THE UNIVERSITY OF HONG KONG

Department of Mathematics

COLLOQUIUM

Permanent function: an instrument to solve algebraic and combinatorial problems

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Abstract

The talk will be based on our recent results with M. Budrevich.

Two important functions in matrix theory, determinant and permanent, look very similar:

$$\det A = \sum_{\sigma \in S_n} (-1)^{\sigma} a_{1\sigma(1)} \cdots a_{n\sigma(n)} \quad \text{and} \quad \mathrm{per}\ \mathrm{A} = \sum_{\sigma \in \mathrm{S_n}} \mathrm{a}_{1\sigma(1)} \cdots \mathrm{a}_{\mathrm{n}\sigma(\mathrm{n})}$$

here $A = (a_{ij}) \in M_n(\mathcal{F})$ is an $n \times n$ matrix and S_n denotes the set of all permutations of the set $\{1, \dots, n\}$.

While the computation of the determinant can be done in a polynomial time, it is still an open question, if there exists a polynomial algorithm to compute the permanent. Due to this reason, starting from the work [2] by Pólya, 1913, different approaches to convert the permanent into the determinant were under the intensive investigation.

Among our results we prove the following theorem:

Theorem 1. Suppose $n \geq 3$, and let F be a finite field with char $F \neq 2$. Then, no bijective map $T : M_n(F) \to M_n(F)$ satisfies per $A = \det T(A)$.

In the second part of the talk we discuss the bounds for permanents of ± 1-matrices. The class of $(-1, 1)$-matrices is very important in algebra and combinatorics and in various their applications. For example, well-known Hadamard matrices are of this type.

In 1974 Wang [3, Problem 2] posed a problem to find a decent upper bound for |per (A)| if A is a square ± 1-matrix of rank k. In 1985 Kräuter [1] conjectured some concrete upper bound.

We prove the Kräuter's conjecture and thus obtain the complete answer to the Wang's question. In particular, we characterized matrices with the maximal possible permanent for each value of k.

References:

[1] A.R. Kräuter, Recent results on permanents of (+1, -1)-matrices, Ber. No. 249, Berichte, 243-254, Forschungszentrum Graz, Graz, 1985.

[2] G. Pólya, Aufgabe, **424**, Arch. Math. Phys. **20**, 3, (1913) 271.

[3] E.T.H. Wang, On permanents of (+1, -1)-matrices, Israel J. Math., **18** (1974) 353-361.

Date:	**June 4, 2018 (Monday)**
Time:	**3:00 – 4:00pm**
Venue:	**Room 210, Run Run Shaw Bldg., HKU**

All are welcome