Graduate Courses
(Updates Spring 2008)

MATH6101 Intermediate Complex Analysis

Meeting Date / Time: Lecture 1-6 every Thursday Jan 17 - Feb 28, 2008, 3:00 - 5:45pm

Lecture 7-12 every Thursday Mar 13 - Apr 17, 2008, 3:00 - 5:45pm

Venue: Room 517, Meng Wah Complex, HKU starting Jan 17, 2008

Part I: Lectures 1-6 by Professor Ngaiming Mok
Part II: Lectures 7-12 by Dr. Tuen Wai Ng

Part I: Meromorphic Functions
Professor Ngaiming Mok

We will first of be dealing with questions of existence of holomorphic and more generally meromorphic functions with various properties on plane domains. We start with the inhomogeneous Cauchy-Riemann equation on plane domains and derive a proof of Runge's Theorem on approximating holomorphic functions. The latter will be applied to prove, in the case of plane domains, two fundamental existence results: existence of meromorphic functions with prescribed principal parts, given by the Mittag-Leffler Theorem, and presciption of meromorphic functions with prescribed zeros and poles.

If time permits, we will also discuss a special class of meromorphic functions on the complex plane, viz., those that are doubly-periodic with respect to a lattice. This includes the construction of elliptic functions and of theta functions, and the representation of elliptic functions in terms of translates of the sigma function.

References:

1. R. Narasimhan: Complex Analysis in One Variable, Birkhäuser 2001 (2nd edition).
2. O. Forster: Lectures on Riemann Surfaces, Springer-Verlag 1981.
3. J.B. Conway: Functions of One Complex Variable I, Springer-Verlag 1995.
4. K. Chandrasekharan: Elliptic Functions, Springer-Verlag 1985.

Part II: Normal Families
Dr. Tuen Wai Ng

The notion of a normal family of meromorphic functions has played an important role in the development of complex function theory. In fact, many important results, like Picard's theorem, Schottky's theorem, and the Riemann mapping theorem, can be proved by the normal family approach.

In the first part of this course, we shall first introduce the notion of a normal family of meromorphic functions and give several necessary and sufficient conditions on the normality (these include the Montel's three points theorem and Marty's theorem). We then apply this results to prove the classical results mentioned above. One guiding principle for discovering new normality tests is the so-called Bloch principle which is only a heuristic principle. We shall make it a rigorous principle by proving the Robinson-Zalcman theorem. One main ingredient of the proof of Robinson-Zalcman theorem is the Zalcman's rescaling lemma. It turns out that this lemma has enormous applications in complex analysis and complex dynamics and we shall explore a few of them.

References:

1. Joel L. Schiff: Normal Families,Springer-verlag,1993.
2. Lawerence Zalcman: Normal Families: New Perspectives, Bull. Amer. Math. Soc. 35 (1998), 215-230.
3. Walter Bergweiler: Rescaling principles in function theory. Analysis and its applications (Chennai, 2000), 11-29, Allied Publ., New Delhi, 2001.