When “Mr. Ou (Euclid)” came to China …

SIU Man Keung, Department of Mathematics, University of Hong Kong

ABSTRACT

The Italian Jesuit Matteo Ricci and the Chinese scholar-official XU Guang-qi of the Ming Dynasty collaborated to produce a translation of the first six books of *Elements* (more precisely, the fifteen-book-version *Euclidis Elementorum Libri XV* compiled by Christopher Clavius in the latter part of the sixteenth century) in Chinese in 1607, with the title *Ji He Yuan Ben* [Source of Quantity]. This paper attempts to look at the historical context that made *Elements* the first European text in mathematics to be translated into Chinese, and how the translated text was received at the time as well as what influence the translated text exerted in various domains in subsequent years, if any, up to the first part of the twentieth century. This first European text in mathematics transmitted into China in the Ming Dynasty led the way of the first wave of transmission of European science into China, while a second wave and a third wave followed in the Qing Dynasty, but each in a rather different historical context. Besides comparing the styles and emphases of mathematical pursuit in the Eastern and the Western traditions we try to look at the issue embedded in a wider intellectual and cultural context.

1. Introduction

In early-twentieth century the revolutionary socialist intellectual CHEN Du-xiu (陳獨秀 1879-1942), later to become a co-founder of the Chinese Communist Party, introduced to the Chinese a “Mr.De (德先生) [Democracy]” and a “Mr.Sai (賽先生) [Science]”. Actually, about three centuries earlier another foreigner “Mr.Ou (歐先生) [Euclid]” came to China, the influence of whom, in some sense, paved the way for the latter pair. This paper tells this story.

In a symposium on the quatercentenary of the Chinese translation of *Elements* held at the Institute of Mathematics of Academia Sinica in November of 2007 I gave a talk with the title “Mr. Ou (Euclid) in China for four hundred years” (Siu, 2007). Three years later I gave a public lecture in the Hong Kong University with the title “1607, a year of (some) significance: Translation of the first European text in mathematics — *Elements* — into Chinese” (Siu, 2011). The title of that talk is inspired by that of a well-received book by the historian Ray Huang (黃仁宇 1918-2000). Huang’s book *1587, A Year of No Significance* was translated into Chinese soon after its publication and was given a more informative but perhaps less pithy title *Wanli Shiwu Nian* [萬曆十五年 In the Fifteenth Year of the Reign of Emperor Wanli] (Huang, 1981). Huang begins his book with the passage: “Really, nothing of great significance happened in 1587, the year of the Pig. […] Let me begin my account with what happened on March 2, 1587, an ordinary working day.” His intention is to give an account of history from a “macrohistory” viewpoint, which he further exemplifies in a subsequent book *China: A Macro History* (Huang, 1988). The purpose is to give an analysis of events that occurred in a long span of time, viewed from a long distance with a broad perspective. In this respect events, some of which might not reveal its true significance when it initially happened, cumulated in time to produce long-term effects. It is in a similar vein that I try to tell the story of the event that occurred in 1607 depicted in the title.

Clearly, this paper will be based on those two previous talks with some modification. We will look at the historical context that made Euclid's *Elements* the first European text in mathematics to be translated into Chinese, and how the translated text was received at the time as well as what influence the translated text exerted in various domains in subsequent years, if any, up to the first part of the twentieth century.

2. The Beginning

The story started with the "era of exploration" when Europeans found a way to go to the East via sea route. Various groups took the path for various reasons, among them the missionaries. As a byproduct of the evangelical efforts of the missionaries an important page of intellectual and cultural encounter between two great civilizations unfolded in history. From around 1570 to 1650 the most prominent group of missionaries that came to spread Christian faith in China were the Jesuits sent by the Society of Jesus, which was founded by Ignatius of Loyola (1491-1556) in 1540. Of the many Jesuits this paper focuses attention on only one, Matteo Ricci (利瑪竇 1552-1610), and of the many contributions of Ricci in the transmission of Western learning into China this paper focuses attention on only one, his collaboration with XU Guang-qi (徐光啟 1562-1633) in translating Euclid's *Elements* into Chinese.

On August 7, 1582 Ricci arrived in Macau, which was a trading colony in China set up by the Portugese with the consent of the Ming Court in 1557. Macau is the first as well as the last European colony in East Asia, being returned to Chinese sovereignty as a Special Administrative Region of China in 1999, four hundred and forty-two years later. Together with its neighbouring city of Hong Kong, which became a British colony in 1842 and was returned to Chinese sovereignty in 1997, the two places played an important role in the history of the rise of modern China in a rather subtle way.

Ricci studied at St. Paul's College in Macau before proceeding to mainland China and finally reached Peking (Beijing) in January of 1601. He became the most prominent Catholic missionary in China. When he passed away on May 11, 1610, he was the first non-Chinese that was granted the right to be buried on Chinese soil, an indication of the high esteem he was held in at the time.

Ricci learnt mathematics from Christopher Clavius (1538-1612) at Collegio Romano in Rome where he studied from September 1572 to May 1578 before being sent to the East for missionary work. The translation of *Elements* was based on the version compiled by Clavius in 1574 (with subsequent editions), a fifteen-book edition titled *Euclidis Elementorum Libri XV* (Clavius, 1574). Ricci left with us a very interesting and informative account of his life and missionary work in China in the form of a journal that was prepared for publication by a contemporary Jesuit Nicolas Trigault (1577-1628) in 1615 (Ricci, 1953). Let us quote a few passages from this journal of Ricci's regarding the translation of *Elements*.

"[…] Whoever may think that ethics, physics and mathematics are not important in the work of the Church, is unacquainted with the taste of the Chinese, who are slow to take a salutary spiritual potion, unless it be seasoned with an intellectual flavouring. […] All this, what we have recounted relative to a knowledge of science, served as seed for a future harvest, and also as a foundation for the nascent Church in China. […] but nothing pleased the Chinese as much as the volume on the *Elements* of Euclid. This perhaps was due to the fact that no people esteem mathematics as highly as the Chinese, despite their method of teaching, in which they propose all kinds of propositions but without demonstrations." (Ricci, 1953) Is that really the case? In Section 4 we will come back to this point.

