The Emperor's new lessons: How and why did Emperor Kangxi learn Western science and mathematics?

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In a way this public lecture is a sequel to the public lecture I gave on October 7, 2010 ["1607, a year of (some) significance: Translation of the first European text in mathematics --- *Elements* --- into Chinese"]. Actually I have in mind a story in trilogy. The first part was published under the same title as the public lecture given in 2010. The second part was written up and submitted a few months ago, with the title "Facing the change and meeting the challenge: Mathematics curriculum in *Tongwen Guan* (同文館) in China in the second half of the nineteen century". This part is sandwiched in between!

A couple of months after setting the title of this public lecture I learnt of the new book by Catherine Jami which bears a similar title. [Catherine Jami, *The Emperor's New Mathematics : Western Learning and Imperial Authority During the Kangxi Reign (1662-1722)*, OUP, 2011.]

I have not read Jami's book, but have throughout the past two decades studied many of her papers written on the subject, some jointly with HAN Qi (韓琦), from which I benefit much. Indeed, this lecture borrows heavily from the dedicated and meticulous research done by them. I look forward to reading Jami's new book, which surely will give a much more detailed and scholarly analysis of the topic than what this lecture can offer. For me the title of Jami's book suggests a pun on the so-called "*new maths*", an eventful but controversial movement in mathematics education which started in the late 1950s and went on in different parts of the world in the 1960s and 1970s. For my title I make a pun on a different story, the well-known tale (1873) by Hans Christian Andersen about an honest little boy who commented on the Emperor's "*new clothes*".

It is commonly agreed among historians that the transmission of Western learning in China occurred mainly in two waves, the first one occurring in the late-16th to the mid-17th century in the latter part of the Ming Dynasty (明), the second one occurring in the second half of the 19th century in the Qing Dynasty (清). It is not the purpose of this lecture to discuss these two waves. Rather, we will talk about what happened in between, a wake of the first wave occurring in the mid-17th to the mid-18th century, particularly what happened in the Imperial Court of Kangxi Emperor (康熙皇帝)

It is to be noted that the three phases of transmission took place within quite different historical contexts and with quite different mentality, as exemplified by the slogans of those days:

"In order to surpass we must try to understand and to synthesize (欲求超勝必須會通)." (late-16th to mid-17th century)

"Western learning has its origin in Chinese learning (西學中源)." (mid-17th to mid-18th century)

"Learn the strong techniques of the [Western] barbarians' in order to control them (師夷長技以制夷)." (second half of the 19th century) --- no offence to non-Chinese friends in the audience!

In this public lecture we discuss what was done by Kangxi Emperor, who came to the throne before he turned seven in 1661 and reigned for near to sixty-two years until he died at the age of sixty-eight in 1722. We give an overview of his activities in connection with the study of mathematics, astronomy and science, roughly divided into three periods, interrupted by other activities he was engaged in as the ruler of a vast empire.

The so-called "calendar case" (曆獄) started in 1664 and ended in 1669, when Kangxi Emperor really began his reign free of the strong domination of his regents. As a result of the "calendar case" Kangxi Emperor applied himself diligently to the study of astronomy, mathematics and science. He went on field trips in 1682/1683. In 1688 the so-called "*King's Mathematicians*" sent by King Louis XIV of France arrived in China, prompting another period of intensive study of astronomy, mathematics and science in the Imperial Court. In 1713 the *Suanxue Guan* (Office of Mathematics 算學館) was established to deal with the compilation of *Shuli Jingyun* (Collected Basic Principles of Mathematics 數理精蘊), which was completed in 1722/1723, while Kangxi Emperor died in 1722. This year is the 290th anniversary of the publication of this monumental treatise.

We start with the first period, which spanned the 1660s and 1670s. Following the practice of the Ming Court the Qing Court employed the service of foreign missionaries as official astronomers to make an accurate calendar, which was held by many as a major tool of legitimization of the rule in traditional Chinese thinking.

