Sub-additive functionals, information theory, and non-convex optimization

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Abstract: Network information theory studies optimal communication rates, or capacity regions, in multiuser settings. Establishing the optimality of certain achievable strategies required clever arguments, a la Gallager's converse, or the use of fundamental geometric inequalities like the Entropy Power inequality. An inspection of the arguments for converses of coding theorems or those in outer bounds reveal that the crux of these arguments lies in establishment of the sub-additivity of certain functionals. For certain kind of functionals a method is developed that implies the optimality of Gaussian distributions and consequently one can obtain alternate proofs of inequalities such as the entropy power inequalities and Brascamp-Lieb inequalities, as well as obtain new inequalities.

I will provide functionals for which the establishment of sub-additivity (or lack there-of) would answer the optimality of the best-known achievable schemes in fundamental network information their settings. The sub-additivity of these functionals are in turn related to establishing properties of global maximizers of a certain family of non-convex optimization problems. I will present some unifying observations and conjectures across the family of optimization problems.