To Ricci, who studied mathematics under Clavius, the treatise *Elements*, compiled by Euclid (c.325-265 B.C.E.) in the early third century B.C.E., was the basis of any mathematical study. He therefore suggested to his Chinese friend XU Guang-qi that *Elements* should be the first mathematical text to be translated. XU set himself to work very hard on this project. He went to listen to Ricci's exposition of *Elements* every day in the afternoon (since he could not read Latin, while Ricci was well versed in Chinese) and studied laboriously, and at night he wrote out in Chinese everything he had learnt by day. We are told according to an account by Ricci: "When he [XU Guang-qi] began to understand the subtlety and solidity of the book, he took such a liking to it that he could not speak of any other subject with his fellow scholars, and he worked day and night to translate it in a clear, firm and elegant style. […] Thus he succeeded in reaching the end of the first six books which are the most necessary and, whilst studying them, he mingled with them other questions in mathematics. […] He would have wished to continue to the end of the Geometry; but the Father [Matteo Ricci] being desirous of devoting his time to more properly religious matters and to rein him in a bit told him to wait until they had seen from experience how the Chinese scholars received these first books, before translating the others." (Bernard, 1935) The translated text was published in 1607 and was given the title *Ji He Yuan Ben*[1] [幾何原本 Source of Quantity]. In the preface Ricci said, "[……] but I said: "No, let us first circulate this in order that those with an interest make themselves familiar with it. If, indeed, it proves of some value, then we can always translate the rest." Thereupon he [XU Guang-qi] said, "Alright. If this book indeed is of use, it does not necessary have to be completed by us." Thus, we stopped our translation and published it, […]" (Xu, 1984).

But in his heart XU Guang-qi wanted very much to continue the translation. In a preface to a revised edition of *Ji He Yuan Ben* in 1611 he lamented, "It is hard to know when and by whom this project will be completed." (Xu, 1984) This deep regret of XU was resolved only

[1] The term "*ji he*" becomes the modern Chinese terminology for geometry. A suggestion by some that it is a transliteration of the Western word *geometria* sounds unlikely for various reasons. By looking at the translated definitions in Book V, which is on Eudoxus' theory of proportion, we see that "*ji he*" is the technical term for "magnitude". In traditional Chinese mathematical classics the term "*ji he* [幾何 how much, how many]" frequently appears to begin a problem. Apparently, XU Guang-qi was familiar with this term from his knowledge of traditional Chinese mathematics and borrowed it to translate "magnitude". He also perceived the significance of the notion of "magnitude" in *Elements* so that he put the term in the title. See (Siu, 1995/1996; Siu, 2011) for more details.

two and a half centuries later when the Qing mathematician LI Shan-lan (李善蘭 1811-1882) in collaboration with the English missionary Alexander Wylie (偉烈亞力 1815-1887) translated Book VII to Book XV in 1857 (Liu, 1989) based on the English translation of *Elements* by Henry Billingsley published in 1570 (Xu, 2005).

For an in-depth analysis of the translation of *Ji He Yuan Ben* readers are strongly recommended to consult the book by Peter Engelfriet (Engelfriet, 1998), which is a revised and expanded version of the author's 1996 doctoral dissertation at Leiden University.

3. Euclid's *Elements* in the Western world

Euclid's *Elements* compiled circa the third century B.C.E. is regarded as a milestone in the history of mathematics and more generally in the history of thought, and has been exerting enormous influence throughout the Western world since it was compiled, much more than just as a book on mathematics (Grabiner, 1988; Kline, 1953).

A mathematician, Saul Stahl, once said, "Geometry in the sense of mensuration of figures was spontaneously developed by several cultures and dates to several millennia B.C.E. The science of geometry as we know it, namely, a collection of abstract statements regarding ideal figures, the verification of whose validity requires only pure reason, was created by the Greeks." (Stahl, 1993) The fifth century commentator of Euclid's *Element* Proclus (ca 410-485) said, "Its name [μαθηματική] thus makes clear what sort of function this science performs. It arouses our innate knowledge, awakens our intellect, purges our understanding, brings to light the concepts that belong essentially to us, takes away the forgetfulness and ignorance that we have from birth, sets us free from the bonds of unreason;[…]" (Proclus, 1970)

Indeed, the curriculum for higher education of Plato's Academy included arithmetic and logistic, plane geometry and solid geometry, astronomy, harmonics (music theory), which comprised what was named as the *quadrivium* by the Roman philosopher-mathematician Boethius (ca 480-524). Plato (427-347 B.C.E.) said in his *Republic*, "When they reach thirty they will be promoted to still higher privileges and tested by the power of Dialectic, to see which can dispense with sight and the other senses and follow truth into the region of pure reality." (Plato, 1942) Likewise in the curriculum for the aristocracy in ancient China there were the *liu yi* [六藝 six (gentlemanly) arts --- Rites (禮), Music (樂), Archery (射), Charioteering/Horsemanship (御), History/Writing (書), Arithmetic/Mathematics (數)]. In the medieval times the *quadrivium* (arithmetic, geometry, music, astronomy) together with three more subjects, the *trivium* (rhetoric, dialectic, grammar) became the seven liberal arts, which enriched the human mind to make a free man. The English polymath Francis Bacon (1561-1626) said in his *Of Studies*, "Histories make men wise; poets, witty; the mathematics, subtle; natural philosophy, deep; moral, grave; logic and rhetoric, able to contend." (Bacon, 1906)

The French philosopher Bernard le Bovier de Fontenelle (1657-1757) said in 1699, "The spirit of geometry is not only confined to geometry that it cannot be taken out and transferred to other domains of knowledge. A work of morality, politics, criticism, perhaps even eloquence, will become more elegant, other things being equal, if it is touched by the hand of geometry." (Fontenelle, 1785) The German philosopher Immanuel Kant (1724-1804) said, "Now it has not gone so well for human reason in this case. One can point to no single book, as for instance one presents a *Euclid*, and say: this is metaphysics, […]" (Kant, 2004)

So, the influence of Euclid's *Elements* in the Western world was, right from the beginning, more than just a book on mathematics. Let us read the quotes from two first-rate minds of the twentieth century on the respective impact they received from studying *Elements*.

