The University of Hong Kong – Shanghai Jiao Tong University
Joint Workshop

March 9 & 10, 2024
Room 210, Run Run Shaw Building, The University of Hong Kong

March 9, 2024 (Saturday)	
12 30 - 14 30	*Lunch*
14:30 - 15:00 **Siran Li**	**The isometric immersions problem: from perspectives of PDE, geometry, and physics**
15:00 - 15:30 **Ngaiming Mok**	**From the Bergman Kernel to Functional Transcendence**
15:30 - 16:00 **Jinyan Fan**	**Derivative-free Levenberg-Marquardt method via smoothing technique**
16 00 - 16 30	*Coffee break*
16:30 - 17:00 **Dong Li**	**To be announced**
17:00 - 17:30 **Shenggao Zhou**	**Thermal Electrokinetics in Charging and Discharging Processes of Supercapacitors: Modeling and Computation**
18 30 - 20 30	*Dinner*

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March 10, 2024 (Sunday)	
09:00 -09:30 Chunjing Xie	The rigidity of steady solutions of Navier-Stokes system and its applications
09:30 - 10:00 Zhiwen Zhang	Efficient Interacting Particle Methods for Computing Near Singular Solutions of Keller-Segel Chemotaxis Systems and High-Dimensional Eigenvalue Problems
10:00 - 10:30 Songting Li	The Mathematical Modeling, Analysis, and Simulation of Single Neuron Dynamics
10 30 - 11 00	*Coffee break*
11:00 - 11:30 Tak-Kwong Wong	Hamilton-Jacobi-Bellman Equations in Macroeconomics
11:30 - 12:00 Min Tang	Multiscale inverse problem in chemotaxis models
12:00 - 12:30 Qidi Zhang	Finite-time singularity formation for the Landau-Lifshitz-Gilbert equation in dimension two
12 30 - 14 30	*Lunch*
14:30 - 15:00 Jiefei Yang	Gradient enhanced sparse Hermite polynomial expansions for pricing and hedging high-dimensional American options
15:00 -15:30 Guanglian Li	On Sparse Grid Interpolation for American Option Pricing with Multiple Underlying Assets
15:30 - 16:00 Guangyue Han	An Introduction to Information Theory
16 00 - 16 30	*Coffee break*

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Program	**March 9** **Saturday**
12 30 - 14 30	Lunch
14 30 - 15 00	**Siran Li**, SJTU *The isometric immersions problem: from perspectives of PDE, geometry, and physics*
15 00 - 15 30	**Ngaiming Mok**, HKU *From the Bergman Kernel to Functional Transcendence*
15 30 - 16 00	**Jinyan Fan**, SJTU *Derivative-free Levenberg-Marquardt method via smoothing technique*
16 00 - 16 30	*Coffee Break*
16 30 - 17 00	**Dong Li**, HKU *To be announced*
17 00 - 17 30	**Shenggao Zhou**, SJTU *Thermal Electrokinetics in Charging and Discharging Processes of Supercapacitors: Modeling and Computation*
18 30 -	*Dinner*

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Program	**March 10** **Sunday**
09 00 - 09 30	**Chunjing Xie,** SJTU *The rigidity of steady solutions of Navier-Stokes system and its applications*
09 30 - 10 00	**Zhiwen Zhang**, HKU *Efficient Interacting Particle Methods for Computing Near Singular Solutions of Keller-Segel Chemotaxis Systems and High-Dimensional Eigenvalue Problems*
10 00 - 10 30	**Songting Li**, SJTU *The Mathematical Modeling, Analysis, and Simulation of Single Neuron Dynamics*
10 30 - 11 00	*Coffee break*
11 00 - 11 30	**Tak-Kwong Wong**, HKU *Hamilton-Jacobi-Bellman Equations in Macroeconomics*
11 30 - 12 00	**Min Tang**, SJTU *Multiscale inverse problem in chemotaxis models*
12 00 - 12 30	**Qidi Zhang**, HKU *Finite-time singularity formation for the Landau-Lifshitz-Gilbert equation in dimension two*
12 30 - 14 30	*Lunch*
14 30 - 15 00	**Jiefei Yang**, HKU *Gradient enhanced sparse Hermite polynomial expansions for pricing and hedging high-dimensional American options*
15 00 - 15 30	**Guanglian Li**, HKU *On Sparse Grid Interpolation for American Option Pricing with Multiple Underlying Assets*
15 30 - 16 00	**Guangyue Han**, HKU *An Introduction to Information Theory*
16 00 - 16 30	*Coffee break*

