who do well in mathematics competitions tend to develop a liking for solving problems by very clever but sometimes quite *ad hoc* means, but lack the patience to do things in a systematic but hard way or view things in a more global manner. They tend to look for problems that are already well-posed for them and they are not accustomed to dealing with vague situations. Pursuing mathematical research is not just to obtain a prescribed answer but to explore a situation in order to understand it as much as one can. It is far more important to be able to raise a good question than to be able to solve a problem set by somebody else who knows the answer already. One may even change the problem (by imposing more conditions or relaxing the demand) in order to make progress. This is, unfortunately, not what a contestant is allowed to do in a mathematics competition!

Of course, many strong contestants in various mathematics competitions go on to become outstanding mathematicians, but many stay at the level of being good competition problem solvers even if they go on to pursue mathematics. Many leave the field altogether. That is not a problem in itself, because everybody has his or her own aspiration and interest, and there is no need for everybody to become a research mathematician. On the other hand, it would be a pity if they leave the field because they get tired of the subject or acquire a lopsided view of the subject as a result of over-training during the youthful years they spent on mathematics competitions.

Looking at the history of several famed mathematics competitions we see a host of winners in the Eötvös Mathematics Competition of Hungary, started in 1894[2], went on to become eminent mathematicians [5]; we see many medalists in the IMO's, since the event started in 1959, received in their subsequent career various awards for their important contribution to the field of mathematics, including the Fields Medal, Navanlinna Prize, Wolf Prize,… [6]; we see the same happens for many Putnam Fellows in the William Lowell Putnam Mathematical Competition in the USA for undergraduates [8]. On the other hand, the Fields Medalist, Crafoord Prize and Wolf Prize recipient, YAU Shing-Tung, is noted for his public view against mathematics competitions. The eminent Russian mathematician of the last century, Pavel Segeevich Aleksandrov (1896-1982), was reported to have once said that he would not have become the mathematician he was had he joined the Mathematical Olympiad! An explanation of this polarity in opinions is to be sought in the way how one regards this activity known as a mathematics competition from the impression one gets in witnessing how it is run.

When I worked as a coordinator in the 35th IMO in 1994 I noticed that some teams scored rather high marks, but all six contestants in the team worked out the problem in the same way, indicating solid training on the team's part. However, there were some teams, not all of whose members scored as high marks, but each of whom approached the same problem in a different way, indicating a free and active mind that works independently and imaginatively. It made me wonder: will such qualities like independence and imagination be hampered by over-training, and if so, does that mean over-training for mathematics competitions defeats the purpose of this otherwise meaningful activity? Rather than over-training would an extended follow-up investigation of a competition problem enable the youngsters to better appreciate what mathematical exploration is about? I am sure many contestants who go on to become outstanding mathematicians followed this practice of follow-up investigation during the youthful years they spent on mathematics competitions.

I will now illustrate with two examples. The first example is a rather well-known problem in one IMO. We will see how one can view it as more than just a competition problem begging for just an answer. The other example is on a research topic where the main problem is still open to this date (as far as I am

[2] The role played by the journal *Középiskolai Matematikai és Fizikai Lapok* on mathematics and physics for secondary school founded in that same year merits special attention. For more detailed information readers are invited to visit the website of the journal [7].

aware of). We will see how a research problem differs from a problem viewed in the context of a mathematics competition problem.

It was natural that I paid some special attention to the questions set in the 29th IMO, although I did not take part in the actual event in July of 1988. Question 6 of the 29th International Mathematical Olympiad reads:

"Let *a* and *b* be positive integers such that $ab + 1$ divides $a^2 + b^2$. Show that $\frac{a^2+b^2}{ab+1}$ is the square of an integer."

