Planning and Scheduling Staff Duties by Goal Programming

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We propose goal programming (GP) models for an integrated problem of staff duties planning and scheduling, for baggage services section staff at the Hong Kong International Airport. The problem is solved via its decomposition into a GP planner, followed by a GP scheduler. The results can be adopted as a good crew schedule in the sense that it is both feasible, satisfying various work conditions, and "optimal" in minimizing overtime shifts.

Keywords: planning, scheduling, rostering, goal program modeling

INTRODUCTION

This paper advocates a general modeling framework for a complete crew assignment system. It arises naturally as a mathematical description for the staff deployment problem of their baggage handling agents at BSS-HAS, the Baggage Services Section of the Hongkong Airport Services, Ltd. HAS of the (new) Hong Kong International Airport (at Chak Lap Kok of Lantau Island) is the primary handler of all ground services and support functions, including aircrafts and passengers alike.

Our project of optimization modeling for staffing is motivated by the need to produce daily work plan of the baggage service agents at the passenger terminal. Our complete BSS crew system consists of its three component GP models: the Duties Generation Problem (DGP), the Crew Scheduling Problem (CSP) and the Crew Rostering Problem (CRP). While such modeling may well be regarded as one among the vast literature of the commonly known area of workforce planning/scheduling (an excellent review is given by Bodin et al, 1983), our decomposition approach has, for the actual case study, exhibited its significant impact albeit its modeling simplicity. The resulting preemptive goal programming formulations have very satisfactorily addressed the planning/scheduling/rostering issues to handle frequent changes of flight schedules by flexibility in work patterns of agent duties.

Crew Scheduling

In the general area of routing and scheduling of vehicle and crew (Bodin et al, 1983), it is common to separate the overall problem into two steps consisting of the determination of the time tables – vehicle routing, followed by the staff assignment – crew scheduling.

Various useful models for crew scheduling problem (CSP) aiming at differing merits and purposes have been proposed, such as (matching based) heuristics models of Ball et al, 1983; network models of Carraresi and Gallo, 1984; and set partitioning models of Falkner and Ryan, 1987. Among the mathematical programming approaches, there are work of Lessard et al, 1981;

column generation approach of Desrochers and Soumis, 1989, Desrochers et al, 1992; integer programming approach of Ryan and Foster, 1981, Ryan and Falkner, 1987; decomposition approaches of Patrikalakis and Xerocostas, 1992, Vance et al, 1997; and complementary approaches of Wren et al, 1985.

These quoted above constitute only a tiny fraction of the vast literature, not to mention techniques of implementation for practical applications, notably computerized scheduling such as the various reported systems of "HASTUS" of Lessard et al, 1981, "CREW-OPT" by Desrochers, et al, 1992, "EXPRESS" by Falkner and Ryan, 1992; and that of Chu and Chan, 1998.

Successful real applications are extremely significant for the airlines. Besides the "household name" of SABRE, we mention two most recent "milestone" works of Vance et al, 1997 and of Mason et al, 1998.

Crew Rostering

The outcome of the crew scheduling phase is typically a set of daily staff assignments required to cover the (actual or forecast) demand. "In the (next) *crew rostering* phase, a set of *working rosters* is constructed that determine the sequence of duties that each single crew has to perform ..., to cover everyday all the duties selected in the first phase" (quoted from Caprara et al, 1998). This has been referred to as the Crew Rostering Problem (CRP) by Caprara et al in their FARO prize winning work for the Italian Railway *Ferrovie dello Stato SpA*, jointly sponsored by the Italian Operational Research Society during 1994-1995.

Similar to the case of crew scheduling, past work on CRP has seen numerous approaches and applications. There are optimization approaches such as that of Gamache and Soumis, 1993; network model of Balakrishnan and Wong, 1990; and column generation approach of Gamache et al, 1994. Novel heuristics approaches integrating Set Covering and/or Assignment Problem are reported by Hagberg, 1985, Carpaneto and Toth, 1987, and Caprara et al, 1995. More recently, Valouxis and Housos, 2002, propose a quick heuristics for combined bus and driver scheduling, consisting of minimum cost matching, set partitioning and shortest path.

