Mathematical experience in school

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My early learning experience in mathematics in school does not leave with me a strong impression. The only vivid memory is perhaps the reciting of the multiplication table in Primary 2 (between 6-year-old and 7-year-old), mainly because I lay snugly in the arms of my mother while reciting it.

When I came to Primary 5 (in 1954) classes adopted English as a teaching medium (which is not my mother tongue). Basically I did not learn anything new in mathematics except learning terms in English. Lessons in mathematics were even less exciting than before. I was not really interested in the textbook problems on the number of innings in cricket, a game that was totally foreign to me, or in calculating a sum in pounds, shillings and pence, having to memorize that twenty-one shillings made a guinea! (Hong Kong was then a British colony, but in Hong Kong the currency is in dollars and cents.) Again, the only vivid memory is something unrelated to mathematics. I was attracted by a textbook problem on cakes, scones and pastries mainly because there was a picture of the delicacies in the textbook, but I tasted a real scone only some twenty-five years later while travelling in the UK!

Then I entered secondary school. (Again, the teaching medium was English, which is not my mother tongue). Mathematics still did not make a deep impression upon me in the first two years. On the contrary I was rather apprehensive of lessons in arithmetic because the teacher usually spent the first ten minutes of class to test us on mental arithmetic. Not that I could not do the sums, but I could not catch the question in English fast enough. By the time I got it, the teacher was giving the next question. So I got almost a blank answer sheet each time, and that made me rather unhappy. Fortunately, that teacher was only there for a month on teaching practice; there was no more mental arithmetic test after she left.

The situation improved tremendously when I got to Form 3, and came across synthetic geometry, more or less in the classical fashion of Euclid's *Elements* (using the textbook *Essentials of Geometry* by A.B. Mayne). I was then a 13-year-old. What struck me was the fascination in arriving at an explanation of a result that I could see (and conjecture) from drawing a figure (or many such figures) but could not at first understand why it was so. The logical deduction was so clear-cut and lucid. The first example that gave me this strong impression was the result that the three medians of a triangle are concurrent. By drawing many such figures I saw the astonishing fact but did not see why it was so. Then a proof came, bingo! It was particularly impressive at the time because we were then learning centre of mass in physics lesson. Explaining via this physical aspect gave the same result. This integration of mathematics and physics was an "intellectual impact" for me.

The topics I loved most at the time (from Form 3 to Form 5) were geometric constructions (by straight-edge and compasses) and loci. One problem is still vivid in my mind. The problem is to construct an equilateral triangle with its vertices on three given parallel lines respectively. I spent quite some time on the problem. After doing that I asked myself what happens when the three given lines are concurrent instead of being parallel, or when they become three concentric circles, or more generally are just three straight lines. I do not remember solving them all (maybe I still would not be able to solve them all today), but I remember I had a lot of enjoyment during the working. Too bad I had no access to dynamic geometric softwares (such as Cabri) in those days (the late1950s), or else I would spend even more time on geometric constructions.

Although we began to learn algebra in Form 1, the teaching at the time focused too much on the technical part that I missed the importance of the rationale in learning such techniques (in order to make use of them in solving equations later) that I did not like the subject. Again, geometry came to the rescue. In Form 3 we learnt graphs in connection with solving equations — a rudimentary version of coordinate geometry, which we did not have until we got to Upper Six — in which I could see an interplay between geometry and algebra. I still remember vividly the one problem in the textbook (*Essentials of Algebra* by A.B. Mayne): "Draw the graphs of $y = x^2$ and $5y = 6x + 4$ on the same diagram for values of x from -2 to 3. From the graphs solve $5x^2 = 6x + 4$. Also find out roughly from the graphs, by drawing the appropriate parallel line, for what value of a the equation $5x^2 = 6x + a$ will have equal roots."

Another incident in the learning of algebra that offered me a revelation was when I came to the Remainder Theorem. By carrying out tedious long division of various polynomials by $X - c$ and looking at the answers I could guess that the remainder was obtained from the original polynomial by plugging in c for X, but I found it tedious to write out the full detail in general. Then my teacher taught me to write the division as $P(X) = Q(X)(X - c) + R$ and showed me what happened if one plugged in c for X. I was struck by the power of abstraction, so to speak.

I learnt by myself coordinate geometry and calculus in Form 4, before the subjects were taught in Upper Six. Because I also loved working on problems in geometric constructions, I vaguely formed an idea that whatever quantity can be constructed one can calculate it from certain initially given quantities. Many years later when I studied the topic of geometric constructions and field extensions in abstract algebra as an undergraduate I saw the same idea, and I felt as if I met an old friend! That was another revealing incident in my learning experience in mathematics in school.

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