Abstract

Erdos and Volkmann showed in 1966 that there exist additive subgroups in the real line with arbitrary Hausdorff dimension. The situation of subrings is very different, as it was shown in 2001 by Edgar and Miller and independently by Bourgain, that any proper measurable subring of the reals must have zero Hausdorff dimension. In the talk, I will discuss how Bourgain related this problem to that of Fourier decay of multiplicative convolutions, and an optimal entropy condition for Fourier decay recently obtained in collaboration with Tuomas Orponen and Pablo Shmerkin.