Optimal Advertising Outsourcing Strategy with Different Effort Levels and Uncertain Demand

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Abstract

We study the issue of advertising outsourcing and production planning for a manufacturer facing asymmetric advertising cost and uncertain market demand. To improve product sales, a manufacturer would hire an advertising agency to provide professional service on product advertising before the production takes place. A contract taking into account both advertising effort level and payment is introduced to incentivize the advertising agency to report the exact cost to the manufacturer. Furthermore, a model with the goal of maximising the manufacturer's net profit is proposed, in which both product demand and payment to the advertising agency are affected by the advertising effort level. Analytical solutions of the optimal strategies including the optimal advertising effort level and the optimal payment to the advertising agency are derived. Optimal retail price and the optimal production quantity are also obtained for the manufacturer in making managerial decisions.

Keywords: Production; Advertising outsourcing; Principal-agent problem; Revelation principle; Uncertain demand.

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Outline

1. Motivations : Problems and Objectives
2. The Basic Model : Notations, Sequence of Events, Assumptions and Contract
3. Model Analysis
4. Numerical Examples
5. Conclusions

Problems

- A higher level of advertising effort can develop trendier products. However, the manufacturer **may not be experienced** in product advertising design.
- Due to the globalization of economics and the consideration of the core competencies and cost issues, manufacturers are **inclined to outsource** their product advertising to other experienced and professional agent companies.

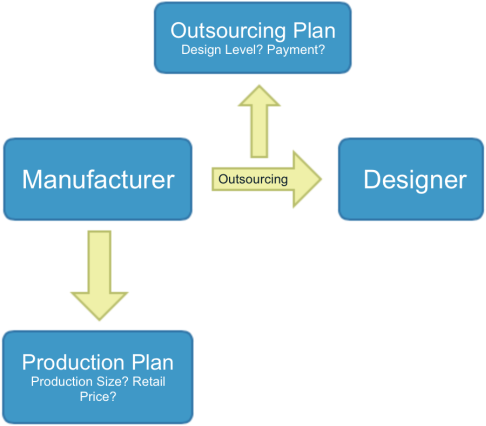
  • There are three major strategies for companies adopting outsourcing: **business improvement**, **business impact** and **commercial exploitation**, see for instance, Diromualdo and Gurbaxani (1998).

Problems

- In most outsourcing models, **one popular assumption** is that both the **principal (manufacturer)** and the **agent** **know the information during the whole process**.
- However, the principal may not react to market changes as fast as the agent, since the agent is more professional in some areas. The **information** between the manufacturer and the advertising agency is **asymmetric**.
- Therefore the advertising agency (agent) may tend to **report a cost higher than the actual cost** for making more profit.

Problems

- Here in this project, we consider the problem of finding the **optimal strategy** for the manufacturer to **maximize his profit**, including the **outsourcing plan** and **production plan**.
- Taking into account that the cost information between the manufacturer and the advertising agency is asymmetric.
- Furthermore, in our study, the **product demand** is assumed to be dependent on the **retail price** and the **advertising effort level**.


Outsourcing Plan
Design Level? Payment?
Manufacturer
Outsourcing
Designer
Production Plan
Production Size? Retail Price?

Objectives

Our model aims to address the manufacturer's **outsourcing and production planning problems** when he has **asymmetric information** about the advertising agency's **advertising cost**. To achieve this

- A contract is introduced to induce truthful information based on the revelation principle, Myerson (1979).
- When asymmetric information occurs regarding the agent's cost, the principal can design a **mechanism** or a **contract** under which the agent would get her **maximum profit when she reports the true cost**.
- A manufacturer-led advertising outsourcing model is presented to maximize the manufacturer's profit based on the principal-agent framework.

- **The revelation principle** is a fundamental principle in mechanism design. It states that if a social choice function can be implemented by an arbitrary mechanism (i.e., if that mechanism has an equilibrium outcome that corresponds to the outcome of the social choice function), then the same function can be implemented by an incentive compatible direct mechanism (i.e., in which players truthfully report type) with the same equilibrium outcome (payoffs).
- **A social choice function** is a function that maps a set of individual preferences to a social outcome. An example function is the utilitarian function, which says “give the item to a person that values it the most”.

