AS/2905/WKC/04

THE UNIVERSITY OF HONG KONG
DEPARTMENT OF MATHEMATICS
MATH 2905 QUEUEING THEORY AND SIMULATION
Assignment 4

Due Date: 23 April 2014. (5:00 pm) [2%]

1. (a) Consider the mixed congruential method for the generation of pseudorandom numbers $[x_n = (ax_{n-1} + c) \bmod m]$ with $a = 17$, $c = 43$, $m = 100$ and $x_0 = 2$. Generate four random numbers $\{r_1, r_2, r_3, r_4\}$ in $(0, 1)$ such that $r_i = x_i/100$.
(b) By using the four random numbers in (a) to obtain an approximation of the definite integral:
$$I = \int_0^2 e^x dx = e^2 - 1.$$
(c) Obtain an approximation of I by using four more random numbers generated by the method described in (a) (i.e. $\{x_1, x_2, \ldots, x_7, x_8\}$). Compare the answer with the one you obtained in (b). What did you find?

2. Initially, $P_1 = 1, P_2 = 2, \cdots, P_n = n$. Explain what does the following algorithm do to $P_1, P_2, \cdots, P_n$.
Step 1. Set $k = n$.
Step 2. Generate a random number U and let $I = \lfloor kU \rfloor + 1$.
Step 3. Interchange the values of P_I and P_k.
Step 4. Let $k = k - 1$
Step 5. If $k > 1$ go to Step 2; otherwise stop.

3. The negative binomial probability mass function with parameters (r, p), where r is a positive integer and $0 < p < 1$, is given by
$$p_j = \frac{(j-1)!}{(j-r)!(r-1)!} p^r (1-p)^{j-r}, \quad j = r, r+1, \cdots$$
(a) Use the fact that a negative binomial random variable can be regarded as the sum of r independent, identically distributed geometric random variables, obtain an algorithm for simulating from this distribution.
(b) Verify the relation
$$p_{j+1} = \frac{j(1-p)}{j+1-r} p_j \ .$$
Use this relation to give a second algorithm for generating negative binomial random variables.

4. Give an algorithm for generating a random variable having p.d.f.
(i) $f(x) = \frac{e^x}{(e-1)}, \quad 0 \le x \le 1.$
(ii)
$$f(x) = \begin{cases} e^{-2|x|}, & x \le 0 \\ e^{-2x}, & x > 0 \end{cases}$$