The German Jesuit Johann Adam Schall von Bell (湯若望) convinced Shunzhi Emperor (father of Kangxi Emperor 順治皇帝) to adopt a new calendar, which was actually the fruition of a huge project accomplished by XU Guang-qi (徐光啟) and his team in the Ming Dynasty but could not be implemented because of the collapse of the Ming Dynasty in 1644. When Kangxi Emperor came to the throne, Adam Schall von Bell lost the support of the late emperor and was accused of treason by the conservative ministers backed up by the powerful regents of the boy-emperor (who was then only ten-year-old). Adam Schall von Bell, along with some other Chinese Catholic converts, was sentenced to death and would have met his tragic ending were it not for a strong earthquake that shook the capital for several days (in 1665). People took this to be a warning from heaven that told them it was wrong to accuse Adam Schall von Bell of treason. Furthermore, the Grand Empress Dowager (grandmother of Kangxi Emperor) intervened so that Adam Schall von Bell was spared the death sentence but was expelled to Macau instead, where he died there soon afterwards in 1666. This event, known as the "calendar case", was rehabilitated in 1669 at the initiation of another foreign missionary, the Belgian Jesuit Ferdinand Verbiest (南懷仁).

Verbiest, who spent two years at the famous University of Coimbra before he was sent to China for missionary work, succeeded Adam Schall von Bell as the Head of the Imperial Astronomical Bureau (欽天監), serving from 1669 to 1688. The way Verbiest managed to turn the tide was to challenge his opponent, the conservative minister YANG Guang-xian (楊光先), to compete in measuring the shadow of the sun on one December day of 1668. This event left a deep impression on the young Kangxi, as he later recounted the story:

"You all know that I am good at mathematics but do not know the reason why I study it. When I was very young there were frequent disputes between Han officials and Westerners in the Imperial Astronomical Bureau. They accused each other so badly that somebody might get beheaded! Yang Guangxian and Tang Ruowang [Adam Schall von Bell --- apparently Kangxi's memory did not serve him well here!] competed in predicting the sun shadow at the Wu Gate in the presence of the nine chief ministers. However, none of them knew what the astronomers were doing. In my opinion, how can a person who

lacks knowledge judge who is right or wrong? Hence I determined to study with all my might and main the subject of mathematics. Now that the methods were compiled and explained clearly in books, learners find it easy. Heaven knows how difficult it was for me to learn it in those days!"

Verbiest was very much respected as the teacher of Kangxi Emperor in mathematics, astronomy and science. Realizing that he was himself getting old, Verbiest wrote and asked the Society of Jesus to send more younger missionaries learned in mathematics and astronomy to China. His call was answered by the French. As a result of the Treaty of Tordesillas (1494) and the Treaty of Zaragoza (1529) the new world of America and the old but till then exotic world of Asia were divided between Spain and Portugal in land and resources, in political power as well as in Christian power, particularly missionary work in the so-called "infidel" countries, later known as "protectorate of missions". To counteract the Portuguese influence and control over missionary work in China at the time, in 1685 King Louis XIV of France sent the "*King's Mathematicians*" to China. These were French Jesuits, but in order not to cause problem with the Portuguese authority they were sent under a sort of disguise as "*Kings' Mathematicians*".

[The six "*King's Mathematician*", who left Brest in France in March of 1685, were: Jean de Fontaney (1643-1710 洪若(翰)), Joachim Bouvet (1656-1730 白晉), Jean-Françoise Gerbillon (1654-1707 張誠), Claude de Visdelou (1656-1737 劉應), Louis Le Comte (1655-1728 李明), Guy Tachard (1651-1712 塔夏爾). Tachard stayed in Siam, so only the other five reached Ningbo in July of 1687 and Peking in 1688.]

By the time the five French Jesuits settled in the Imperial Court in 1688, Kangxi Emperor's first teacher, Ferdinand Verbiest, had passed away. Kangxi Emperor continued to learn assiduously Western mathematics and astronomy from these "*Kings' Mathematicians*" in the second period.

