THE UNIVERSITY OF HONG KONG

Institute of Mathematical Research
Department of Mathematics

Number Theory Seminar

Gap Principle of Divisibility Sequences of Polynomials

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Date:	October 30, 2023 (Monday)
Time:	4:00 – 5:00pm
Venue:	Room 210, Run Run Shaw Bldg., HKU

Abstract

Let $f \in \mathbb{Z}[x]$ and $\ell \in \mathbb{N}$. Consider the set of all $(a_0, a_1, \ldots, a_\ell) \in \mathbb{N}^{\ell+1}$ with $a_i < a_{i+1}$ and $f(a_i) \mid f(a_{i+1})$ for all $0 \leq i \leq \ell - 1$. We say that f satisfies the gap principle of order ℓ if $\lim a_\ell / a_0 = \infty$ as $a_0 \to \infty$ for any such $(a_0, a_1, \ldots, a_\ell)$. We also define the gap order of $f(x)$ to be the smallest positive integer ℓ such that $f(x)$ satisfies the gap principle of order ℓ. If such ℓ does not exist, we say that $f(x)$ does not satisfy the gap principle. In this talk, we will discuss a conjecture by Chan, Choi and Lam that $f(x)$ does not satisfy the gap principle if and only if $f(x)$ is in the form of $f(x) = A(Bx + C)^n$ for some $A, B, C \in \mathbb{Z}$. Moreover, we completely determine the gap order of any polynomial that if $f(x)$ is not in the form of $A(Bx + C)^n$, then $f(x)$ has gap order 2 if $f(x)$ is a quadratic polynomial or a power of a quadratic polynomial; and has gap order 1 otherwise. Related to the proof of above results, the multiplicative order of the fundamental solution of Pell's equation $X^2 - DY^2 = 1$ in $\mathbb{Z}[\sqrt{D}]/ < D >$ will also be discussed. These are joint work with Tsz Ho Chan, Peter Cho-Ho Lam and Daniel Tarnu.

All are welcome