Duties Generation

The modeling formulation of DGP that we put forth here can be interpreted as the basic core – the planner – of a more sophisticated DGP/CSP/CRP integrated model in the following sense. DGP in its simplest form (computes and) allocates duties (of given fixed structure of work pattern, rather than crew or staff needing further varying requirements of scheduling) to cover known demands. Demands are given, for equally spaced (such as half-hourly) time intervals of (the working time of) a day. As such, DGP is the *prerequisite* to CSP and CRP in that it provides the planning inputs needed in subsequent scheduling and rostering of staff. The logical flow of their relationships can be summarized below, where t = hour of day, j = day of week, p = weekly work pattern and a = agent,

Hourly Demands $D(t)$
<<**Planner: DGP Model**>>
Duties $X(i,k)$
Allocations $R(j,t)$
$\downarrow$
Daily Requirements $R(j) = \sum_t R(j,t)$
<<**Scheduler: CSP Model**>>
Daily Staffing $S(j)$
Allocations $R(j,t)$
$\downarrow$
<<**Roster: CRP Model**>>
Duty Rosters $I(p,a,j,t)$

GOAL PROGRAMMING MODELS

As its name implies, DGP allocates duties (performed by crew) in an optimal way to meet known demand over a contiguous number of time intervals. We describe only its *extended* formulation below. A detailed account of DGP formulations is given in an earlier paper of Chu, 2001.

DGP Model

We use the following common notations for all the subsequent models. Let H be the working time horizon, and $h = 1, \cdots, H$ index the individual hours (or half-hours). R_h denotes the demand for interval h and d_h represents the over allocation (or over-achievement deviation variable in a goal programming context) at interval h.

The length of a duty is denoted by J. The primary decision variable x_{ij} is the number of allocated staff that starts duty from interval i and breaks at the j^{th} interval after the start of duty, $j = 1, \cdots, J$. Hence for a working horizon of intervals $1 \cdots H$, we have for the index $i = S, \cdots, T$. The earliest start interval S is such that $S \geq 1$ whereas the latest start interval T is limited to $T \leq H - J + 1$ (to finish work at interval H). Note that normally $S = 1$ as long as $R_1 > 0$ (there is demand for the very first interval); and $T = H - J + 1$ whenever $R_H > 0$ (there is demand for the very last interval).

As noted by Mason et al, 1998, personnel scheduling problems (or referred to as *workforce allocation* problems by Baker, 1976) have been studied for many years. Network flow formulations, such as in Segal, 1974 and Bartholdi and Ratliff, 1978 can well handle their simplest forms. Additional side constraints such as break requirements demand more complex procedures.

One advantage of the DGP model is its ease of extension in various ways. One such concern is the inclusion of flexibility in staffing mode: introducing over-time (OT) for any number of on-duty staff. This simply calls for adding another decision variable y_{mn} representing the number of allocated OT staff who start work at interval m and finish work in interval n. Defined generally as such, OT work can take different modes: a (limited) number of intervals immediately before a regular time (RT) duty only, or a (similarly limited) number of intervals immediately after an RT duty only.

As an illustration, the DGP model with OT allocation is given below.

$$\text{Min} \quad \sum_{m=1}^{H} \sum_{n=m}^{m+L-1} \left(\sum_{h=m}^{n} T_h\right) y_{mn} \tag{1a}$$

$$\text{Min} \quad \sum_{i=S}^{T} \sum_{j=1}^{J} c_{ij} x_{ij} \tag{1b}$$

$$\text{Min} \quad WD \tag{1c}$$

Subject to

$$\sum_{i=p}^{q} \sum_{j \neq h-i+1} x_{ij} + \sum_{m=h-L+1}^{h} \sum_{n=h}^{m+L-1} y_{mn} - d_h = R_h , \quad h = 1, \cdots, H \tag{2}$$

$$\sum_{m=(i-1)-L+1}^{i-1} y_{m,i-1} + \sum_{n=i+J}^{(i+J)+L-1} y_{i+J,n} \leq \sum_{j=1}^{J} x_{ij} , \qquad i = S, \cdots, T \tag{3}$$

$$\sum_{i=S}^{T} \sum_{j=1}^{J} x_{ij} \leq \text{MaxRT} \tag{4}$$

$$\sum_{m=1}^{H} \sum_{n=1}^{H} y_{mn} \leq \text{MaxOT} \tag{5}$$

$$d_h \leq D , \qquad h = 1, \cdots, H \tag{6}$$

Here $p \equiv \max \{h - J + 1, S\}$, $q \equiv \min \{h, T\}$, and both the RT allocation $\{ x_{ij} \}$ and the OT allocation $\{ y_{mn} \}$ are non-negative integer variables. Note that L stands for the maximum number of (additional) OT intervals allowed before or after the J RT intervals.

