THE UNIVERSITY OF HONG KONG

Institute of Mathematical Research
Department of Mathematics

Numerical Analysis Seminar

The discrete maximum principle, and positivity-preservation, in finite element methods

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Abstract

The quest for physical consistency in the discretisation of PDEs started as soon as the numerical methods started being proposed. By physical consistency we mean a discretisation that, by design, satisfies a property also satisfied by the continuous PDE. This property might be positivity of the discrete solution, or preservation of some bounds (e.g., concentrations should belong to the interval [0,1]), or can also be energy preservation, or exactly divergence-free velocities for incompressible fluids, etc.

Regarding positivity preservation, this topic has been around since the pioneering work by Ph. Ciarlet in the late 1960s and early 1970s. In the context of finite element methods, it was shown in those early works that in order for a finite element method to preserve positivity the mesh needs to satisfy certain geometrical restrictions, e.g., in two space dimensions with simplicial elements the triangulation needs to be of Delaunay type (in higher dimensions or quadrilateral meshes the restrictions are more involved). Throughout the years several conclusions have been reached in this topic, but in the context of finite element methods the discretisations tend to be of first order in space, and tend to be nonlinear. So, many important problems still remain open. In particular, one open problem is how to build a discretisation that will lead to a positive solution regardless of the geometry of the mesh and the order of the finite element method.

This talk will be divided in two parts. In the first one I will give a very quick summary of some nonlinear discretisations that respect the discrete maximum principle. I will focus mostly in the convection-diffusion equation, and in the discretisation known as Algebraic Flux Correction (AFC) scheme. I will review some of the results that I have obtained in collaboration with several collaborators (E. Burman, V. John, and P. Knobloch, mainly), results that include stability, and convergence (or lack of convergence) of the method depending on the geometry of the mesh, and the precise definition of the method. In the second part of the talk I will present a method that enforces bound-preservation (at the degrees of freedom) of the discrete solution. The method is built by first defining an algebraic projection onto the convex closed set of finite element functions that satisfy the bounds given by the solution of the PDE. Then, this projection is hardwired into the definition of the method by writing a discrete problem posed for this projected part of the solution. Since this process is done independently of the shape of the basis functions, and no result on the resulting finite element matrix is used, then the outcome is a finite element function that satisfies the bounds at the degrees of freedom. Another important observation to make is that this approach is related to variational inequalities, and this fact will be exploited in the error analysis. The core of the second part will be devoted to explaining the main idea in the context of linear (and nonlinear) reaction-diffusion equations. Then, I will explain the main difficulties encountered when extending this method to convection-diffusion equations.

Date:	February 22, 2023 (Wednesday)
Time:	5:00 – 6:00pm
Venue:	ZOOM: https://hku.zoom.us/j/ Meeting ID: 913 6532 3891 Password: 310656

All are welcome