Notations

p	the unit retail price (a decision variable);
Q	the production size (a decision variable);
c	the unit manufacturing cost;
x	the true time cost parameter of the designer;
y	the reported cost parameter of the designer;
m	the advertising effort level.

Sequence of Events

Step 1 The manufacturer (principal) offers a contract to the advertising agency (agent), which includes different advertising cost parameters, the corresponding advertising effort levels and the corresponding payments.

Step 2 The advertising agency reports an advertising cost parameter, and then chooses an option which includes the advertising effort level and the payment promised by the manufacturer.

Step 3 The manufacturer decides the production size Q when the advertising effort level is revealed, and carries out the production. The retail price p is then determined before the selling season.

Assumptions

- The manufacturer knows the **probability distribution of the true cost parameter** x that defined in the interval $[\underline{b}, \bar{b}]$ with the **probability density function** $f(\cdot)$ and **cumulative distribution** $F(\cdot)$.
- The manufacturer decides the **advertising effort level** $m(\cdot)$ which is a function of the **reported cost** y:

$$\underline{m} \leq m(y) \leq \overline{m}, \quad \forall y \in [\underline{b}, \bar{b}] \tag{1}$$

and also the advertising cost $t(y)$ to the advertising agency based on the reported value y.

- We assume that the **demand** $\Lambda(p, m(\cdot))$ satisfies

$$\boxed{\Lambda(p, m(y)) = A - \lambda p + km(y)} \quad (2)$$

where A, λ, and k are positive quantities.

- The **linear demand function** has been widely used in literature, see for instance, He et al. (2009), Perdikaki et al. (2016).
- It is natural to assume that the demand is positive and $A >> \lambda c$. Here λ and k are measures for market sensitivity in response to the change of the retail price and the advertising effort, respectively.

- The cost of **advertising effort** is supposed to be **quadratic** in m and is given by

$$\boxed{C(x, m) = xm^2}$$

where x is the advertising cost parameter. This function has been widely used to describe the cost of advertising effort, see for instance, Jorgensen and Zaccour (2014).
• We adopt the quadratic function in our discussion to describe the cost of advertising effort because it is the simplest function that satisfies two important features.
(i) It is **strictly increasing** because higher effort level means higher costs in human resources, materials and time, etc.
(ii) With the **convexity** of the function, one is able to show that the marginal cost of advertising effort is increasing, see for instance, Huang et al. (2011).

Contract Based on the Revelation Principle

Designer's Profit

Denote the advertising **agency's profit** by $\beta(x, y)$, and it is assumed to be

$$\boxed{\beta(x, y) = t(y) - x m(y)^2.} \tag{3}$$

- $t(y)$ **is the advertising cost** paid by the manufacturer to the advertising agency based on the reported value y.
- The manufacturer decides the **advertising effort level** $m(y)$ **based on the reported value** y.
- $x m(y)^2$ is **the cost of advertising effort**, and it is assumed to be quadratic in advertising effort level $m(y)$.

Contract Based on the Revelation Principle

Constraints

- The manufacturer has to ensure that the designer getting her maximal profit when she reports the true cost parameter, i.e., the Incentive Compatibility (IC) constraint, namely,

$$\beta(x,y) = t(y) - xm(y)^2 \leq \beta(x,x), \quad \text{for all } y,x \in [\underline{b},\bar{b}]. \quad (4)$$

- The Participation Constraint (PC) is to ensure the designer gets the reservation profit zero when she reports the true cost parameter, i.e.,

$$\beta(x,x) = t(x) - xm(x)^2 \geq 0, \quad \text{for all } x \in [\underline{b},\bar{b}]. \quad (5)$$

- Decision Constraint (DC)

$$\underline{m} \leq m(x) \leq \overline{m}, \ \forall x \in [\underline{b},\bar{b}] \quad (6)$$

