THE UNIVERSITY OF HONG KONG

Department of Mathematics

COLLOQUIUM

Convergence of numerical solutions for stochastic partial integro-differential equations subject to space-time white noise

Dr. Jilu Wang

Florida State University, USA

Abstract

The stability and convergence of numerical methods for the stochastic partial integro-differential equation

$$\partial_t \psi - \Delta \partial_t^{1-\alpha} \psi = f + \varepsilon \dot{W}$$

is considered in a convex polygon/polyhedron $\mathcal{O} \subset \mathbb{R}^d$, $d \in \{1, 2, 3\}$, where $\partial_t^{1-\alpha} \psi$ (for given $\alpha \in (0, 1) \cup (1, 2)$) denotes the Caputo fractional derivative/integral, $f(x, t)$ a given deterministic source function, ε a given positive parameter, and $\dot{W}$ space-time white noise. Such problems arise naturally by considering the heat transfer in a material with thermal memory, subject to white noise. For the above model, both the time-fractional derivative and the stochastic process result in low regularity of the solution. Hence, the numerical approximation of such problems and the corresponding numerical analysis are very challenging. In this work, the stochastic partial integro-differential equation is discretized by a backward Euler convolution quadrature in time with piecewise continuous linear finite element method in space. The sharp-order convergence

$$\mathbb{E}\|\psi(\cdot, t_n) - \psi_n^{(h)}\|_{L^2(\mathcal{O})} = \begin{cases} O\big(\tau^{\frac{1}{2} - \frac{\alpha d}{4}} + \ell_h^{\frac{1}{2}} h^{\frac{1}{\alpha} - \frac{d}{2}}\big) & \text{if } \alpha \in \Big[\dfrac{1}{2}, \dfrac{2}{d}\Big), \\ O\big(\tau^{\frac{1}{2} - \frac{\alpha d}{4}} + h^{2 - \frac{d}{2}}\big) & \text{if } \alpha \in \Big(0, \dfrac{1}{2}\Big), \end{cases}$$

up to a logarithmic factor $\ell_h^{\frac{1}{2}} = (\ln(e + 1/h))^{\frac{1}{2}}$, is established in general d-dimensional spatial domains, where $\psi_n^{(h)}$ denotes the approximate solution at the nth time step, and $\mathbb{E}$ the expectation operator. Numerical results are presented to illustrate the theoretical analysis.

Date:	**March 8, 2018 (Thursday)**
Time:	**9:00 – 10:00am**
Venue:	**Room 210, Run Run Shaw Bldg., HKU**

All are welcome