The two great monarchs in the West and the East admired each other. There were a number of striking similarities between them. Louis XIV came to the throne at the tender age of less than four (in 1643); Kangxi at the age of less than seven (in 1661). Louis XIV was brought up under the strong influence of his mother, Queen Anne of Austria; Kangxi by his grandmother, the Grand Empress Dowager Xiaozhuang (孝莊皇太后). The royal education they received ensured that they were well versed in both the military and the literary aspect, observant of benevolence in ruling, and as a result both were fond of fine arts. Both were raised in a multi-cultural background, Louis XIV of French, Austrian, Spanish and Italian; Kangxi of Han Chinese, Manchurian and Mongolian. The initial phase of their reign was dominated by powerful ministers, Louis XIV by Cardinal Jules Mazarin (1602-1661) for the first eighteen years ; Kangi by Aobai [Obei 鰲拜] (1610?-1669) for the first eight years. But once they regained true power they both exhibited diligence to consolidate their rule. Louis XIV ruled for seventy-two years and one hundred and one days; Kangxi for sixty-one years and three hundred and eighteen days.

King Louis XIV wrote a letter to Kangxi Emperor in 1688. Last December I visited a special exhibition on Kangxi Emperor and the Sun King Louis XIV at the Palace Museum in Taipei. One of the exhibits is the hand-written draft of this letter, now kept in the Archives of the French Ministry of Foreign and European Affairs. The letter can also be read in the journal of the French Jesuit, Philippe Avril (1654-1698?), published in 1692 as the book *Voyage en divers etats d'Europe et d'Asie : entrepris pour découvrir un noveau chemin à la Chine* (Travels into divers parts of Europe and Asia, undertaken by the French King's order to discover a new way by land to China).

"Most high, most Excellent, most Puissant, and most Magnanimous Prince, Our Dearly Beloved Good Friend, may God increase your Grandeur with a happy end. Being inform'd, that Your Majesty, was desirous to have near your Person, and in your Dominions, a considerable number of Learned Men, very much vers'd in the European Sciences, we resolv'd some Years ago, to send you six Learn'd

Mathematicians Our Subjects, to show Your Majesty what ever is most curious in Sciences, and especially the Astronomical Observations of the Famous Academy we have establish'd in our good City of Paris:

But whereas the length of the Sea Voyage, which divides Our Territories from Yours, is lyable to many accidents, and cannot be perform'd without much time and danger: We have form'd the design, out of a desire to contribute towards Your Majesties satisfaction, to send you some more of the same Father Iesuits who are Our Mathematicians, with Count Syri, by Land, which is the shortest, and safest way, to the end they may be the first, near Your Majesty, as so many Pledges of our Esteem and Friendship, and that at the return of the said Count Syri, we may have a faithful account of the admirable and most extraordinary Actions that are reported of your Life. Whereupon We beseech God, to augment the Grandeur of Your Majesty, with an end altogether happy. Written at Marly, the 7th of August, 1688.

Your most Dear, and Good Friend, Louis."

You may wonder why the letter is not kept in China but in France. The reason is because this letter never reached the hands of Kangxi Emperor. In Book Four of his book Avril told this story. King Louis XIV and his minister Jean-Baptiste Colbert wrote a letter to Peter the Great of Russia in October of 1687 to seek his permission and protection of an envoy led by Father Avril to China through the Great Steppe (what is now Siberia, Turkistan, Mongolia). Without obtaining the permission and protection from Russia Father Avril attempted but failed to reach China after six year's effort, reaching only Istanbul and had to return to France with the letter!

Two of the French Jesuits, Jochaim Bouvet and Jean-François Gerbillon, left us with their diaries which gave a detailed account of their days spent in the Imperial Court. Let us read some parts of what Bouvet wrote in his diary, published in France in 1697 and soon to be translated into English in 1699 [*Histoire de l'empereur de la Chine: presentée au Roy* (The history of Cang-hy, the present emperour of China: presented[sic] to the most Christian King)].

"His natural genius is such as can be parallel'd but by few, being endow'd with a quick and piercing Wit, a vast memory, and great Understanding; His constancy is never to be shaken by any sinister Event, which makes him the fittest Person in the World, not only to undertake, but also to accomplish Great designs.[…] But, what may seem most surprising, is, that so great a Monarch, who bears upon his shoulders the weight of so vast an Empire, should apply himself with a great deal of Assiduity to, and have a true relish of all Sorts of useful Arts and Sciences."