Manufacturer's Net Profit

- The manufacturer's net profit is denoted by π, and it is given as follows:

$$\begin{aligned} \pi &= p \cdot \min\{Q, \Lambda(p, m(x))\} - c \cdot Q - t(x) \\ &= G(p, Q, m(x), x) - t(x) \end{aligned} \tag{7}$$

where

$$G(p, Q, m(x), x) = (p - c)Q - p[Q - (A - \lambda p + km(x))]^{+}.$$

- It consists of the sales revenue, the manufacturing cost and the **payment to the designer.**

Manufacturer-led Advertising Outsourcing Model

- The manufacturer's task can be described as a two-stage problem. We analyse this two-stage problem by backward induction:
  • The first step is to determine the retail price p and the production size Q with the assumption that $m(\cdot)$ and $t(\cdot)$ are given.
  • The second step is to determine the functions $m(\cdot)$ and $t(\cdot)$.
  • Furthermore, the first step is also analysed by backward induction, which implies that p is determined firstly with the assumption that Q is given and then Q is determined with the aim of maximising the total net profit π.

Manufacturer-led Advertising Outsourcing Model

- The following is the optimization problem:

$$\left\{ \begin{array}{ll} & \max_{m(\cdot),t(\cdot)} \left\{ E\left[\max_{Q}\{\max_{p}\{\pi\}\}\right]\right\} \\ \text{subject to} & \\ & (IC): \ \beta(x,x) \geq \beta(x,y), \ \ \forall x,y \in [\underline{b},\bar{b}], \\ & (PC): \ \beta(x,x) \geq 0, \ \ \forall x \in [\underline{b},\bar{b}], \\ & (DC): \ \underline{m} \leq m(x) \leq \overline{m}, \ \ \forall x \in [\underline{b},\bar{b}]. \end{array} \right. \tag{8}$$

Proposition (1)

$$\text{Denote} \quad \Pi = \max_{Q}\left\{\max_{p}\{G(p,Q,m(x),x)\}\right\} - sm(x)^2 \qquad (9)$$

where $s = x + h(x)$ and $h(x)$ is the inverse hazard rate, i.e., $h(x) = F(x)/f(x)$ [1].

Optimization problem (8) can be converted to the following optimization problem:

$$\begin{cases} & \max_{m(x)}\left\{\int_{\underline{b}}^{\bar{b}} \Pi f(x)dx\right\} \\ \text{subject} & \text{to} \\ & m'(x) \le 0, \quad \forall x \in [\underline{b}, \bar{b}], \\ & m(\underline{b}) = \overline{m}, \quad m(\bar{b}) = \underline{m}, \end{cases} \qquad (10)$$

[1] Recall that $f(x)$ and $F(x)$ are the probability density and cumulative distribution functions of the true cost parameter x, respectively.

The optimal value of $t(\cdot)$ in optimization problem (8) is:

$$\boxed{t^*(x) = xm^*(x)^2 + \int_x^{\bar{b}} m^*(\tau)^2 d\tau.} \tag{11}$$

• Eq. (11) implies that the manufacturer has to give an additional payment to the advertising agency with the aim of inducing the advertising agency to report the true value of the advertising cost parameter.

• The integral term in Eq. (11) is the additional payment that can be considered as the **information rent**[2]. Without it, the advertising agency may tend to report a higher value of the advertising cost parameter to get more profit.

[2] Information rent is rent an agent derives from having information not provided to the principal.

Two forms of advertising are considered:

(a) **Informative advertising**, in which the firm provides real information about the product's existence, features and quality. **In Case (a)**, **the parameter k tends to be a constant**, since consumers receiving an informative advertising would decide to buy the products according to their actual needs.

(b) **Image advertising**, in which the firm communicates an image for the product that allows buyers to associate themselves with. Then the effectiveness of an image advertising would be uncertain, and hence the **parameter k in Case (b) is random**.