One of them is Bertrand Russell (1872-1970), who said, "At the age of eleven, I began Euclid, with my brother as tutor. This was one of the great events of my life, as dazzling as first love. […] I had been told that Euclid proved things, and was much disappointed that he started with axioms. At first, I refused to accept them unless my brother could offer me some reason for doing so, but he said, "If you don't accept them, we cannot go on", and as I wished to go on, I reluctantly admitted them *pro temp*." (Russell, 1967) Indeed, in the translation by Matteo Ricci and XU Guang-qi the Postulates were rendered as requests accompanied by the remark "if it is requested to construct this, it is not allowed to say that it cannot be done". The Greek word "axioma" has the meaning of making a request, quite in line with what Russell's elder brother told him about!

The other modern figure is no other than Albert Einstein (1879-1955), who said, "At the age of twelve I experienced a second wonder of a totally different nature: in a little book dealing with Euclidean plane geometry, which came into my hands at the beginning of a school year. […] The lucidity and certainty made an indescribable impression upon me. […] it is marvellous enough that man is capable at all to reach such a degree of certainty and purity in pure thinking as the Greeks showed us for the first time to be possible in geometry." (Einstein, 1957)

Let us now come to Isaac Newton (1642-1727). In a memorandum of John Conduitt (1688-1737), husband of Newton's niece and successor to Newton's office as Master of the Mint, who in turn learnt of it from Abraham de Moivre (1667-1754) in November of 1727, we read: "Got Euclid to fit himself [that is, Newton] for understanding the ground of Trigonometry. Read only the titles of the propositions, which he found so easy to understand that he wondered how anybody could amuse themselves to write any demonstrations of them. Began to change his mind when he read that Parallelograms upon the same base and between the same Parallels are equal, and that other proposition that in a right angled Triangle the square of the Hypotenuse is equal to the squares of the other two sides." (Whiteside, 1970) In a recollection of Newton by Henry Pemberton (1694-1771), who superintended the third edition of Newton's *Principia*, it is said that Newton once expressed his opinion of the ancients: "Of their taste, and form of demonstration Sir Isaac always professed himself a great admirer: I have heard him even censure himself for not following them yet more closely than he did; and speak with regret of his mistake at the beginning of his mathematical studies, in applying himself to the works of Descartes and other algebraic writer, before he had considered the elements of Euclide with that attention, which so excellent a writer deserves." (Pemberton, 1728) Indeed, in his monumental work *Philosophiæ Naturalis Principia*

Mathematica of 1687 Newton started the exposition with three axioms of motion from which the theory was developed through Euclidean geometry even though the results were obtained through the calculus that he newly developed! (These accounts about the influence of Euclid on Newton have been questioned by some historians of science, in view of the mathematics curriculum in Cambridge of that period and especially because Newton was such a genius who learnt on his own. An in-depth study and discussion can be found in the works of Derek Thomas Whiteside (1932-2008) based on Newton's notes and papers and the copy of John Barrow's rendition of *Elements* that Newton studied together with his own marginal notes (Whiteside, 1970; Whiteside, 1982).)

Another example is *Ethics* of 1675 by Baruch Spinoza (1632-1677), which was written in the style of Euclid's *Elements* with terms like definitions, axioms, propositions, corollaries and even the abbreviation QED (*quod erat demonstrandum*) at the end of a proof! (Spinoza, 1994) Still another famous book of the early nineteenth century with the mark of influence by the style of Euclid's *Elements* is *An Essay on the Principle of Population* of 1798 (with many subsequent editions) by Thomas Malthus (1766-1834) with two starting postulata, the first being that "food is necessary for the existence of man", the second being that "the passion between the sexes is necessary, and will remain nearly in its present state". Malthus then drew the pessimistic conclusion that assuming these two postulata as granted, "the power of population is indefinitely greater than the power in the earth to produce subsistence for man" (Malthus, 1914).

Thomas Jefferson (1743-1826), one of the founders of the United States of America and its third president, once said, "Science is my passion, politics my duty." (Bedini, 1990) He told his friends in his letters, "[…] and I suppose I can pursue my studies in the Greek and Latin as well there as here, and likewise learn something of the Mathematics." and also, "I have given up newspapers in exchange for Tacitus and Thucydides, for Newton and Euclid; and I find myself much the happier." (Jefferson, 1984) Indeed, when Jefferson drafted the *Declaration of Independence* he began it with the sentence: "We hold these truths to be self-evident, that all men are created equal, that they are endowed by the Creator with certain unalienable Rights, that among these are Liberty and the pursuit of Happiness. […]" (Lemay, 1988) Based on this "axiom" he gave the justification for the rising up against King George III of England.

Abraham Lincoln (1809-1865), the sixteenth president of the United States of America, spoke of his own experience with the study of Euclid, "What he [that is, Lincoln] has in the way of education he has picked up. After he was twenty-three and had separated from his father, he studied English grammar --- imperfectly, of course, but so as to speak and write as he now does. He studied and nearly mastered the six books of Euclid since he was a member of the Congress." (Lincoln, 1989) He also once said, "One would start with great confidence that he could convince any sane child that the simpler propositions of Euclid are true; but nevertheless, he would fail, utterly, with one who should deny the definitions and axioms. The principles of Jefferson are the definitions and axioms of free society." (Lincoln, 1989) In his famous Gettysburg Address of 1863 Lincoln said, "Four score and seven years ago our fathers brought forth on this continent, a new nation, conceived in Liberty, and dedicated to the proposition that all men are created equal […]" (Lincoln, 1989)

Not everybody see Euclid's *Elements* in the same light. The nineteenth century mathematician James Joseph Sylvester (1814-1897) once said, "The early study of Euclid made me a hater of Geometry, which I hope may plead my excuse if I have shocked the opinions of any in this room […] by the tone in which I have previously alluded to it as a schoolbook; […]" (Baker, 1904-1910) However, he continued with: "[and] yet, in spite of this repugnance, which had become a second nature in me, whenever I went far enough into any mathematical question, I found I touched, at last, a geometrical bottom." (Baker, 1904-1910)