"[…] so there is not any Science in Europe that ever came to his Knowledge, but he showed a great Inclination to be instructed in it. The first Occasion which had a more than ordinary Influence upon his Mind, happened (as he was pleased to tell us himself) upon a Difference arisen betwixt Yang quansien [Yang Guang-xian], the Famous Author of the last Persecution in China, and father Ferdinand Verbiest, of the Society of Iseus.[…] As this Tryal of Skill in the Mathematiks was the first Occasion that introduced the Father Missionaries into the Emperor's acquaintance; so from that time, he always shew'd a great inclination to be instructed in the Mathematical Sciences, which in effect, are in great Esteem among the Chineses."

"During the space of two Years, Father Verbiest instructed him in the Usefulness of the best of the Mathematical Instruments, and in what else was most Curious in Geometry, the Statique, and Astronomy; for which purpose he wrote several Treatises.[…] He did the Honour to us four Iesuits, Missionaries then at Peking, to receive our Instructions, sometimes in the Chinese, sometimes in the Tartarian Language; […] Much about the same time, Father Anthony Thomas, did give him further Instruction concerning the Use

of the best Mathematical Instruments, in the Chinese language, and the Practical part of Geometry and Arithmatik, the principles of which he had formerly been taught by Father Verbiest. He would also have us explain him the Elements of Euclid in the Tartarian Language, being desirous to be well instructed in them, as looking upon them to be the Foundation, upon which to build the rest."

"After he was sufficiently instructed in the Elements of geometry, he ordered us to compile a whole System of both the Theorick and Practick of Geometry, in the Tartarian Language, which we afterwards explain'd to him in the same manner as we had done with the Elements of Euclid. At the same time, Father Thomas made a Collection of all the Calculations of geometry and Arithmaticks (in the Chinese language) containing most of the Curious Problems extant, both in the European and Chinese Books, that treat this matter. He was so much delighted in the pursuit of these Sciences, that besides betwixt two and three Hours, which were set aside every day on purpose to be spent in our Company, he bestowed most of his leisure time, both in the day and at night in his Studies."

Some of the mathematical instruments Kangxi Emperor used and an Imperial desk for lessons in mathematics are now kept in the Palace Museum in Beijing.

However, not all missionaries portrayed Kangxi Emperor so favourably. The Italian Matteo Ripa (1682-1746 馬國賢), who was not a Jesuit but was sent to China from the Propaganda Fide of the Papacy [renamed as the Congregation for the Evangelization of the People in 1982 in view of the negative meaning of "propaganda"!], arrived in the Imperial Court in 1710. He wrote in his memoir the following passages:

"The emperor supposed himself to be an excellent musician, and still better mathematician; but though he had a taste for the sciences and other acquirements in general, he knew nothing of music, and scarcely understood the first elements of mathematics. There was a cymbal or a spinet in almost every apartment, but neither he nor his ladies could play upon them: sometimes indeed with one of his fingers he touched a note, which was enough, according to the extravagant flattery, practiced at the court of China, to throw the by-standers into ecstasies of admiration, as I myself have often witnessed."

This is somewhat like the remark made by the little boy in the tale of Hans Christian Andersen! This kind of behavior of ingratiating oneself with the wealthy and powerful people is definitely not unfamiliar, in the old days just as it is now. Ripa continued to explain what he thought was the reason.

"I must say that I was not a little surprised to find how Kang-hy [Kangxi], who was really a man of enlarged understanding, believed all the exaggerated praises of his courtiers, and was childishly vain. This was perhaps to be attributed to the flattery that had been continually lavished upon him since the eight years of his age, when he began to reign."

On the other hand it is to be noted that the Chinese Emperor known to Ripa might not be like the same person who so much impressed Verbiest, Bouvet and the other Jesuits a quarter of a century earlier. Besides, perhaps as a result of the caliber of Ripa himself or as a result of a general disagreement between the Jesuits and the Papacy in Vatican, Ripa was far less close to the Emperor than the Jesuits. Thus, we may have to read Ripa's account with some caution.