Case I: k is Constant

Proposition (2)

If k is constant and $4\lambda \underline{b} - k^2 > 0$, then the optimal solutions of Q, p and $m(x)$ in optimization problem (10) are given by

$$\begin{cases} p^* = \dfrac{A + km^*(x)}{2\lambda} + \dfrac{c}{2}, \\ Q^* = \dfrac{A + km^*(x)}{2} - \dfrac{\lambda c}{2}, \end{cases} \tag{12}$$

$$m^*(x) = \begin{cases} \overline{m}, & \text{if } s < \frac{(A-c\lambda)k+\overline{m}k^2}{4\lambda\overline{m}} \\ \dfrac{(A-c\lambda)k}{4\lambda s - k^2}, & \text{if } \frac{(A-c\lambda)k+\overline{m}k^2}{4\lambda\overline{m}} \le s < \frac{(A-c\lambda)k+\underline{m}k^2}{4\lambda\underline{m}} \\ \underline{m}, & \text{if } s \ge \frac{(A-c\lambda)k+\underline{m}k^2}{4\lambda\underline{m}}. \end{cases} \tag{13}$$

Case II: k is a Random Variable

- Suppose k is a random variable with probability density function $g(\cdot)$, $k \in [k_L, k_H]$. Let

$$\boxed{k_a = \int_{k_L}^{k_H} k g(k) dk}$$

be the **mean (average value)** of k.

- Denote

$$\pi_r = \max_Q E\left\{\max_p \{\pi\}\right\} = \max_Q E\{\max_p G(p, Q, m(x), x)\} - t(x) \tag{14}$$

as the **optimal net profit** of the manufacturer at the beginning of the second stage.

- Similarly, set

$$\Pi_r = \max_{Q} E\left\{\max_{p}\left\{G(p, Q, m(x), x)\right\}\right\} - sm(x)^2. \tag{15}$$

- The optimization problem with random k is given as follows:

$$\begin{cases} & \max_{m(x)}\left\{\int_{\underline{b}}^{\bar{b}} \Pi_r f(x)dx\right\} \\ \text{subject} & \text{to} \\ & m'(x) \leq 0, \quad \forall x \in [\underline{b}, \bar{b}], \\ & m(\underline{b}) = \overline{m}, \quad m(\bar{b}) = \underline{m}, \end{cases} \tag{16}$$

Lemma (2)

Suppose k is a random variable. Denote

$$\begin{aligned}
\pi_{r1} &= \frac{(A+k_a m(x)-\lambda c)^2}{4\lambda} - t(x), \\
\pi_{r2} &= \int_{k_q}^{k_H} \frac{A+km(x)}{\lambda} Qg(k)dk - \int_{k_q}^{k_H} \frac{Q^2}{\lambda} g(k)dk - t(x) - cQ \\
&\quad + \int_{k_L}^{k_q} \frac{(A+km(x))^2}{4\lambda} g(k)dk, \\
\Pi_{r1} &= \frac{(A+k_a m(x)-\lambda c)^2}{4\lambda} - sm(x)^2, \\
\Pi_{r2} &= \int_{k_q}^{k_H} \frac{A+km(x)}{\lambda} Qg(k)dk - \int_{k_q}^{k_H} \frac{Q^2}{\lambda} g(k)dk - sm(x)^2 - cQ \\
&\quad + \int_{k_L}^{k_q} \frac{(A+km(x))^2}{4\lambda} g(k)dk,
\end{aligned}$$

where $k_q = \frac{2Q-A}{m(x)}$.

There are two cases for the optimal solution of optimization problem (16).