4. *Elements* and traditional Chinese mathematics

How did *Elements* blend in with traditional Chinese mathematics? Let us first look at what Ricci said about traditional Chinese mathematics: "The result of such a system is that anyone is free to exercise his wildest imagination relative to mathematics, without offering a definite proof of anything. In Euclid, on the contrary, they recognized something different, namely, propositions presented in order and so definitely proven that even the most obstinate could not deny them." (Ricci, 1953)

Is it really true that the notion of a mathematical proof was completely absent from ancient Chinese mathematics as Ricci remarked? This is a debatable issue. We shall look at one example, which would have made Ricci think otherwise, had he the opportunity of having access to the commentaries of LIU Hui (劉徽) of the third century. In particular we will discuss the following problem: Given a right-angled triangle *ABC* with *AC* as its hypotenuse, inscribe a square in it, that is, construct a square *BDEF* with *D* on *AB*, *E* on *AC*, and *F* on *BC*? (For a more detailed discussion on the mathematics and the relevant texts in the primary sources see (Engelfriet, 1998; Engelfriet, Siu, 2001; Siu, 2011). For the primary sources see (Chemla, Guo, 2004; Clavius, 1574; Guo, 2009; Heath, 1925). For further discussion on the comparison of the styles and roles of proofs in the Western and Eastern worlds see (Chemla, 1996; Chemla, 2012; Siu, 1989; Siu, 1993; Siu, 2008; Siu, 2012).)

This problem does not appear in Euclid's *Elements*. Were it there, the solution would have probably looked like this: Bisect $\angle ABC$ by *BE* (*E* on *AC*) [Book I, Proposition 9]. Drop perpendiculars *ED*, *EF* (*D* on *AB*, *F* on *BC*) [Book I, Proposition 12]. Prove that *BDEF* is the inscribed square we want. The seventeenth century English mathematician John Speidell treated this problem as problem 69 in his 1616 book *A geometrical extraction, or, A compendium collection of the chiefest and choisest problems* by a different method which still carries a strong "Euclidean flavor". The problem (in a more general version) appears as Added Proposition 15 of Book VI in *Euclidis Elementorum Libri XV*, which was translated by Matteo Ricci and XU Guangqi : Divide *AB* at *D* such that $AD : DB = AB : BC$ [Book VI, Proposition 10]. Draw *DE* parallel to *BC* and *EF* parallel to *AB*, (*E* on *AC*, *F* on *BC*). *DBFE* is the inscribed square we want.

Now that we know such an inscribed square exists we can ask what the length of its side is. It can be shown from the construction that the side x of the inscribed square in a right-angled triangle with sides of length *a*, *b* containing the right angle is given by $x = \frac{ab}{a+b}$. More generally, for an arbitrary triangle *ABC* with base *BC* = *b* and altitude *AH* = *h*, the side *x* of the inscribed square *IFEG* (with *I*, *F* on *BC*, *G* on *AB* and *E* on *AC*) is given by $x = \frac{hb}{h+b}$.

For a change let us look at the same problem as phrased in Chapter 9 of *Jiu Zhang Suan Shu* [九章算術 Nine Chapters on the Mathematical Art] compiled between 100 B.C.E. and 100 C.E. Problem 15 says: "Now given a right-angled triangle whose *gou* is 5 *bu* and whose *gu* is 12 *bu*. What is the side of an inscribed square? The answer is 3 and 9/17 *bu*.
Method: Let the sum of the *gou* and the *gu* be the divisor; let the product of the *gou* and the *gu* be the dividend. Divide to obtain the side of the square." (See Figure 1.)

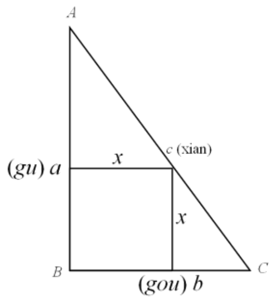

Figure 1

A lecture[2] on mathematics should at least contain one proof, so let me now give that one proof here, a dissection-re-assemble proof by LIU Hui. The line of thinking and the style of presentation are quite different from that in *Elements*. Readers may like to try their hands on explaining the formula by two different proofs given in the commentary of LIU Hui in the mid-third century. The first method is a "visual proof" of the formula $x = \frac{ab}{a+b}$ by dissecting and re-assembling coloured pieces. (See Figure 2.). LIU's commentary actually describes the coloured pieces so that were the original diagram extant it would provide the making of a set of useful teaching aid! A similar but more interesting computation was devised by LIU Hui for the next problem in that same chapter in *Jiu Zhang Suan Shu*, which is on an inscribed circle of a right-angled triangle (Siu, 1993).

[2] This paper is the text of an invited lecture given in July 2013 at the 6th International Congress of Chinese Mathematicians held in Taipei

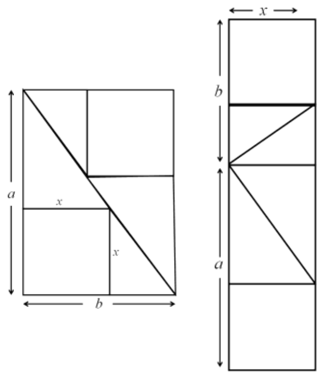

Figure 2

The second method is based on the theory of proportion, making use of the so-called *jin you shu* [今有術 method of *jin you*, known as the *Rule of Three* in the Western world] and the *principle of invariant ratio* (which is basically the same as the content of Proposition 43 of Book I of *Elements*) (Siu, 2011). Although there was no theory of similar triangles developed in ancient Chinese mathematics, a special case of it in the situation of right-angled triangles was frequently employed with dexterity and proved to be rather adequate for most purposes.

