THE UNIVERSITY OF HONG KONG

Institute of Mathematical Research
Department of Mathematics

Numerical Analysis Seminar

Multilevel Markov Chain Monte Carlo Methods for Bayesian Inversion of Partial Differential Equations

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Abstract

The Bayesian approach to inverse problems, in which the posterior probability distribution on an unknown field is sampled for the purposes of computing posterior expectations of quantities of interest, is starting to become computationally feasible for partial differential equation (PDE) inverse problems. Balancing the sources of error arising from finite-dimensional approximation of the unknown field, the solution of the forward PDE and the sampling of the probability space under the posterior distribution are essential for the design of efficient computational methods. We study Bayesian inversion for a model elliptic PDE with an unknown diffusion coefficient. We consider both the case where the PDE is uniformly elliptic with respect to all the realizations, and the case where uniform ellipticity does not hold, i.e. the coefficient can get arbitrarily close to 0 and arbitrarily large as in the log-normal model. We provide complexity analysis of Markov chain Monte Carlo (MCMC) methods for numerical evaluation of expectations with respect to the posterior measure, in particular bounds on the overall work required to achieve a prescribed error level. We first bound the computational complexity of 'plain' MCMC where a large number of realizations of the forward equation is solved with equally high accuracy. The work versus accuracy bounds show that the complexity of this approach can be quite prohibitive. We then present a novel multi-level Markov chain Monte Carlo strategy which utilizes sampling from a multi-level discretization of the posterior and the forward PDE. The strategy achieves an optimal complexity level that is equivalent to that for performing only one step on the plain MCMC. The optimal accuracy and complexity are mathematically rigorously proven. Numerical results confirm our analysis. This is a joint work with Jia Hao Quek (NTU, Singapore), Christoph Schwab (ETH, Switzerland) and Andrew Stuart (Caltech, US).

References

V. H. Hoang, Ch. Schwab and A. M. Stuart (2013), Complexity analysis of accelerated MCMC methods for Bayesian inversion, *Inverse Problems*, 29, 085010, 37 pp.

V. H. Hoang, J. H. Quek, Ch. Schwab (2020), Analysis of a multilevel Markov chain Monte Carlo finite element method for Bayesian inversion of log-normal diffusion, *Inverse Problems*, 36 035021, 47 pp.

V. H. Hoang, J. H. Quek, Ch. Schwab (2021), Multilevel Markov Chain Monte Carlo for Bayesian inversion of parabolic partial differential equations under Gaussian prior, *SIAM/ASA Journal on Uncertainty Quantification*, 9, 384-419.

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Venue: Room 210, Run Run Shaw Bldg., HKU
and
ZOOM: https://hku.zoom.us/j/
Meeting ID: 913 6532 3891
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