(i) If $(k_a - k_L)m(x) \le c\lambda$, then

$$p^* = \frac{A + km(x) - Q^*}{\lambda}, \tag{17}$$

$$Q^* = \frac{A + k_a m(x) - \lambda c}{2}, \tag{18}$$

$$\pi_r = \pi_{r1}, \tag{19}$$

$$\Pi_r = \Pi_{r1}. \tag{20}$$

(ii) If $(k_a - k_L)m(x) > c\lambda$, then

$$p^* = \begin{cases} \frac{A + km(x) - Q^*}{\lambda}, \text{ if } k \in [k_q, k_H], \\ \frac{A + km(x)}{2\lambda}, \text{ if } k \in [k_L, k_q], \end{cases} \tag{21}$$

$$Q^* = \operatorname{argmax}_Q \{\pi_{r2}\}, \text{ where } Q \in \left[\frac{A + k_L m(x)}{2}, \frac{A + k_H m(x)}{2}\right], \tag{22}$$

$$\pi_r = \pi_{r2}(Q^*). \tag{23}$$

$$\Pi_r = \Pi_{r2}(Q^*). \tag{24}$$

It is assumed that
$A - \lambda c > (\sqrt{4\lambda \overline{b}} - k_a)\underline{m} + ((k_a - k_L)\overline{m} - \lambda c) > (\sqrt{4\lambda \overline{b}} - k_a)\underline{m} > 0$.
Then $\pi_{r1} > 0$ and $\pi_{r2} > 0$.

Lemma (3)

Suppose that $\underline{m} \leq \frac{c\lambda}{k_a - k_L}$ *and* $4\underline{b}\lambda > k_a^2$. *Denote by* Π_{r1}^* *the maximum value of* Π_{r1} *under* $m(x) \in \left[\underline{m}, \frac{c\lambda}{k_a - k_L}\right]$. *We have*

$$\Pi_{r1}^* = \begin{cases} \Pi_{r1}\left(\frac{c\lambda}{k_a - k_L}\right), & \text{if } m_{r1}(x) \geq \frac{c\lambda}{k_a - k_L}, \\ \Pi_{r1}(m_{r1}(x)), & \text{if } \underline{m} \leq m_{r1}(x) < \frac{c\lambda}{k_a - k_L}, \\ \Pi_{r1}(\underline{m}), & \text{if } m_{r1}(x) < \underline{m} \end{cases} \tag{25}$$

where

$$m_{r1}(x) = \frac{(A - c\lambda)k_a}{4s\lambda - k_a^2}.$$

An algorithm for solving problem (16)

Finding the optimal solution of the problem based on Lemmas 2 and 3:

- **Step 1** We employed grid search method
  Set $m(x) \in [\frac{c\lambda}{k_a - k_L}, \overline{m}]$ and $Q \in [\frac{A + k_L m(x)}{2}, \frac{A + k_H m(x)}{2}]$.
  For each $m(x)$, using discrete methods to find the optimal Q such that π_{r2} is maximized;
- **Step 2** With the optimal Q in Step 1, find the optimal $m(x)$ such that Π_{r2} is maximized. The maximum value is denoted as Π^*_{r2}. The optimal p is given in Eq. (21);
- **Step 3** Find Π^*_{r1} and the optimal $m(x)$ according to Lemma 3. The corresponding optimal values of Q and p are given by Eq. (17);
- **Step 4** The optimal function of Π_r in Eq. (16) is $\Pi^*_r = \max\{\Pi^*_{r1}, \Pi^*_{r2}\}$, and find the corresponding optimal $m(x)$, Q and p.

K Follows a Bernoulli Distribution

We assume that there are two values of k, corresponding to different states of the economy, namely,

$$k = \begin{cases} k_H, & \text{if the economy is in High (H) state,} \\ k_L, & \text{if the economy is in Low (L) state.} \end{cases} \tag{26}$$

- Let θ be the probability that the economy is in State L, $0 < \theta < 1$.
- Then the probability that the economy is in State H will be $(1 - \theta)$.

