Scientific Contributions : An Overview

Ngaiming Mok

Mok's career in Complex Geometry spans over more than 3 decades in the US, France and China (since he joined the University of Hong Kong in 1994), and his scientific works link up a wide spectrum of subject areas including Complex Differential Geometry, Several Complex Variables, Complex Algebraic Geometry and most recently Arithmetic Geometry.

Capitalizing on his works in Kähler Geometry especially on the compactification of complete Kähler manifolds of finite volume (Mok [*Ann. Math.* 1989], Mok-Zhong [*Ann. Math.* 1989]) and on holomorphic isometries with respect to the Bergman metric (Mok [*JEMS* 2012]), Mok has recently developed a novel methodology for tackling problems in Functional Transcendence Theory (Mok [*Compositio Math.* 2019], Chan-Mok [*J. Diff. Geom.,* to appear]) using a combination of differential geometric, complex analytic and algebraic geometric methods, which, in collaboration with researchers in Number Theory and Model Theory (in Mathematical Logic), has led to a surprising resolution in the affirmative of the Ax-Schanuel Conjecture for Shimura varieties (Mok-Pila-Tsimerman [*Ann. Math.* 2019]) in Functional Transcendence Theory with striking applications to Arithmetic Geometry.

In connection with the study of minimal rational curves on uniruled projective manifolds, Mok [*J. Diff. Geom.* 1986] introduced the concept of the variety of minimal rational tangents (without the terminology), alias VMRTs, in his resolution in the affirmative of the Generalized Frankel Conjecture (on the characterization of compact Kähler manifolds of nonnegative holomorphic bisectional curvature) using a combination of non-linear partial differential equations (the Ricci flow) and the deformation theory of rational curves.

In collaboration with J.-M. Hwang, Mok has developed a geometric theory of uniruled projective manifolds modelled on varieties of minimal rational tangents, resolving a number of important classical problems in Complex Algebraic Geometry regarding complex structures and rigidity phenomena (Hwang-Mok [*Invent Math.*1998, 1999, 2004, *Crelle* 1997, *Ann. ENS* 2002]), proving especially the rigidity of rational homogeneous manifolds $S = G/P$ of Picard number 1 under Kähler deformation with 1 identificable exception (for which Kähler deformation rigidity fails), and resolving in the affirmative the Lazarsfeld Problem (on the characterization of nonsingular holomorphic images of $S = G/P$ of Picard number 1).

More recently, Mok has initiated a study of local complex submanifolds of uniruled projective manifolds X and developed a theory of uniruled projective subvarieties on them. Hong-Mok [*J. Diff. Geom.* 2010, *J. Alg. Geom.*2013, *Selecta Math.* 2020] has in particular established the Schur rigidity of non-linear Schubert cycles Z on uniruled projective manifolds $S = G/P$ of Picard number 1 (i.e., that any effective cycle homologous to a positive integral multiple of Z must be a sum of translates of Z by elements $g \in G$). He has developed in Mok-Zhang [*J. Diff. Geom.* 2018] a general theory of geometric substructures modelled on sub-VMRTs on a uniruled projective manifold X of Picard number 1 using methods of Complex Differential Geometry, giving in particular sufficient differential geometric conditions in terms of the projective second fundamental form on VMRTs to guarantee that certain analytic germs of complex submanifolds $\Sigma \subset S$ are in fact *algebraic*.

Mok's works [*Proc. Nat. Acad. Sci.* 1986, *Ann. Math.* 1987] on Hermitian metric rigidity on finite-volume quotients of bounded symmetric domains have been instrumental in the formulation and solution of rigidity problems for holomorphic maps, and in the study of harmonic maps in Kähler geometry (Cao-Mok [*Invent Math.* 1990], Mok [*Invent Math.* 1992], Mok [*Invent Mok.* 2004] and Koziarz-Mok [*Amer. J Math.* 2010]). On the subject of harmonic maps Mok-Siu-Yeung [*Invent Math.* 1993] proved geometric superrigidity of compact Riemannian symmetric manifolds of negative Ricci curvature other than real and complex hyperbolic space forms.

The different components of Mok's research contributions are all integrated as a whole, and he has put forth a dynamic perspective on the important subject area of Complex Geometry with its rich contents (Complex Differential Geometry, Several Complex Variables, Complex Algebraic Geometry, and Partial Differential Equations) and multiple interfaces with Algebraic Groups, Functional Transcendence Theory, Arithmetic Geometry, etc. In his research endeavors Mok has been focusing on uniformization problems, rigidity phenomena, deformation theory, curvature conditions and geometric structures. His research works have consistently and in particular most recently impacted the study of Kähler manifolds, uniruled projective manifolds, Schubert cycles, Shimura varieties, totally geodesic subsets and unlikely intersections arising from a variety of vibrant research areas in Mathematics.