How did XU Guang-qi perceive Euclidean geometry which he newly learnt from Clavius' rendition of Euclid's *Element*, and to what extent did he understand the thinking, approach and presentation of the book, which are so very different from those of traditional Chinese mathematics that he was familiar with? Despite XU's emphasis on utility of mathematics, he was sufficiently perceptive to notice the essential feature about *Elements*. In a preface to *Ji He Yuan Ben* of 1607 he wrote: "As one proceeds from things obvious to things subtle, doubt is turned to conviction. Things that seem useless at the beginning are actually very useful, for upon them useful applications are based. […] It can be truly described as the envelopment of all myriad forms and phenomena, and as the erudite ocean of a hundred schools of thought and study." (Xu, 1984)

In a preface to another book *Ce Liang Fa Yi* [測量法義 Methods and Principles in Surveying] of 1608, which is an adapted translation by Matteo Ricci and XU Guang-qi of parts of *Geometria practica* compiled by Christopher Clavius in 1606, he wrote: "It has already been ten years since Master Xitai [西泰子 that is, Matteo Ricci] translated the methods in surveying. However, only started from 1607 onwards the methods can be related to their principles. Why do we have to wait? It is because at that time the six books of *Ji He Yuan Ben* were just completed so that the principles could be transmitted. […] As far as the methods are concerned, are they different from that of measurement at a distance in *Jiu Zhang* [*Suan Shu*] and *Zhou Bi* [*Suan Jing*]? They are not different. If that is so, why then should they be valued? They are valued for their principles." (Xu, 1984)

He elaborated this point in an introduction to his 1608 book *Ce Liang Yi Tong* [測量異同 Similarities and Differences in Surveying] by saying, "In the chapter on *gou gu* of *Jiu Zhang Suan Shu* there are several problems on surveying using the gnomon and the trysquare, the methods of which are more or less similar to those in the recently translated *Ce Liang Fa Yi* (Methods and Principles in Surveying). […] The y*i* [義 principles] are completely lacking. Anyone who studies them cannot understand where they are derived from. I have therefore provided new *lun* [論 proofs] so that examination of the old text becomes as easy as looking at the palm of your hand." (Xu, 1984) In connection with this he wrote in a memorial submitted to the Emperor in 1629 in his capacity as the official in charge of the Astronomical Bureau, "[not knowing that] there are *li* [理 theory], *yi* [義 principle], *fa* [法 method] and *shu* [數 calculation] in it. Without understanding the theory we cannot derive the method; without grasping the principle we cannot do the calculation. It may require hard work to understand the theory and to grasp the principle, but it takes routine work to derive the method and to do the calculation." (Xu, 1984)

With this perception XU tried hard to assimilate Western mathematics and to synthesize it with Chinese traditional mathematics. One example is his work on Problem 15 in Chapter 9 of *Jiu Zhang Suan Shu* and Added Proposition 15 of Book VI in *Euclidis Elementorum Libri XV* reported in his 1609 book *Gou Gu Yi* [勾股義 Principle of the Right-angled Triangle]. He explained this as Problem 4: "*Gu AB* is 36, *gou BC* is 27. It is required to produce its inscribed square. Let the product of *gou* and *gu* be the dividend. Let the sum of *gou* and *gu* be the divisor, which is *AE* equal to 63. Divide and obtain each side of the inscribed square, *HB* and *BJ*, to be 15.428 […]"(Xu, 1984) His complicated reasoning may seem rather round-about and unnecessary, or perhaps it indicates a kind of incompatibility between the two styles of doing mathematic so that it would be unnatural to force one into the mould of the other. However, despite its shortcoming this also indicates an admirable attempt of XU Guang-qi to synthesize Western and Chinese mathematics (Engelfriet, Siu, 2001; Siu, 2011).

Why would XU Guang-qi work so hard in synthesizing Western and Chinese mathematics? XU was a Chinese scholar brought up in the Confucian tradition, upholding the basic tenet of self-improvement and social responsibility, leading to an aspiration for public service and an inclination to pragmatism. He first got to know Catholic missionaries by an incidental encounter with the Jesuit Lazzaro Cattaneo (1560-1640) in the southern province of Guangdong, who probably introduced him to Ricci. He saw in Western religion and Western science and mathematics an excellent way to cultivate the mind and a supplement to Confucian studies. He also saw in Western science and technology the significant role it would play in improving the well-being of his countrymen. This eagerness on his part to study Western learning was very much welcomed by Ricci as it was in line with the tactics adopted by the Jesuit missionaries in making use of Western science and mathematics to attract and convert the Chinese literati class who usually occupied important positions in the Imperial Court. Ricci impressed the Chinese intellectuals as an erudite man of learning, thereby commanding their trust and respect (Siu, 1995/1996).

By the Ming Dynasty the mathematical legacy in China was no longer preserved and nurtured in the way it should be. Quite a number of important mathematical classics were either completely lost or left in an incomplete form. As a scholar brought up in the Confucian tradition XU Guang-qi was aware that mathematics had once occupied a significant part of education and statecraft in China and should be restored to its former position of importance. He ascribed the unsatisfactory state of the subject at his time to two factors, which he expressed in 1614 in the preface to another translated European mathematical text *Epitome Arithmeticae Practicae* compiled by Christopher Clavius in 1583, translated by LI Zhi-zao (李之藻 1565–1630) also in collaboration with Matteo Ricci, "There are two main causes for negligence and dilapidation of mathematics that set in only during several past centuries. Firstly, scholars in pursuit of speculative philosophical studies despise matters of practical concern. Secondly, sorcery encroaches upon mathematics to turn it into a study filled with mysticism.” (Xu, 1984)

He saw in the introduction of Western mathematics, which was novel to him, a way to revive the indigenous mathematical tradition. He had a wider vision of mathematics, not just as an intellectual pursuit but as a subject of universal applications as well. In an official memorial submitted to the Emperor in 1629, he said, “Furthermore, if the study of measure and number [mathematics] is understood, then it can be applied to many problems [other than astronomy] as a by-product.” (Xu, 1984) Such problems were labelled by him in ten categories (度數旁通十事): (1) weather forecast, (2) irrigation, (3) musical system, (4) military equipment, (5) accounting, (6) building, (7) machine, (8) topography, (9) medical practice, (10) timepieces. In this memorial we can witness his emphasis on “study of measure and number (度數之學)” in that “all subjects with shapes and substances can be explained in terms of mathematics”.