Lemma (4)

Suppose that k follows the distribution that is described in Eq. (26). For any fixed $m(\cdot)$, there are two cases for the optimal solution of optimization problem (16):
(i) If $(1-\theta)(k_H - k_L)m(x) \leq c\lambda$, then

$$\begin{cases} p^* = \dfrac{A + km(x) - Q^*}{\lambda}, \\ Q^* = \dfrac{A + k_a m(x) - \lambda c}{2}, \end{cases} \tag{27}$$

$$\begin{aligned} \pi_r &= \frac{(A + k_a m(x) - \lambda c)^2}{4\lambda} - t(x), \\ \Pi_r &= \frac{(A + k_a m(x) - \lambda c)^2}{4\lambda} - sm(x)^2. \end{aligned}$$

Lemma (4)

(ii) If $(1-\theta)(k_H - k_L)m(x) > c\lambda$, *then*

$$p^* = \begin{cases} \dfrac{A + k_H m(x) - Q^*}{\lambda}, & \text{if } k = k_H, \\ \dfrac{A + k_L m(x)}{2\lambda}, & \text{if } k = k_L, \end{cases} \tag{28}$$

$$Q^* = \frac{A + k_H m(x)}{2} - \frac{c\lambda}{2(1-\theta)}, \tag{29}$$

$$\pi_r = \theta\frac{(A + k_L m(x))^2}{4\lambda} + (1-\theta)\frac{(A + k_H m(x)^2)}{4\lambda} + \frac{c^2\lambda}{4(1-\theta)} - c\frac{A + k_H m(x)}{2}$$

$$\Pi_r = \theta\frac{(A + k_L m(x))^2}{4\lambda} + (1-\theta)\frac{(A + k_H m(x)^2)}{4\lambda} + \frac{c^2\lambda}{4(1-\theta)} - c\frac{A + k_H m(x)}{2}$$

Proposition (3)

If k is Bernoulli distributed as described in Eq. (26) and $4\lambda \underline{b} - E(k^2) > 0$, then the optimal solution of $m(x)$ in optimization problem (16) is given by $m^*(x)$:

$$m^*(x) = \begin{cases} \underline{m}, & \text{if } m_3(x) \leq \underline{m}, \\ m_3(x), & \text{if } \underline{m} < m_3(x) \leq \overline{m}, \\ \overline{m}, & \text{if } m_3(x) > \overline{m}, \end{cases} \tag{30}$$

where

$$m_3(x) = \begin{cases} m_{34}(x) = \dfrac{Ak_a - c\lambda k_H}{4s\lambda - (\theta k_L^2 + (1-\theta)k_H^2)}, & \text{if } s < \dfrac{D}{4c\lambda^2}, \\ m_{31}(x) = \dfrac{(A - c\lambda)k_a}{4s\lambda - k_a^2}, & \text{if } s \geq \dfrac{D}{4c\lambda^2}, \end{cases} \tag{31}$$

with $D = k_a[Ak_a - (A - c\lambda)k_L]$. Furthermore, $m_{34}(x)$ and $m_{31}(x)$ satisfy the following conditions:
(i) If $s < \frac{D}{4c\lambda^2}$, then $(1-\theta)(k_H - k_L)m_{34}(x) > c\lambda$;
(ii) If $s \geq \frac{D}{4c\lambda^2}$, then $(1-\theta)(k_H - k_L)m_{31}(x) \leq c\lambda$.

K Follows a Uniform Distribution

- In particular, for a “new” product, no relevant information is available, one may assume that k follows a uniform distribution.
- we solve optimization problem (10) under the assumption that k is uniformly distributed, i.e., k follows $U(k_L, k_H)$. In this case, the average value of k is $k_a = \frac{k_L+k_H}{2}$.

Lemma (5)

If k follows $U(k_L, k_H)$ and $m(\cdot)$ is given, then the optimal solution of optimization problem (16) is given by the following two cases: Case I: If $(k_H - k_L)m(x) \leq 2c\lambda$, then

$$\begin{cases} p^* &= \frac{A+km(x)-Q^*}{\lambda}, \\ Q^* &= \frac{A+k_a m(x)-\lambda c}{2}, \end{cases} \tag{32}$$

and

$$\begin{aligned} \pi_r &= \frac{(A+k_a m(x)-\lambda c)^2}{4\lambda} - t(x), \\ \Pi_r &= \frac{(A+k_a m(x)-\lambda c)^2}{4\lambda} - sm(x)^2. \end{aligned}$$

Lemma (5)