The third period began with the establishment of the Office of Mathematics in Mengyangzhai [Studio for the Cultivation of the Youth 蒙養齋], a place situated in the garden Changchunyuan [Garden of the Exuberant Spring 暢春園] inside the Imperial Palace. Prince Yinzhi (胤祉), third son of Kangxi Emperor, was made the Head of Office of Mathematics. Besides serving as a school for taking lessons in mathematics, astronomy and science, the main task of this Office was to compile the monumental treatise

Lüli Yuanyuan [Origin of Mathematical Harmonics and Astronomy 律曆淵源], comprising three parts: *Lixiang kaocheng* [Compendium of Observational Computational Astronomy 曆象考成] in forty-two volumes, *Shuli Jingyun* [Collected Basic Principles of Mathematics 數理精蘊] in fifty-three volumes, and *Lülü Zhengyi* [Exact Meaning of Pitchpipes 律呂正義] in five volumes. The compilers were all Chinese officials and scholars, but the book mentioned the contribution of foreign missionaries at the beginning of *Shuli Jingyun*.

The treatise *Shuli Jingyun* includes both traditional Chinese mathematics, the part that was still in extant and was understood at the time, as well as Western mathematics, highly likely from the "lecture notes" prepared by the missionaries for Kangxi Emperor during his first period of ardent study in the 1670's.

[We show some illustrative pages of the monumental treatise. In the beginning the contribution of many foreign missionaries is mentioned, although the whole book was compiled by the hard work of Chinese scholars. The part on geometry is interesting. It is not the six books of *Elements* translated by Matteo Ricci and XU Guang-qi in 1607, but is a translation of the French version, *Elémens de géométrie* by the French Jesuit Ignace Gaston Pardies (1636-1673) published in 1671 with a second edition in 1705. That is probably the textbook Kangxi Emperor learnt geometry from the French Jesuits. Here are pages on trigonometric tables and the logarithmic table, which do not belong to traditional Chinese mathematics but are of European origin, brought into China by the missionaries.]

In the remaining part we will discuss one single topic in more detail, namely, the solving of algebraic equations taught by the French Jesuit Antoine Thomas (1644-1709 安多).

Antoine Thomas, who studied at University of Coimbra from 1678 to 1680 was ordered to go to Peking in 1685. [University of Coimbra, established at the end of the thirteenth century, was an academic centre of the time, and also a place where many Jesuits received their education as students or taught as teachers before they went to missionary work in Asia. Last October I attended a conference in Portugal and made a special trip to pay homage to this very old (but still thriving) university in Europe. When I sat in the lecture room, I had a strange feeling that Verbiest and Thomas might have stood in that same room to deliver their lessons more than three centuries ago!]

By the time Thomas came to China he had compiled *Synopsis mathematica*, which was based on the book *De numerosa potestatum ad exegesim resolution* (On the numerical resolution of powers by exegetics) written by François Viète in 1600. Thomas later revised it as *Suanfa Zuanyao Zonggang* (Outline of the essential calculations 算法纂要總綱) and *Jiegenfang Suanfa* (Method of borrowed root and powers 借根方算法), to be used as lecture notes for the mathematics lessons in the Imperial Court and later incorporated into Books 31-36 of *Shuli Jingyun*.

A public lecture in mathematics should contain some mathematics, so I am going to lay before you a little bit of mathematics. You may doze off for a few minutes if you prefer to skip it. For ease of comprehension we will adopt our accustomed mathematical language instead of what people used to write several hundred years ago.

I will illustrate first with an example in the book of Viète, namely, to solve the equation "*x* squared plus A times *x* equals B". Let x_1 be a first approximation of a root, which is *x*_1 plus *x*_2. Substitute into the equation and neglect the comparatively much smaller term *x*_2 squared. We obtain *x*_2 in terms of *x*_1, A and B. So we have a better approximation *x*_1 plus *x*_2. Keep reiterating the process to obtain a better and better approximation. Let us look at a similar problem in Book 33 of *Shuli Jingyun*.: "If the cube [of root] and eight roots are equal to 1824 *che*, how much is one root?" [Here is a page from the draft on

computation by Kangxi Emperor when he was learning the method.] In modern day mathematical language we want to solve *x* cubed plus eight times *x* equals 1824, which is accomplished by a method called "extraction of cube root with accompanying number". The basic idea is the same as that in the first example. If the answer is not exact, the process will give a better and better approximation to any number of decimal places.