A slick solution to this problem, offered by a Bulgarian youngster (Emanouil Atanassov) who received a special prize for it, starts by supposing that $k = \frac{a^2+b^2}{ab+1}$ is **not** a perfect square and rewriting the expression in the form

$$a^2 - kab + b^2 = k \text{, where k is a given positive integer} \quad (*) .$$

Note that for any integral pair (a, b) satisfying (*) we have $ab \geq 0$, or else $ab \leq -1$, and $a^2 + b^2 = k(ab + 1) \leq 0$, implying that $a = b = 0$ so that $k = 0$! Furthermore, since *k* is not a perfect square, we have $ab > 0$, that is, none of a or b is 0. Let (a, b) be an integral pair satisfying (*) with $a > 0$ (and hence $b > 0$) and $a + b$ *smallest*. We may assume $a > b$. (By symmetry we may assume $a \geq b$. Note that $a \neq b$ or else k is a number lying strictly between 1 and 2!) Regarding (*) as a quadratic equation with a positive root a and another root a', we see that $a + a' = kb$ and $aa' = b^2 - k$. Hence a' is also an integer and (a', b) is an integral pair satisfying (*). Since $a'b > 0$ and $b > 0$, we have $a' > 0$. But

$$a' = \frac{b^2-k}{a} \leq \frac{b^2-1}{a} \leq \frac{a^2-1}{a} < a,$$

so that $a' + b < a + b$, contradicting the choice of (a, b)! This proves that $\frac{a^2+b^2}{ab+1}$ must be the square of an integer. [Having no access to the original answer script I try to reconstruct the proof based on the information provided by a secondary source [1, p. 505]. The underlying key ideas are (i) choice of a minimal solution (a, b), and (ii) the expression (*) viewed in the context of a quadratic equation.]

Slick as the proof is, it also invites a couple of queries. (1) What makes one suspect that $\frac{a^2+b^2}{ab+1}$ is the square of an integer? (2) The argument by *reductio ad absurdum* should hinge crucially upon the condition that *k* is not a perfect square. In the proof this condition seems to have slipped in casually so that one does not see what really goes wrong if k is *not* a perfect square. More pertinently, this proof *by contradiction* has **not explained** why $\frac{a^2+b^2}{ab+1}$ must be a perfect square, even though it **confirms** that it is so.

In contrast let us look at a less elegant solution, which is my own attempt. When I first heard of the problem, I was on a trip in Europe and had a 'false insight' by putting $a = N^3$ and $b = N$ so that

$$a^2 + b^2 = N^2(N^4 + 1) = N^2(ab + 1) .$$

Under the impression that any integral solution (a, b, k) of $k = \frac{a^2+b^2}{ab+1}$ is of the form (N^3, N, N^2) I formulated a strategy of trying to deduce from $a^2 + b^2 = k(ab + 1)$ the equality

$$[a - (3b^2 - 3b + 1)]^2 + [b - 1]^2 = \{k - [2b - 1]\}\{[a - (3b^2 - 3b + 1)][b - 1] + 1\}.$$

Were I able to achieve that, then I could have reduced b in steps of one until I got down to the equation $k = \frac{a^2+1}{a+1}$ for which $a = k = 1$. By reversing steps I would have solved the problem. I tried to carry out this strategy while I was travelling on a train, but to no avail. Upon returning home I could resort to systematic brute-force checking and look for some actual solutions, resulting in a (partial) list shown below.

a	**1**	**8**	**27**	30	**64**	112	**125**	**216**	240	**343**	418	**512**	…
b	**1**	**2**	**3**	8	**4**	30	**5**	**6**	27	**7**	112	**8**	…
k	**1**	**4**	**9**	4	**16**	4	**25**	**36**	9	**49**	4	**64**	…

Then I saw that my ill-fated strategy was doomed to failure, because there are solutions other than those of the form (N^3, N, N^2). However, not all was lost. When I stared at the pattern, I noticed that for a fixed k, the solutions could be obtained recursively as (a_i, b_i, k_i) with

$$a_{i+1} = a_i k_i - b_i,\ b_{i+1} = a_i,\ k_{i+1} = k_i = k.$$

It remained to carry out the verification. Once that was done, all became clear. There is a set of 'basic solutions' of the form (N^3, N, N^2) where $N \in \{1,2,3,\dots\}$. All other solutions are obtained from a 'basic solution' recursively as described above. In particular, $k = \frac{a^2+b^2}{ab+1}$ is the square of an integer. I feel that I understand the phenomenon much more than if I just learn from reading the slick proof.

