

THE UNIVERSITY



OF HONG KONG

*Institute of Mathematical Research
Department of Mathematics*

GEOMETRY SEMINAR

Hodge theory, unlikely intersections, o-minimal geometry, and Diophantine geometry

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and

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Date: Tuesday, February 20, 27 & March 12, 2024

Time: 2:00 – 3:30pm

Venue: Rm 210, Run Run Shaw Bldg., HKU

Abstract

We will begin with an introduction of the key notions in Hodge theory. Firstly, we will motivate pure Hodge structures and mixed Hodge structures, which can be regarded as certain linearizations of complex geometric objects. We will then discuss variations of mixed Hodge structures and their period mappings, which are used to investigate families of quasi-projective varieties. Examples include Shimura varieties, and period mappings obtained from the exponential function and the j -function.

We will then explain how Hodge theory is used to prove results (called Ax-Schanuel) about transcendental degrees of function fields. When phrased geometrically, it roughly says that if the intersection of a given algebraic variety and the graph of a period mapping is “unlikely”, then this intersection should be related to certain group orbits of Hodge structures.

Since period mappings are transcendental, to prove Ax-Schanuel results one needs a theory that interpolates algebraic geometry and analytic geometry. We will give an introduction of o-minimal geometry and mention their applications.

If time permits, we will explain and also speculate, how Hodge theory including Ax-Schanuel results, can be used in Diophantine geometry, e.g. sparsity/non-density/finiteness of integral points on moduli spaces (in particular moduli spaces of affine hypersurfaces in tori defined by Laurent polynomials), Hast's higher dimensional Chabauty-Kim and how it depends on the Bloch-Kato conjecture, uniform bound for the number of rational points on elliptic curves by Dimitrov-Gao-Habegger, André-Oort conjecture and Zilber-Pink conjecture.