Case II: If $(k_H - k_L)m(x) > 2c\lambda$, then

$$p^* = \begin{cases} \dfrac{A + km(x) - Q^*}{\lambda}, & \text{if } \dfrac{2Q - A}{m(x)} < k \leq k_H, \\ \dfrac{A + km(x)}{2\lambda}, & \text{if } k_L \leq k \leq \dfrac{2Q - A}{m(x)}, \end{cases} \tag{33}$$

$$Q^* = \frac{A + k_H m(x) - \sqrt{2\lambda c m(x)(k_H - k_L)}}{2}, \tag{34}$$

and

$$\pi_r = \frac{1}{12\lambda}[(A + k_H m(x))^2 + (A + k_H m(x))(A + k_L m(x)) + (A + k_L m(x))^2] + \frac{c\sqrt{2\lambda c m(x)(k_H - k_L)}}{3} - c\frac{A + k_H m(x)}{2} - t(x).$$

$$\Pi_r = \frac{1}{12\lambda}[(A + k_H m(x))^2 + (A + k_H m(x))(A + k_L m(x)) + (A + k_L m(x))^2] + \frac{c\sqrt{2\lambda c m(x)(k_H - k_L)}}{3} - c\frac{A + k_H m(x)}{2} - sm(x)^2.$$

Proposition (4)

If k follows $U(k_L, k_H)$ and $4\lambda \underline{b} - E(k^2) > 0$, then the optimal solution of $m(x)$ in optimization problem (16) is given by $m^*(x)$:

$$m^*(x) = \begin{cases} \underline{m}, & \text{if } m_4(x) \le \underline{m}, \\ m_4(x), & \text{if } \underline{m} < m_4(x) \le \overline{m}, \\ \overline{m}, & \text{if } m_4(x) > \overline{m}, \end{cases} \tag{35}$$

where $D = k_a[Ak_a - (A - \lambda c)k_L]$, and

$$m_4(x) = \begin{cases} m_{44}(x) & , \text{ if } s < \dfrac{D}{4c\lambda^2}, \\ m_{41}(x) = \dfrac{(A - c\lambda)k_a}{4s\lambda - k_a^2} & , \text{ if } s \ge \dfrac{D}{4c\lambda^2}. \end{cases} \tag{36}$$

Furthermore, $m_{44}(x)$ and $m_{41}(x)$ satisfy the following conditions:
(i) If $s < \frac{D}{4c\lambda^2}$, then $(k_H - k_L)m_{44}(x) > 2c\lambda$;
(ii) If $s \ge \frac{D}{4c\lambda^2}$, then $(k_H - k_L)m_{41}(x) \le 2c\lambda$.

Numerical Examples

- In this section, we present numerical examples to give further insights of our model. We assume that the true design cost x is uniformly distributed over $[30, 80]$, $A = 500$ units, $\lambda = 10$, $c = 5$ dollars, $\underline{m} = 1$, $\overline{m} = 20$.
- We consider five different cases of k:
  (i) k is a constant, $k = 15$;
  (ii) k is a constant, $k = 20$;
  (iii) k is Bernoulli distributed, $k_L = 10$, $k_H = 30$, $\theta = \frac{1}{3}$;
  (iv) k is Bernoulli distributed, $k_L = 10$, $k_H = 30$, $\theta = \frac{1}{2}$;
  (v) k is uniformly distributed, $k_L = 10$, $k_H = 30$.
- Here we discuss the outsourcing contract designed above. Case (v) is used as an example and other cases can be analyzed using the same method.

Table 1: Outsourcing contract: m is continuous.

y	30	35	40	45	50	55	60	65	70	75	80
$m(y)$	11.4	7.5	5.6	4.5	3.8	3.2	2.8	2.5	2.3	2.0	1.9
$t(y)$	4942.6	2611.9	1688.0	1206.6	914.1	718.9	580.1	476.6	396.6	333.0	281.3

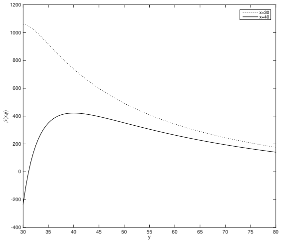


Figure 1: Advertising agency's net profit $\beta(x, y)$ when $x = 30$ and $x = 40$.