— End of Program —

Titles and Abstracts

Jinyan Fan Shanghai Jiao Tong University

Derivative-free Levenberg-Marquardt method via smoothing technique

In this talk, we present a derivative-free Levenberg-Marquardt method for nonlinear least squares problems, where the Jacobian matrices are approximated via spherical smoothing. It is shown that the gradient models which use the approximate Jacobian matrices are probabilistically first-order accurate. The almost-sure global convergence of the method is also given.

Guangyue Han HKU

An Introduction to Information Theory

I will give in this talk an introduction to information theory. If time permits, I will briefly talk about some recent developments and future directions in information theory.

Dong Li HKU

To be announced

Guanglian Li HKU

On Sparse Grid Interpolation for American Option Pricing with Multiple Underlying Assets

In this work, we develop a novel efficient quadrature and sparse grid based polynomial interpolation method to price American options with multiple underlying assets. The approach is based on first formulating the pricing of American options using dynamic programming, and then employing static sparse grids to interpolate the continuation value function at each time step. To achieve high efficiency, we first transform the domain from $\mathbb{R}^d$ to $(-1,1)^d$ via a scaled tanh map, and then remove the boundary singularity of the resulting multivariate function over $(-1,1)^d$ by a bubble function and simultaneously, to significantly reduce the number of interpolation points. We rigorously establish that with a proper choice of the bubble function, the resulting function has bounded mixed derivatives up to a certain order, which provides theoretical underpinnings for the use of sparse grids. Numerical experiments for American

arithmetic and geometric basket put options with the number of underlying assets up to 16 are presented to validate the effectiveness of our approach. This is a joint work with my PhD student Miss Jiefei Yang.

Siran Li Shanghai Jiao Tong University

The isometric immersions problem: from perspectives of PDE, geometry, and physics

We report our recent work on a classical problem in differential geometry: isometric immersions and/or embeddings of Riemannian and semi-Riemannian manifolds. The underlying PDE is the system of Gauss–Codazzi–Ricci equations. Existence of isometric immersions is studied under various curvature conditions, via elliptic and hyperbolic PDE techniques. Weak continuity of isometric immersions is investigated with the help of the theory of compensated compactness. Connections to other problems in mathematical physics, including fluid dynamics, harmonic maps, and nonlinear elasticity, will be discussed.

Songting Li Shanghai Jiao Tong University

The Mathematical Modeling, Analysis, and Simulation of Single Neuron Dynamics

Our brain comprises billions of neurons, each receiving thousands of input signals on dendrites and integrating these signals spatiotemporally to generate action potentials that encode information. Due to the highly complex geometric structure and nonlinear ion channels of neuronal dendrites, there is currently a lack of quantitative mathematical theories to describe their computational functions. In this talk, I will first introduce the biological characteristics of individual neurons and some of their intricate computational functions. Subsequently, to understand signal integration in neurons, we establish a PDE cable model describing neuronal dynamics with spatial dendrites. Using asymptotic analysis, we derive a quantitative bilinear rule that characterizes the neuronal integration of multiple spatiotemporal input signals. Furthermore, through dimensionality reduction analysis, we derive a simplified ODE model that effectively captures the voltage dynamics at the cell body, with experimental verifications. Our model forms the basis of developing fast algorithms for simulating large-scale neuronal networks, providing a novel computational framework for future brain simulation.