[Thanks to Peter Shiu we can turn the indirect proof into a more transparent direct proof based on the same key ideas. Proceed as before and set $c = \min(a, b)$ and $d = \max(a, b)$. Look at the quadratic equation

$$x^2 - kcx + (c^2 - k) = 0\,,$$

for which d is a positive root with another root d'. Since $d + d' = kc$ and $dd' = c^2 - k < c^2 \leq dc$, we know that d' is an integer less than c so that $d' + c < 2c \leq a + b$. By the choice of (a, b), d' cannot be positive. On the other hand,

$$(d+1)(d'+1) = dd' + (d + d') + 1 = (c^2 - k) + kc + 1 = c^2 + (c-1)k + 1 \geq c^2 + 1 > 0\,.$$

Therefore, $d' + 1 > 0$, implying that $d' = 0$. Hence, $k = c^2 - dd' = c^2$ is the square of an integer.]

The next example is a research problem on the so-called Barker sequence, which is a binary sequence of length s for which the sequence and each off-phase shift of itself differ by at most one place of coinciding entries and non-coinciding entries in their overlapping part. Technically speaking we say that the sequence has its aperiodic autocorrelation function having absolute value 0 or 1 at all off-phase values. For instance, 11101 is a Barker sequence of length 5, while 111010 of length 6 is not a Barker sequence. Neither is 11101011 of length 8 a Barker sequence, but 11100010010 is a Barker sequence of length 11. For application in group synchronization digital system in communication science R.H. Barker first introduced the notion in 1953. Such sequences for s equal to 2,3,4,5,7,11,13 were soon discovered and in 1961 R. Turyn and J. Storer proved that there is no Barker sequence of *odd* length s larger than 13. A well-known conjecture in combinatorial designs says that there is no Barker sequence of length s larger than 13, which has withstood the effort of many able mathematicians for more than half a century. Although the conjecture itself remains open, it has stimulated much research in combinatorial designs and in the design of sequences and arrays in communication science. In order to better understand the original problem researchers change the problem and look at the 2-dimensional analogue of arrays or even analogues in higher dimensions, or other variations such as non-binary sequences and arrays over an alphabet set with more than two elements, or instead of a single sequence a pair of sequences (known as Golay complementary sequence pair) satisfying some suitably formulated modification on their aperiodic autocorrelation functions. In this sense the problem, instead of looking like an interesting piece of curio, opens up new fields and generates new methods and techniques which prove useful elsewhere. (I would recommend an excellent survey paper to those who wish to know more about this topic [2]).

School mathematics and "Olympiad mathematics"

Since many mathematics competitions aim at testing the contestants' ability in problem solving rather than their acquaintance with specific subject content knowledge, the problems are set in some general areas which can be made comprehensible to youngsters of that age group, independent of different school syllabi in different countries and regions. That would cover topics in elementary number theory, algebra, combinatorics, sequences, inequalities, functional equations, plane and solid geometry and the like. Gradually the term "Olympiad mathematics" is coined to refer to this conglomeration of topics. One question that I usually ponder over is this: why can't this type of so-called "Olympiad mathematics" be made good use of in the classroom of school mathematics as well? If one aim of mathematics education is to let students know what the subject is about and to arouse their interest in it, then interesting non-routine problems should be able to play their part well when used to supplement the day-to-day teaching and learning. In the preface to *Alice in Numberland: A Students' Guide to the Enjoyment of Higher Mathematics* (1988) the authors, John Baylis and Rod Haggarty remark, "The professional mathematician will be familiar with the idea that entertainment and serious intent are not incompatible: the problem for us is to ensure that our readers will enjoy the entertainment but not miss the mathematical point, […]"