The Chinese mathematician MEI Jue-cheng (1681-1763 梅彀成) told the story on how he learnt the new method. In his book *Chishui Yizhen* (Pearls remaining in the red river 赤水遺珍) of 1761 he said,

"While I was serving in the Imperial Court of the late Emperor canonized as Shengzu Humane [Kangxi], I was instructed on the method of *jiegenfang* (borrowed root and powers 借根方) by the late Emperor. He issued an edict to say that the Westerners called the book *aerrebala* (algebra 阿爾熱巴拉) that means "Method from the East". I respectfully learnt the method, which is really marvelous and is the guide to mathematics.

I suspected that the method resembles that of *tianyuan* (celestial unknown 天元), so I took up the book *Shoushi Licao* (Calculation draft of the *Shoushi* calendar 授時曆草) and studied it, thereby clarifying the matter. Though the terminologies are different the two methods are the same, not just a mere resemblance."

This is an indication of how the saying "Western learning has its origin in Chinese learning" got promulgated in those days. In making his subjects believe that Western learning originated in older Chinese learning, Kangxi Emperor knew that the Chinese would be more than willing to learn it and would not regard it as something opposing traditional value. Or, maybe he really thought that the method originated in older Chinese learning. Indeed, similar methods were explained in mathematical classics of earlier days, most of which became less known by the Ming and early Qing period. As early as in the Qin-Han period (3rd century B.C. E. to 2nd century 秦漢) the famous mathematical classics *Jiuzhang Suanshu* (Nine chapters on the mathematical art 九章算術) explained the method called "extraction of square root with accompanying number", which calculated a root of a quadratic equation to any degree of accuracy. This method was extended by later mathematicians of the 12th/13th centuries during the Song-Yuan period (宋元) to solve equations of higher degree. For instance, in the mathematical classics *Shushu Jiuzhang* (Mathematical treatise in nine sections 數書九章) of 1247 QIN Jiu-shao (秦九韶) treated an equation of degree ten. The method was the same as what is known in the Western world as Horner's method, devised by William George Horner (1788-1837) in 1819. However, it is grossly wrong to say that *aerrebala* (algebra) means "Method from the East". For the Europeans the method was transmitted to them from the Islamic world --- indeed "east" to them, but not from China! The word "algebra" itself does not connote anything about "Method from the East" but comes from the Arabic word *al-jabr*, meaning "restoration", which is to be understood together with another Arabic word *al-muqābala*, meaning "reduction". These two words appear in the title of a famous Arabic treatise of the 9th century from the pen of Muhammad ibn Mūsā al-Khwārizmī. The two words together describe the method in solving an algebraic equation by transposing terms and simplifying the expression, something a school pupil of today would be quite familiar with.

François Viète wrote another book in 1591 with more important influence, namely, *In Artem Analyticem Isagoge* (Introduction to the analytical art). In his book he introduced what he called "*logistica numerosa*" and "*logistica speciosa*", that is, numerical calculation and symbolic calculation. Viète was so pleased with his idea that he concluded his book by the exclaimation: "*Quod est, nullum non problema solver* (There is no problem that cannot be solved)"! It led to subsequent work of René Descartes, *La géométrie* (Geometry)

of 1637 and that of Isaac Newton, *Arithmetica Universalis* (Universal arithmetic) of 1707 (with the work actually done about forty years earlier), by which time mathematicians in Europe were familiar with the use of symbolic calculation. Again, a school pupil of today would be quite familiar with that too, but when Viète first introduced it in his book, it was a very novel idea. We will now see how Kangxi Emperor reacted to it when another French Jesuit, Jean-François Foucquet (1665-1741 傅聖澤) taught him this new method, which Foucquet called the "new method of *aerrebala*".