We see that the LHS of constraint (2) is the total work contribution as a function of both RT and OT staff. The RT (or x_{ij}) portion is straightforward, while the OT (or y_{mn}) part picks out the total number of OT staff for a maximum span of L intervals which cover h. Constraint (3) ensures that each y_{mn} is indeed an OT allocation, by stipulating that an OT is assigned only if there is already an RT x_{ij} allocation (before or after). The single parameter *MaxRT* of constraint (4) denotes the maximum number (or strength) of RT staff and that *MaxOT* of constraint (5) is the maximum permitted number of OT staff. Suitably mixed (i.e. RT+OT) duties allocation can be obtained by varying these two parameters in repeated runs of the model, preemptively with the three goals (1a) to (1c) in that order.

Finally, the coefficients $\{ T_h \}$ in (1a) represent the unit OT pay rates, possibly different for different time intervals of the day, whereas the coefficients $\{ c_{ij} \}$ in (1b) represent the usual unit RT pay rates. The single variable D of constraint (6) records the maximum (i.e. over achievement) deviation over all time intervals. Its non-smoothed penalty term WD in (1c) is treated as a lowest priority goal in search of an "optimal" mixed duties allocation plan that includes both (positive) x_{ij} and y_{mn}.

CSP Model
Next, the CSP model, which is often referred to as the (cyclic or weekly) staffing model (see, for example, Schrage, 1999) is stated below.

Inputs: (from DGP Model)
$R(j) \equiv \sum_t R(j,t)$ Required no. of start duties on day j
Constratints:

$$\sum_{1 \le i \le 5} \mathrm{START}(@wrap(j-i+1,7)) - \mathrm{OVER}(j) = R(j), \; j = 1, \ldots, 7$$

Objective functions:

$$\mathrm{Min} \sum_i \mathrm{START}(i)$$

$$\mathrm{Min} \;\; \mathrm{MaxOVER} \;\; (\equiv \mathrm{Max}_j \; \mathrm{OVER}(j))$$

CRP Model
Finally, the CRP model, which in many ways has the interpretation of a set-covering formulation is given below.

Indices:
- $p =$ roster pattern $(1, \ldots, 7)$
- $a =$ baggage service agent (BSA)
- $j =$ start time half-hour $(1, \ldots, 11$, or $12, \ldots, 22)$
- $t =$ day of week $(1, \ldots, 7)$

Inputs:
$R(j,t) =$ required no. of start duties (at half-hour j on day t)
— output from DGP Model
$S(t) =$ required no. of starting crew (on day t)
— output from CSP Model
Covering (Roster) Variables:
$I(p,a,j,t) = 1$, if agent a is assigned to cover roster pattern p at time j on day t
$I(p,a,j,t) = 0$, if one or more of the following conditions hold:
i) $a > S(p)$
ii) $t = @\ \mathrm{warp}(p+5,7)$ $[\equiv t1(p)]$
iii) $t = @\ \mathrm{warp}(p+6,7)$ $[\equiv t2(p)]$
iv) $R(j,t) = 0$

Covering (Roster) Constraints:

1) Each (assigned) agent gets 1 duty on each working day

$$\sum_{j|R(j,t)\geq 1} I(p,a,j,t) \;=\; 1, \quad \forall p, a \leq S(p), t \neq t1(p), t2(p)$$

2) Each (assigned) agent gets 5 duties each week

$$\sum_{j,t|R(j,t)\geq 1, t\neq t1(p), t2(p)} I(p,a,j,t) \;=\; 5, \quad \forall p, a \leq S(p)$$

3) Start duties ($R(j,t)$) of each slot are covered

$$\sum_{p,a\leq S(p)} I(p,a,j,t) \;\geq\; R(j,t), \quad \forall j,t|R(j,t) \geq 1; t \neq t1(p), t2(p)$$

4) Start rosters ($S(t)$) of each day are allocated

$$\sum_{p,a\leq S(p)} \sum_{j|R(j,t)\geq 1} I(p,a,j,t) \;-\; D(t) \;=\; S(t), \quad \forall t \neq t1(p), t2(p)$$

where $D(\cdot)$ is the over allocation to be minimized in the objective function.

A CONCLUDING REMARK

The purpose of this paper is to illustrate by way of this DGP/CSP/CRP modeling and computational experience, the advantage of its readily producing significant improvement over existing manual staff assignment. Its usefulness is somehow, in our opinion and experience of actually applying it in real situations, rather highly out of proportion with regard to its modeling simplicity. The system's usefulness to the HAS users is indeed decreasing from planning (DGP), to scheduling (CSP), and finally to dispatching (CRP). The last is still influenced regularly by day-to-day actual dispatching and rostering needs (which are left more to the field operational supervisors).

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