By making use of "Olympiad mathematics" in the classroom I do not mean transplanting the competition problems directly there. Rather, I mean making use of the kind of topics, the spirit and the way the question is designed and formulated, even if the confine is to be within the official syllabus. The so-called "higher-order thinking" is (and should be) one of the objectives in school mathematics as well. Sometimes we may have underestimated the capability and the interest of our students in the classroom. It is not true that they only like routine (and hence usually regarded as "easy"?) material. Perhaps they lack the motivation to learn because they find the diluted content dull and are tired of it. Besides, good questions not just benefit the learners in the classroom; it is also a challenging task for the teachers to design good questions, thereby upgrading themselves in the process. In this respect mathematics competitions can benefit teachers as well, if they try to make use of the competition problems to enrich the learning experience of their students. To be able to better reap such benefit carefully designed seminars for teachers and suitably prepared didactical material will be helpful [3, pp. 1596-1597].

There is a well-known anecdote about the famous mathematician John von Neumann (1903-1957). A friend of von Neumann once gave him a problem to solve. Two cyclists A and B at a distance 20 miles apart were approaching each other, each going at a speed of 10 miles per hour. A bee flew back and forth between A and B at a speed of 15 miles per hour, starting with A and back to A after meeting B, then back to B after meeting A, and so on. By the time the two cyclists met, how far had the bee travelled? In a flash von Neumann gave the answer --- 15 miles. His friend responded by saying that von Neumann must have already known the trick so that he gave the answer so fast. His friend had in mind the slick solution to this quickie, namely, that the cyclists met after one hour so that within that one hour the bee had travelled 15 miles. To his friend's astonishment von Neumann said that he knew no trick but simply summed an infinite series! (I leave it as an exercise for you to find the answer by summing an infinite series.)

For me this anecdote is very instructive. For one thing, it tells me that different people may have different ways to go about solving a mathematical problem. There is no point in forcing everybody to solve it in just the same way you solve it. Likewise, there is no point in forcing everybody to learn mathematics in just the same way you learn it. This point dawned on me quite late in my teaching career. For a long time I thought a geometric explanation would make my class understand linear algebra in the easiest way, so I emphasized the geometric viewpoint along with an analytic explanation. I still continue to do that in class to this date, but it did occur to me one day that some students may prefer an analytic explanation because they have difficulty with geometric visualization. To von Neumann, who could carry out mental calculation with lightning speed, maybe an infinite series was the first thing that came up in his mind rather than the time spent by the cyclists meeting each other!

Secondly, both methods of solution have their separate merits. The method of first calculating when the cyclists met is slick and captures the key point of the problem, killing it in one quick and direct shot. The other method of summing an infinite series, which is slower (but not for von Neumann!) and is seemingly more cumbersome and not as clever, goes about solving the problem in a systematic manner, resorting even to brute force. It indicates patience, determination, down-to-earth approach and meticulous care. Besides, it can help to consolidate some basic skills and nurture in a student a good working habit.

It makes me think that there are two approaches in doing mathematics. To give a military analogue one is like positional warfare and the other guerrilla warfare. The first approach, which has been going on in the classrooms of most schools and universities, is to present the subject in a systematically organized and carefully designed format supplemented with exercises and problems. The other approach, which goes on more predominantly in the training for mathematics competitions, is to confront students with various kinds of problems and train them to look for points of attack, thereby accumulating a host of tricks and strategies. Each approach has its separate merit and they supplement and complement each other. Just as in positional warfare flexibility and spontaneity are called for, while in guerrilla warfare careful prior preparation and groundwork are needed, in the teaching and learning of mathematics we should not just teach tricks and strategies to solve special type of problems or just spend time on explaining the general theory and working on problems that are amenable to routine means. We should let the two approaches supplement and complement each other in our classrooms. In the biography of the famous Chinese general and national hero of the Southern Song Dynasty, Yue Fei (岳飛 1103-1142) we find the description: “陣而後戰，兵法之常。運用之妙，存乎一心。 (Setting up the battle formation is the routine of art of war. Manoeuvring the battle formation skillfully rest solely with the mind.)”