XU Guang-qi was a remarkable person whose thinking, insight and perception were way ahead of his time (Bai, 1989; Chen, 1992; Jami, Engelfriet, Blue, 2001; Siu, 1995/1996). He possessed an open, receptive but sceptical mind. He paid attention to the experimental aspect. He viewed scientific pursuit as a collective activity in team work and realized the importance of education in the nurturing of scientific thought, not just that of the transmission of techniques and knowledge. In this respect it would be interesting to raise a few hypothetical questions (Siu, 1995/1996; Siu, 2011):

--- How much would XU Guang-qi have achieved in mathematics if he had concentrated his effort on this one discipline?
--- What would have happened if he had known about the various commentaries on the controversial Fifth Postulate?
--- What would have happened if he had mastered Latin just as Ricci had mastered Chinese?
--- What would have happened if he had the chance and the inclination to actually pay a visit to Europe at the time and to return to China with what he experienced there?

5. Transmission of Western learning into China

History did not (unfortunately) proceed in the way XU Guang-qi would like to see it. The translation of *Elements* by XU Guang-qi and Matteo Ricci led the way of the first wave of transmission of European science into China, with a second wave (or a wake of the first wave as some historians would see it) and a third wave to follow in the Qing Dynasty, but each in a rather different historical context. The gain of this first wave seemed momentary and passed with the downfall of the Ming Dynasty. Looking back we can see its long-term influence, but at the time this small window which opened onto an amazing outside world was soon closed again, only to be forced open as a wider door two hundred years later by Western gunboats that inflicted upon the ancient nation a century of exploitation and humiliation, thus generating an urgency to know more about and to learn with zest from the Western world.

The main features of the three waves of transmission of Western learning into China can be summarized in the prototype slogans of the three epochs. In the late-sixteenth to mid-seventeenth centuries (during the Ming Dynasty) the slogan was: "In order to surpass we must try to understand and to synthesize (欲求超勝必須會通)." In the first part of the eighteenth century (during the Qing Dynasty) the slogan was: "Western learning has its origin in Chinese learning (西學中源)." In the latter part of the nineteenth century (during the Qing Dynasty) the slogan was: "Learn the strong techniques of the '[Western] barbarians' in order to control them (師夷長技以制夷)." (Jami, 1991; Jami, 1992; Siu, 2009; Siu, 2011; Xiong, 1994) As pointed out by Catherine Jami: "[…] the cross-cultural transmission of scientific learning cannot be read in a single way, as the transmission of immutable objects between two monolithic cultural entities. Quite the contrary: the stakes in this transmission, and the continuous reshaping of what was transmitted, can be brought to light only by situating the actors within the society in which they lived, by retrieving their motivations, strategies, and rationales within this context." (Jami, 1999)

The second wave came and lasted from the mid-seventeenth century to the mid-eighteenth century. Instead of Chinese scholar-officials the chief promoter was Emperor Kangxi (康熙) of the Qing Dynasty (reigned 1654-1722). Instead of Italian and Portuguese Jesuits the Western partners were mainly French Jesuits, the so-called "King's Mathematicians" sent by Louis XIV, the "Sun King" of France (reigned 1643-1715), in 1685 (Du, Han, 1992; Han, 1991; Jami, 2002; Jami, 2011; Jami, Han, 2003).

This group of Jesuits led by Jean de Fontaney (1643-1710) reached Peking in 1688. An interesting account of their lives and duties in the Imperial Court was recorded in the journal written by one of the group, Joachim Bouvet (1656-1730), and published in 1697 (Bouvet, 1697). A main outcome was the compilation of a monumental one-hundred-volume treatise *Lü Li Yuan Yuan* [律曆淵源 Origins of Mathematical Harmonics and Astronomy] commissioned by Emperor Kangxi, worked on by a large group of Jesuits, Chinese scholars and official astronomers. The project started in 1713 and the treatise was published in 1722/1723, comprising three parts: *Li Xiang Kao Cheng* [曆象考成 Compendium of Observational Computational Astronomy], *Shu Li Jing Yun* [數理精蘊 Collected Basic Principles of Mathematics], *Lü Lü Zheng Yi* [律呂正義 Exact Meaning of Pitchpipes]. Books 2 to 4 of *Shu Li Jing Yun* are on geometry, which is believed to be based on *Elémens de géométrie* by Ignace Gaston Pardies (1636-1673), first published in 1671 with a sixth edition in 1705 (Han, 1991).

The third wave came in the last forty years of the nineteenth century in the form of the so-called "Self-strengthening Movement" after the country suffered from foreign exploitation during the First Opium War (1839-1842) and the Second Opium War (1856-1860). This time the initiators were officials led by Prince Gong (恭親王 1833-1898) with contribution from Chinese scholars and Protestant missionaries coming from England or America, among whom were LI Shan-lan (李善蘭 1811-1882) and Alexander Wylie (偉烈亞力 1815-1887) who completed the translation of *Elements*. In 1862 *Tong Wen Guan* [同文館 College of Foreign Languages] was established by decree, at first serving as a school for studying foreign languages to train interpreters but gradually expanded into an institute of learning Western science. The slogan of the day, which was "learn the strong techniques of the '[Western] barbarians' in order to control them", reflected the purpose and mentality during that period. In 1866 a mathematics and astronomy section was added to *Tong Wen Guan*, with LI Shan-lan as its head of department. In 1902 *Tong Wen Guan* became part of Peking Imperial University, which later became what is now Beijing University (Siu, 2009; Chan, Siu, 2012; Xiong, 1994).