Ngaiming Mok HKU

From the Bergman Kernel to Functional Transcendence

On a bounded domain $U \Subset \mathbb{C}^n$, starting with any orthonormal basis of the Hilbert space $H^2(U)$ of square-integrable holomorphic functions on U one obtains the Bergman kernel $K_U(z,w)$, which is a reproducing kernel for $H^2(U)$, hence uniquely determined independent of the choice of an orthonormal basis. The strictly plurisubharmonic function $\varphi(z) := \log K(z,z)$ serves as the potential of a Kähler metric ds^2_U called the Bergman metric, which is invariant under $\mathrm{Aut}(U)$. Among bounded domains there are the bounded symmetric domains Ω classified by Élie Cartan such that (Ω, ds^2_Ω) are symmetric, $\Omega = G/K$, in the sense of Riemannian geometry. Here $\Omega \Subset \mathbb{C}^n$ in their standard realizations are bounded domains with semi-algebraic boundaries. In this lecture I will explain: (1) *how the study of holomorphic isometries with respect to the Bergman metric between bounded domains was motivated by problems in arithmetic dynamics*, (2)*how their solutions based on the notion of the diastasis of Eugenio Calabi were generalized to yield algebraicity results for such holomorphic isometries*, (3) *how the study of the asymptotic behavior of holomorphic isometries of the Poincaré disk led to a uniformization theorem for projective varieties covered by algebraic subsets of* Ω, and (4) *how the latter serves as a starting point for research in functional transcendence theory concerning* $X_\Gamma = \Omega/\Gamma$*, where* $\Gamma \subset G$ *is an* arbitrary *lattice.*

Min Tang Shanghai Jiao Tong University

Multiscale inverse problem in chemotaxis models

Models of different scales are used to describe the collective behavior of bacteria motion, including the celebrated Keller-Segel equation and a chemotaxis kinetic equation. While these two models describe the organism movement at the macro- and mesoscopic level respectively, they are asymptotically equivalent in the parabolic regime. One natural question is whether it is possible to use data from one scale to recover the coefficients of the model of the other scale. This can be formulated into multiscale inverse problems and we will show some recent results about chemotaxis models.

Tak-Kwong Wong HKU

Hamilton-Jacobi-Bellman Equations in Macroeconomics

In the work of Arrow et al. (2007, Proc. Natl. Acad. Sci. U.S.A.), they studied a macroeconomic growth model so that the population dynamic was involved in both the total utility (objective function) of the whole population and in the capital

investment process. In essence, they assumed the deterministic evolution for both dynamics, such that the labour force of the population is also incurred through the Cobb-Douglas production function. In this talk, we will first introduce an extension of their problem, particularly over a finite time horizon, in which we also allow more realistic and generic population growth and incorporate a stochastic environment for both the demography and capital investment. For the corresponding Hamilton-Jacobi-Bellman equation, we show the existence and uniqueness of classical solutions by using a hybrid approach that combines techniques in both partial differential equations and stochastic analysis. We believe that the methodology developed in this work can also apply to various sophisticated models arising from economic growth theory and mathematical finance.

Reference:
Arrow, K., Bensoussan, A., Feng, Q., and Sethi, S.P. (2007). Optimal savings and the value of population, Proceedings of the National Academy of Sciences, 47: 18421-6.

Acknowledgement: This work is supported by the Hong Kong General Research Fund (GRF) "Controlling the Growth of Classical Solutions of a Class of Parabolic Differential Equations with Singular Coefficients: Resolutions for Some Lasting Problems from Economics" with project number 17302521.

Chunjing Xie Shanghai Jiao Tong University

The rigidity of steady solutions of Navier-Stokes system and its applications

The Liouville type theorem for stationary Navier-Stokes system in the whole space is longstanding open problem. In this talk, we first discuss the rigidity of steady Navier-Stokes system with dimension bigger than three in a class more general than self-similar solutions, where we do not need any type of self-similarity or smallness of solutions. Furthermore, this rigidity result is used to study the regularity and far field behavior of steady solutions of high dimensional Navier-Stokes system. Finally, we discuss the rigidity for steady Navier-Stokes system in domains with physical boundaries, such as slabs.