In an Imperial Edict issued by Kangxi Emperor between 1712 and 1713 he said:

"Every day soon after getting up I study with the Princes the method of *aerrebala* [algebra] and find it most difficult. He[J.-F. Foucquet] says that it is easier than the old method, but it looks more difficult than the old method and has more errors as well as many awkward features. […] Copy this Imperial Edict and issue the book in the capital to the Westerners for them to study it in details, and to delete those parts that do not make sense. It says something like *Jia* multiplies *Jia*, and *Yi* multiplies *Yi*, without any concrete number appearing. One never knows what the result of the multiplication is. It seems that this man [J.-F. Foucquet] is only mediocre in mathematical skill!"

Jean-Francois Foucquet said in his book *Aerrebala Xinfa* [New Method of *Aerrebala* 阿爾熱巴拉新法]:

"The old method uses numerical values, while the new method uses symbols that are accommodating (*tongrong jihao* 通融記號) […] Using this accommodating notation, it is easy to perform calculation, and it enables one to see the situation clearly so that one can focus on the method and understand the underlying rationale of the calculation. The use of numerical value works only for a particular value, while the use of accommodating notation encompasses all values in general."

The frustration expressed by Kangxi Emperor reminds us of the joke in the classroom in which a teacher announces routinely, "Let x be the age of the boy, then…" only to receive a matter-of-fact protest from a pupil, "What happens if it is not?" The explanation given by Foucquet reminds us of an important underlying message, which is however seldom made apparent to school pupils, namely, that we treat numerical quantities as general objects, and manipulate such general objects as if they were numerical quantities. Although we do not know (prior to solving the equation) what they are, we know that they stand for certain numbers and, as such, obey general rules. For instance we do not know what A and B are, but we know that $A \times B = B \times A$. We can therefore apply these general rules systematically to solve problems which can be formulated in terms of equations.

Of the several methods of solving algebraic equations, the "new method" gradually replaced the other methods by the 18th century and was further developed. However, because of this personal dislike of the subject the transmission into China of the powerful method of symbolic calculation was delayed until nearly one-and-half century later! China had to wait for the translation of Augustus De Morgan's *Elements of Algebra* (1835) by LI Shan-lan (李善蘭) and Alexander Wylie (偉烈亞力) in 1859. For a teacher this phenomenon should not be unfamiliar. Many students who face some difficulty in learning a new subject, instead of putting in more effort, choose the easier way out by blaming the teacher for poor teaching and telling everybody that the teacher knows too little! This will be an even more common phenomenon in the case of giving private tutoring to a spoiled child of a billionaire! In some sense, the episode between Kangxi Emperor and Father Foucquet was like that.

Kangxi Emperor's fervor in learning Western mathematics and science was quite unusual for an emperor. Indeed he was the only emperor in the very long history of China who had committed himself to this kind of study in earnest. One may ask, "What made him do it?" Various reasons can be suggested.

Did vanity play a role? With human nature as it is, this is perhaps true to some extent. From official records or from his own writings one can see that Kangxi Emperor liked to show off before his ministers by teaching them Western mathematics and astronomy, or asking them questions that he knew would be hard for them, followed by his own explanation. He loved going on field trips with an entourage of ministers and displaying his vast knowledge. But mind you, he had to work hard himself to do it, and we will soon see that he had a better reason for doing it than just showing off.

Did statecraft require such kind of study? It surely helps. He well understood that such knowledge helped towards the betterment of his country and subjects.

Was it motivated by intellectual curiosity? To some degree this would be true, judged from the character and personal trait of Kangxi Emperor.