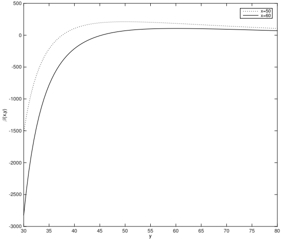


Figure 2: Advertising agency's net profit $\beta(x, y)$ when $x = 50$ and $x = 60$.

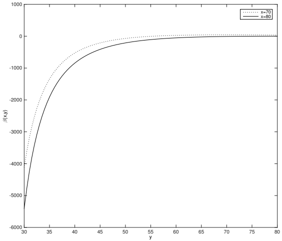


Figure 3: Advertising agency's net profit $\beta(x, y)$ when $x = 70$ and $x = 80$.

- Figures 1, 2 and 3 indicate that the advertising agency's net profit $\beta(x, y)$ reaches the maximum value when $y = x$ for any given x.
- It implies that the manufacturer is able to induce the advertising agency to report the actual advertising cost parameter under the outsourcing contract.
- They also show that a smaller value of x results in a larger value of $\beta(x, x)$.

Sensitivity Analysis

- Figures 4 and 5 depict the optimal advertising effort level $m(x)$ of the product and the optimal payment $t(x)$ to the advertising agency with respect to x.

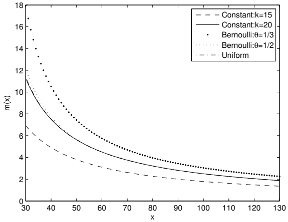


Figure 4: Advertising effort level of the product.

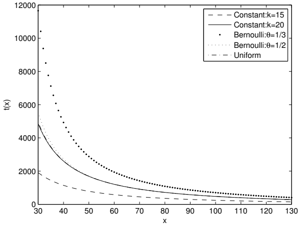


Figure 5: Payment to the advertising agency.

- Figures 4 and 5 show that when x increases, the manufacturer would decrease his advertising effort level accordingly so that he would pay less to the advertising agency.

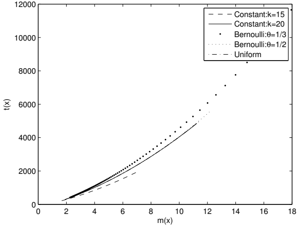


Figure 6: Advertising effort level and payment to the advertising agency.

- Figure 6 shows that there is a positive correlation between $m(x)$ and $t(x)$.
- When the sensitivity of the market demand to the advertising effort level increases, the manufacturer is suggested to pay more attention to promote the coordination with the advertising agency.

- Figures 7 and 8 present the optimal production size Q and the expectation of the optimal retail price $E(p)$. It shows that both of them go down as x goes up.

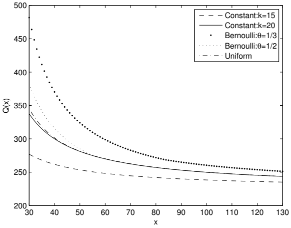


Figure 7: Production size.

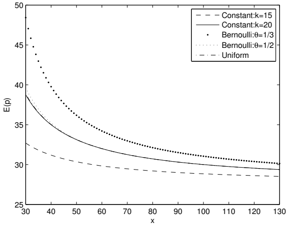


Figure 8: Expectation of the retail price.

- The intuition is that a higher value of x makes it more likely for the manufacturer to lower his advertising effort level and retail price, which makes the market demand of the product to decrease slowly.

Conclusions

- This project studies outsourcing strategies between one manufacturer and one advertising agency with an advertising effort-level-dependent and retail-price-dependent demand.
- The contribution of this work is to propose a mechanism by designing a contract so as to induce the advertising agency to report her actual advertising cost parameter.
- Another problem is the demand uncertainty. We obtain analytical solutions of the optimization problem, taking into account the advertising effort level, payment to the advertising agency, production size and retail price.

Thank You

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