Sometimes the first approach may look quite plain and dull, compared with the excitement acquired from solving competition problems by the second approach. However, we should not overlook the significance of this seemingly bland approach, which can cover more general situations and turns out to be much more powerful than an *ad hoc* method which, slick as it is, solves only a special case. Of course, it is true that frequently a clever *ad hoc* method can develop into a powerful general method or can become a part of a larger picture. A classic case in point is the development of calculus in history. In ancient time, only masters in mathematics could calculate the area and volume of certain geometric figures, to name just a couple of them, Archimedes (c. 287 B.C.E. – c. 212 B.C. E.) and Liu Hui (劉徽 3rd century). Today we admire their ingenuity when we look at their clever solutions, but at the same time feel that it is rather beyond the capacity of an average student to do so. With the development of calculus since the seventeenth century and the eighteenth century, today even an average school pupil who has learnt the subject will have no problem in calculating the area of many geometric figures.

Let me further illustrate with one example, which is a competition problem posed to me by the father of a contestant. In isosceles ΔABC, where $AB = AC$ and the measure of $\angle BAC$ being $20°$, D is taken on the side AC such that $AD = BC$. Find θ, the measure of $\angle ADB$ (see Figure 1).

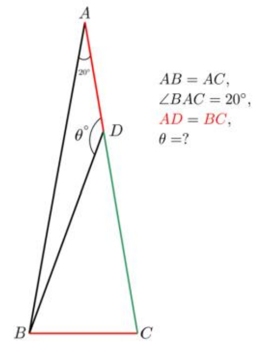

Figure 1

Clearly, if one is to employ the law of sines, then the answer can be readily obtained in a routine manner, namely,

$$\frac{AD}{\sin(\alpha+\theta)} = \frac{AB}{\sin\theta} \ , AD = BC = 2AB\sin(\alpha/2),$$

where α is the measure of $\angle BAC$, thereby arriving at

$$\tan\theta = \frac{\sin\alpha}{2\sin(\alpha/2) - \cos\alpha}\,.$$

When $\alpha = 20°, \theta = 150°$. However, the problem appeared in a primary school mathematics competition in which the contestant was not expected to possess the knowledge of the law of sines! Is there a way to avoid the use of this heavy machinery (for a primary school pupil)? I hit upon a solution by constructing an equilateral ΔFBC with F inside the given ΔABC. Pick a point E on AB such that $AE = CD$ (see Figure 2).

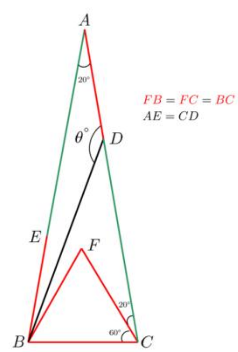

Figure 2

Then it is not hard (by constructing DE, DF) to find out that the measure of $\angle DBE$ is $10°$ (Exercise) so that $\theta = 180° - 20° - 10° = 150°$. Why would I throw in the equilateral ΔFBC as if by magic? It is because I had come across a similar-looking problem before : In an isosceles ΔABC, where $AB = AC$ and the measure of $\angle BAC$ being $20°$, points D and E are taken on AC, AB respectively such that the measure of $\angle DBC$ is $70°$ and that of $\angle ECB$ is $50°$; find φ, the measure of $\angle BDE$ (see Figure 3).

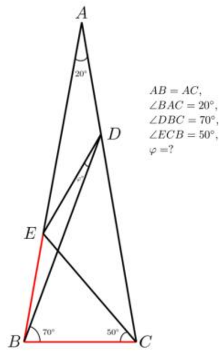

Figure 3

By constructing an equilateral ΔFBC with F inside the given ΔABC we can arrive at the answer $\varphi = 10°$ (Exercise). These two versions are indeed the description of the same situation, because it can be proved that $AD = BC$ in the second problem. Only knowledge of congruence triangles suffices. No knowledge of trigonometry is required. However, if the measure of $\angle DBC$ and that of $\angle ECB$ are not $70°$ and $50°$ respectively, then the geometric proof completely breaks down! But we can still compute the measure of $\angle BDE$ by employing the law of sines, which is within what an average pupil learns in school. It has to be admitted that the method is routine and not as elegant, but it covers the general case and can be handled by an average pupil who has acquired that piece of knowledge.