There are examples of a political leader who has some interest in mathematics. Napoléon Bonaparte (1769-1821) is an example. It was reported by his school teacher that the young Napoléon had a talent for mathematics, and his name has been attached to several nice results in synthetic geometry. One is the Napoléon's Problem that asks for four equal partitions of the circumference of a circle with given centre using only a pair of compasses, or more generally asks for a way to find the centre of a given circle using only a pair of compasses. Another is the so-called Napoléon's Theorem that says: If three equilateral triangles are erected on the three sides of an arbitrary triangle (all outside or all inside), then their centres form another equilateral triangle. Not everybody believes that these results are to be credited to Napoléon. The famous Canadian geometer Harold Scott MacDonald Coxeter (1907-2003) once commented that it is questionable for Napoléon to know enough geometry to think about such problems, just as it is unlikely that his command of English was good enough to compose the famous palindrome (not in French but in English!) "Able was I ere I saw Elba"! However, I think it might be possible for Napoléon to come up with those mathematical results. Trained as an artillery officer in his youth he should know enough geometry to do it. It should not come as a surprise that he developed an interest in mathematics when he was young and kept it up. He made friends with a number of famous French mathematicians of his days and he took a serious interest in re-organizing the Ecole Polytechnique that was famous for its strength in mathematics. However, compared to Kangxi Emperor, Napoléon only dabbled in geometry, with his name attached to a couple of interesting but not as significant oddities in synthetic geometry. Instead, Kangxi Emperor paid much more serious and organized attention to the study of mathematics and science.

[We show here some pages from another notebook of his: *Kangxi Jixia Gewu Bian* (Writings on investigation of things by Kangxi Emperor during his leisure time amidst the dealing with state affairs 康熙幾暇格物論).] Intellectual curiosity to a sufficient extent has to be there to sustain such a strong interest.

Had it to do with heritage? To some extent this is perhaps also the case, to strengthen the impression that he had the mendate to rule and to carry on traditional Chinese learning which he thought was only later improved by the Westerners.

Was there a political motive for control? The answer is definitely in the affirmative. By the late 17th century and the beginning of the 18th century the ethnic group of Manchu people was still regarded by most Han Chinese as alien, just like they regarded the European missionaries as alien people. Other than using a high-handed rule by oppression Kangxi Emperor chose to convince his subjects that he was well-versed in traditional Chinese learning and even more, he had a good command of Western learning as well. Compared with many of his ministers he was much more open to Western learning, perhaps because as I just said, being regarded by the Han Chinese as alien he might felt more akin to the European missionaries than to the Han Chinese, whom he saw as his subordinates.

The historian Robin George Collingwood says,

"History is thus the self knowledge of the living mind. […] For history is not contained in books or documents; it lives only, as a present interest and pursuit, in the mind of the historian when he criticizes and interprets those documents, and by so doing relives for himself the states of mind into which he enquires."

We can only make a guess of the inner mind of Kangxi Emperor. I tend to believe that Kangxi Emperor was very diligent, determined and bright, but also studied hard not without vanity and intention to show off his knowledge with a political motive. In any case, the outcome was several sets of monumental compendium and collection of books that benefit posterity. However, it has to be admitted (sadly) that owing to the limitation in his scope and motive this period of study was also a "missed opportunity" for China. Being confined to a small group within the Imperial Court this period of transmission of Western learning in China did not exert the kind of impact and influence that would have helped the country to move forward and catch up with the Western world which had moved forward by leaps and bounds since the 16th-17th centuries. The first wave in the latter part of the Ming Dynasty came before this happened. Kangxi's period would be a golden opportunity to catch up. History tells us that the catching up had to wait until the second half of the 19th century, by which time China was forced to do it at the threat of gunboats and expeditionary forces and humiliating defeats under foreign exploitation, no more a time to learn in a more peaceful setting as in the two preceding periods.

With his motive in keeping control over Western learning by limiting it to a small circle in the Imperial Court and particularly to himself, the effect of this phase of transmission was not much, definitely far less effective than the second wave that was to come one and half century later. However, the compilation of books like *Shuli Jingyun* did benefit posterity. It also led to a revival of interest in re-discovering traditional Chinese mathematics in the subsequent years before the second wave arrived and helped the Chinese mathematicians to understand the new learning better.

A Chinese idiom says: "Jin wu zu chi, ren wu wan ren (As there is no gold that is completely pure, there is no human being that is completely perfect 金無足赤, 人無完人)." Kangxi Emperor had done his part, and he will be remembered as a good and decent ruler in the long history of China.

Thank you for coming to this lecture!