A magazine *Zhong Xi Wen Jian Lu* [中西聞見錄 Record of News in China and West] with English title *Peking Magazine* was founded in August of 1872 and terminated in August of 1875 (with 36 issues), to be revived as *Ge Zhi Hui Bian* [格致彙編 Compendium for Investigating Things and Extending Knowledge] with English title *The Chinese Scientific and Industrial Magazine* in 1876. In the inaugural issue it was said: "*Zhong Xi Wen Jian Lu* adopts the practice and format of newspapers in the Western world in publishing international news and recent happenings in different countries, as well as essays on astronomy, geography and *ge wu* [格物 science, literally meaning "investigating things"]. The magazine will be published once every month. Any gentleman, Chinese or Westerner who gathers new information or has his own views to express, is invited to submit it to the editors at the *Shi Yi Yuan* [施醫院 Charity Hospital] of *Mi Shi* [*Street*] [米市 Rice Market Street]. The editors will select those items that are considered to be fit for print. In this way, new information will be attained through collective effort to benefit more people so as to enable them to become more and more knowledgeable." (*Zhong Xi Wen Jian Lu* (1872-1875), 1992)

In No. 5 (December 1872) of *Zhong Xi Wen Jian Lu* there appeared a problem: "A plane triangle (acute, right or obtuse) contains thee circles of different radii that touches each other. Want to fix the centres of the three circles. What is the method?" It was followed by a remark: "All students in *Tong Wen Guan* retreated from trying this problem. Whoever can solve the problem should send the diagram [of the solution] to the School of Astronomy and Mathematics and would be rewarded with a copy of *Ji He Yuan Ben* [Chinese translation of Euclid's *Elements*]. The diagram [of the solution] would be published in this magazine so that the author would gain universal fame." (*Zhong Xi Wen Jian Lu* (1872-1875), 1992)

This problem of finding three circles lying within a given triangle having optimal total area was first raised in the Western world in 1803 by the Italian mathematician Gian Francesco Malfatti (1731-1807) who thought that three "kissing" circles give an optimal solution. Attention was then turned to the construction of the three "kissing" circles. Actually, the three "kissing" circles are never an optimal solution! Before 1803 the problem had been raised by Japanese mathematicians, but obviously not to the knowledge of the Westerners in those days. A solution to the Malfatti Problem appeared in No. 8 (March 1873) of *Zhong Xi Wen Jian Lu*, followed by a comment by another reader in Issue No. 12 (July, 1873) together with an acknowledgement of the error and a further comment by the School of Astronomy and Mathematics. This kind of fervent exchange of academic discussion carried on in public domain was a new phenomenon of the time in China. Later, in the *Long Cheng Shu Yuan Ke Yi* [龍城書院課藝 Homework Assignments of the Academy of the Dragon City] there appeared two solutions, one of which is interesting for its employment of the intersection of two hyperbolas, a curve that was new to the Chinese. This episode illustrates the fervent zest of the Chinese of the time in learning mathematics from the West. After all, Euclidean geometry was at the time still quite a novel subject for them (Chan, Siu, 2012; Chan, Siu, 2013). However, non-Euclidean geometry came much later into China, for instance, in the book *Fei Ou Pai Ji He Xue* [非歐派幾何學 Non-Euclidean Geometry] written by CHAN Jin-Min (陳藎民 1895-1981) in 1936.

6. Influence of *Elements* in late nineteenth century China

Despite the enthusiasm on the part of XU Guang-qi to introduce *Elements* into China how was the book received in the Chinese community during the ensuing three centuries? XU Guang-qi himself said in 1607, "This book [the *Elements*] has wide applications and is particularly needed at this point in time. […] In the preface Mister Ricci also expressed his wish to promulgate this book so that it can be made known to everybody who will then study it. Few people study it. I surmise everybody will study it a hundred years from now, at which time they will regret that they study it too late. They would wrongly attribute to me the foresight [in introducing this book], but what foresight have I really?" (Xu, 1984)

However, near to a hundred years later, the situation was still far from what XU would like to see. In *Shu Xue Yao* [數學鑰 The Key to Mathematics] of DU Zhi-geng (杜知耕) that was published in the second part of the seventeenth century (Du, 1984), LI Zi-jin (李子金 1622-1701) said in the preface in 1681, "Even those gentlemen in the capital who regard themselves to be erudite scholars keep away from the book [*Elements*], or close it and do not study its content at all, or study it with incomprehension and perplexity." (*Qing Guang Xu Zhe Cheng Xian Zhi*, 1896)

The Chinese in the seventeenth and eighteenth centuries did not seem to feel the impact of the essential feature of Western mathematics exemplified in Euclid's *Elements* as strongly as XU Guang-qi. Thus, the influence of the newly introduced Western mathematics on mathematical thinking in China was not as extensive and as directly as XU had imagined. The effect was gradual and became apparent only much later. However, the fruit was brought forth elsewhere, not in mathematics but perhaps in a domain of an even higher historical importance.

Three leading figures responsible for the so-called "Hundred-day Reform" of 1898 — KANG You-wei (康有為 1858-1927), LIANG Qi-chao (梁啟超 1873-1929), TAN Si-tong (譚嗣同 1865-1898) — were strongly influenced by their interest in acquiring Western learning (Siu, 2007; Siu, 2011). Towards the end of the nineteenth century KANG You-wei wrote a book titled *Shi Li Gong Fa Quan Shu* [實理公法全書 Complete Book on Concrete Principles and Postulates [of Human Relationship]], later incorporated into his masterpiece *Da Tong Shu* [大同書 Book of Great Unity] of 1913 (Kang, 2012). It carries a shade of the format of *Elements*, as the title suggests.

The book *Ren Xue* [仁學 On Moral Philosophy] written by TAN Si-tong and published posthumously in 1899, carries an even stronger shade of the format of *Elements* (Tan, 1958), reminding one of the book *Ethics* by Baruch Spinoza of 1675 that began with definitions and postulates. To educate his countrymen in modern thinking TAN Si-tong established in 1897 a private academy known as the *Liu Yang Suan Xue Guan* [瀏陽算學館 *Liu Yang* College of Mathematics] in his hometown, stating clearly in a message on the mission of the college that mathematics is the foundation of science, and yet the study starts with mathematics but does not end with it. Apparently, he was regarding mathematics as assuming a higher position than just a technical tool in the growth of a whole-person in liberal education.

In his famous book *Qing Dai Xue Shu Gai Lun* [清代學術概論 Intellectual Trends of the Qing Period], originally published in *Reform Magazine* in 1920/1921, LIANG Qi-chao remarked, "Since the last phase of the Ming, when Matteo Ricci and others introduced into China what was then known as *xi xue* [西學 Western learning], the methods of scholarly research had changed from without. At first only astronomers and mathematicians credited [the new methods], but later on they were gradually applied to other subjects." (Liang, 1959)

The "Hundred-day Reform" ended in failure despite the initiation and support of Emperor Guangxu (光緒, reigned 1875-1908) because of the political situation of the time. TAN Si-tong met with the tragic fate of being arrested and executed in that same year, while KANG You-wei and LIANG Qi-chao had to flee the country and went to Japan. This was one important step in a whole series of events that culminated in the overthrow of Imperial Qing and the establishment of the Chinese Republic in 1911 (Fairbank, Reischauer, 1973; Hsü, 1995).