Jiefei Yang HKU

Gradient enhanced sparse Hermite polynomial expansions for pricing and hedging high-dimensional American options

We propose a simple yet efficient simulation-based method for pricing and hedging (i.e., derivatives of price function) of high-dimensional American options. This

method takes sparse Hermite polynomial expansion as a surrogate model for the continuation value function and computes the expansion coefficients with gradient information. By using sparse Hermite polynomial ansatz space, the derivatives are obtained at almost no extra cost once we have the evaluations of the sparse Hermite polynomials. The coefficents are obtained by solving a linear least squares problem enhanced by gradients with simulated paths. The convergence of this proposed method is analyzed, with an error estimate provided in terms of the best approximation error in the sparse Hermite polynomial ansatz space, the statistical error of solving discrete least squares problems, approximation error in the terminal condition and time stepping size. Extensive numerical experiments demonstrate that the proposed algorithm can outperform the state-of-art least squares Monte Carlo method with more accurate price, Greeks, and optimal exercise strategies in high dimensions up to dimension 100. This is a joint work with my supervisor Professor Guanglian Li.

Qidi Zhang HKU

Finite-time singularity formation for the Landau-Lifshitz-Gilbert equation in dimension two

Landau-Lifshitz-Gilbert equation (LLG) is related to the dynamics of several important physics, including the magnetization of ferromagnets. In mathematics, LLG builds the bridge between two important equations, the harmonic map heat flow and the Schrodinger flow. In this talk, we report the singularity formation of LLG from R^2 to S^2. For any prescribed N distinct points in R^2, we construct a solution that the gradient of the solution to LLG blows up at these N points at a finite time simultaneously. This is a joint work with Juncheng Wei and Yifu Zhou.

Zhiwen Zhang HKU

Efficient Interacting Particle Methods for Computing Near Singular Solutions of Keller-Segel Chemotaxis Systems and High-Dimensional Eigenvalue Problems

Mesh-based methods like finite element methods and spectral methods face challenges when solving PDEs with near singular solutions or in high-dimensional spaces. In this talk, we introduce a particle-based method to compute aggregation patterns and near singular solutions of the Keller-Segel chemotaxis system in three-dimensional space and an interacting particle method to calculate the principal eigenvalues of high-dimensional elliptic operators. We also present numerical experiments to demonstrate

their performance. Furthermore, we introduce the DeepParticle method for learning and generating the distributions of solutions under variations of physical parameters.

Shenggao Zhou Shanghai Jiao Tong University

Thermal Electrokinetics in Charging and Discharging Processes of Supercapacitors: Modeling and Computation

This work proposes a new variational, thermodynamically consistent model to predict thermal electrokinetics in supercapacitors by using an energetic variational approach. The least action principle and maximum dissipation principle from the non-equilibrium thermodynamics are employed to develop modified Nernst-Planck equations for non-isothermal ion transport with temperature inhomogeneity. Laws of thermodynamics are employed to derive a temperature evolution equation with heat sources due to thermal pressure and electrostatic interactions. Property-preserving numerical schemes are discussed as well. Numerical simulations successfully predict temperature oscillation in the charging-discharging processes of supercapacitors, indicating that the developed model is able to capture reversible and irreversible heat generations. The impact of ionic sizes and scan rate of surface potential on ion transport, heat generation, and charge current is systematically assessed in cyclic voltammetry simulations. It is found that the thermal electrokinetics in supercapacitors cannot follow the surface potential with fast scan rates, showing delayed dynamics with hysteresis diagrams. Our work thus provides a useful tool for physics-based prediction of thermal electrokinetics in EDLCs. This is a joint work with Xiang Ji, Jie Ding, Pei Liu, and Chun Liu.