The pleasure (not pressure!) of mathematics competitions

Before working as a coordinator for the 35th IMO I harboured a distrust of the value of mathematics competitions. I still harbour this distrust to some extent, all the more when I witnessed during coordination of the IMO in 1994 how some leaders or deputy leaders over-reacted out of too much concern for winning high scores. Putting strong emphasis on winning/losing will inculcate in the youngsters an unhealthy attitude towards the whole activity. Attaching undue importance to the competition by organizers, teachers, parents, students, is one main source that may cause distortion of the good intention of mathematics competitions, not to mention the more “commercial” consequences that take advantage of this misplaced emphasis. Not only it fails to bring about the ideal outcome of fostering genuine intrinsic interest and enthusiasm in the subject, it takes the fun and meaning out of a truly extracurricular activity as well. Instead of pleasure we are imposing pressure on the youngsters.

Furthermore, the unilateral strengthening of ability to attain high score on these so-called “Olympiad mathematics” problems may have adverse effect on the overall growth of a youngster, not just in terms of academic pursuit in other disciplines (or in mathematics itself!) but even in terms of personal development. In particular, I am disappointed at not finding how mathematics competitions breathe life into a general mathematics culture in the local scene. On the contrary, many people may be misled into believing that those difficult “Olympiad mathematics” problems present the high point in mathematics, and that mathematics is therefore too difficult to lie within reach of an average person.

Concluding remark

On the whole I have great admiration for the talent of those youngsters who take part in a mathematics competition. What little I accomplish in trying out those competition problems with all my might they accomplish at a stroke, and explain it in a clear and lucid manner. I also have great respect for the dedication and enthusiasm of those organizers who believe in the value of a healthy mathematics competition. They are serving the mathematical community in many ways.

One predominant objection to mathematics competition is the requirement to work out the problems within a fixed time span, say three to four hours. Some regard this as an act to undermine the intellectual and intrinsic pleasure of doing mathematics. In a comprehensive paper on mathematics competitions and mathematics education Petar Kenderov points out how this requirement disadvantages those creative youngsters who are "slow workers". Along with it he points out some important features which are not encouraged in a traditional mathematics competition but which are essential for doing good work in mathematics. These include "the ability to formulate questions and pose problems, to generate, evaluate, and reject conjectures, to come up with new and non-standard ideas". Moreover, he points out that all such activities "require ample thinking time, access to information sources in libraries or the Internet, communication with peers and experts working on similar problems, none of which are allowed in traditional competitions." [3, p.1592]

My good friend, Tony Gardiner, who is known for his rich experience in mathematics competitions and had served as the leader of the British IMO team four times, after reading my article in 1995 [4] commented that I should not blame the negative aspects on the mathematics competition itself. He went on to enlighten me on one point, namely, a mathematics competition should be seen as just the tip of a very large, more interesting, iceberg, for it should provide an incentive for each country to establish a pyramid of activities for masses of interested students. It would be to the benefit of all to think about what other activities besides mathematics competitions can be organized to go along with it. These may include the setting up of a mathematics club or publishing a magazine to let interested youngsters share their enthusiasm and their ideas, organizing a problem session, holding contests in doing projects at various levels and to various depth, writing book reports and essays, producing cartoons, videos, softwares, toys, games, puzzles, … . I wish more people will see mathematics competitions in this light, in which case the negative impression, which I might have conveyed in this paper, will no longer linger on!

The good, the bad and the pleasure of mathematics competitions
Are to which we should pay our attention.
Benefit from the good; avoid the bad;
And soak in the pleasure.
Then we will find for ourselves satisfaction!

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