LIANG Qi-chao started the magazine *Shiwu Bao* [時務報 The China Progress] in August of 1896 that lasted until April of 1898. In Issue 8 he published the preface and remarks to his book *Xi Xue Shu Mu Biao Fu Du Xi Xue Shu Fa* [西學書目表附讀西學書法 List of Books on Western Learning and the Way to Study the Books] with an advertisement of the book in the magazine (Liang, 1989). This book introduces over three hundred titles on Western learning. Let us read some excerpts from his book regarding *Elements*:

"XU Jiao-ding [徐交定, that is, XU Guang-qi] translated only the first six books of *Elements*, to be completed by LI Ren-shu [李壬叔, that is, LI Shan-lan]. But the very difficult Book X is incomprehensible to a beginner. Even in Western schools the study is only confined to the first six books." (Liang, 2005)

"Thus, [Alexander] Wylie says that Westerners who wish to read the full original version of the book [*Elements*] would come to China to look for it instead. Learners should begin by reading the translation by XU [Guang-qi]. After a long time of study they will acquire a deeper understanding of the subject and attain an insightful comprehension of its method. Then it comes naturally that they can read the full book. (*Shu Li Jing Yun* [Collected Basic Principles of Mathematics] contains a simplified version of the subject, however it is better to study the original version.)" (Liang, 2005)

"The preface of *Xing Xue Bei Zhi* says that the book contains many important problems that are new and not found in [Euclid's] *Elements*. In Western countries all translations of geometry [*Elements*] would supplement it with new important problems at the end of each chapter, but this was not done in the translation by LI [Shan-lan]. Thus, those who study geometry [*Elements*] must read this book as well." (Liang, 2005) The book *Xing Xue Bei Zhi*

[形學備旨 The Complete Meaning of the Science of Figures] was compiled by the American missionary Calvin Wilson Mateer (狄考文 1836-1908) and the Chinese scholar Zou Li-wen (鄒立文) in 1885. It is believed to be a selected translation of a book by Elias Loomis (1811-1889).

With the promotion by these pioneers *Elements* did have some influence, gradual as it was. By the first part of the twentieth century the Chinese began to appreciate the deeper meaning of *Elements*. An illuminating remark came from an eminent historian CHEN Yin-ke (陳寅恪 1890-1969) who said in an epilogue to the Manchurian translation of *Ji He Yuan Ben* in 1931, "The systematic and logical structure of Euclid's book is of unparalleled preciseness. It is not just a book on number and form but is a realization of the Greek spirit. The translated text in the Manchurian language and the version in *Shu Li Jing Yun* [Collected Basic Principles of Mathematics] are edited to lend emphasis on utility of the subject, not realizing that, by so doing, the original essence has been lost." (Chen, 1931)

7. An afterthought

From the very beginning XU Guang-qi saw in *Elements* its strength over and beyond its technical mathematical content. In his *Ji He Yuan Ben Jia Yi* [幾何原本雜議 Various Reflections on *Ji He Yuan Ben*] he said, "The benefit derived from studying this book is many. It can dispel shallowness of those who learn the theory and improve their concentration. It can supply fixed methods for those who apply to practice and kindle their creative thinking. Therefore everyone in this world should study this book." (Xu, 1984) He continued to say, "Five categories of personality will not learn from this book [the *Elements*]: those who are impetuous, those who are thoughtless, those who are complacent, those who are envious, and those who are arrogant. Thus to learn from this book one not only strengthens one's intellectual capacity but also builds a moral base." (Xu, 1984)

One can find a modern version of this view in the writing of the late Russian mathematics educator Igor Fedorovich Sharygin (1937-2004) who said, "Geometry is a phenomenon of the human culture. […] Geometry, as well as mathematics in general, helps in moral and ethical education of children. […] Geometry develops mathematical intuition, introduces a person to independent mathematical creativity. […] Geometry is a point of minimum for the distance between school mathematics and the mathematics of high level." (Tikhomirov, 2004) In particular, he viewed a mathematical proof in the following manner: "Learning mathematics builds up our virtues, sharpens our sense of justice and our dignity, strengthens our innate honesty and our principles. The life of mathematical society is based on the idea of proof, one of the most highly moral ideas in the world." (Sharygin, 2004) The famed French mathematician André Weil (1906-1998) put it in the succinct remark: "Rigour is to the mathematician what morality is to man." The book *The Education of T.C. Mits: What Modern Mathematics Means to You* [T.C. Mits = The Celebrated Man In The Street] by Lillian R. Lieber (1886-1986), originally published in 1942 and republished in 2007, contains the following passage: "And so you see how mathematics can throw light on various subjects which many people discuss glibly and carelessly since they have never been trained to examine ideas with that METICULOUS CARE with which a mathematician works. […]

There is a model for straight thinking which we all MUST try to imitate. This is not the noisy argumentation of the pseudo-thinkers. Rather it is quiet, honest, careful, COMPETENT. The Moral: Do not be NAÏVE — Use the methods of mathematics." (Lieber, 1944)

By its nature mathematics should be a subject most far away from having an authoritative flavor, but frequently the way it is taught gives a different impression. The mathematics educator Magdalene Lampert says, "These cultural assumptions are shaped by school experience, in which doing mathematics means following the rules laid down by the teacher; knowing mathematics means remembering and applying the correct rule when the teacher asks a question; and mathematical truth is determined when the answer is ratified by the teacher." (Lampert, 1990)

Over four hundred years ago XU Guang-qi already pointed out this moral and intellectual benefit with the study of *Elements*. The Columbia scholar Jacques Barzun (1907-2012), who passed away last year at the ripe old age of 105, said in his 1945 book *Teacher in America,* "Teaching is not a lost art, but the regard for it is a lost tradition." (Barzun, 1945) How many teachers nowadays will continue the tradition of XU Guang-qi in the teaching of geometry in this respect?

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