THE UNIVERSITY OF HONG KONG

Institute of Mathematical Research
Department of Mathematics

Numerical Analysis Seminar

Overcoming the curse of dimensionality: from nonlinear Monte Carlo to the training of neural networks

Professor Arnulf Jentzen
CUHK-SZ and University of Münster, Germany

Date:	April 10, 2024 (Wednesday)
Time:	4:00 – 5:00pm
Venue:	ZOOM: https://hku.zoom.us/j/ Meeting ID: 913 6532 3891 Password: 310656

Abstract

Partial differential equations (PDEs) are among the most universal tools used in modelling problems in nature and man-made complex systems. Nearly all traditional approximation algorithms for PDEs in the literature suffer from the so-called "curse of dimensionality" in the sense that the number of required computational operations of the approximation algorithm to achieve a given approximation accuracy grows exponentially in the dimension of the considered PDE. With such algorithms it is impossible to approximatively compute solutions of high-dimensional PDEs even when the fastest currently available computers are used. In the case of linear parabolic PDEs and approximations at a fixed space-time point, the curse of dimensionality can be overcome by means of Monte Carlo approximation algorithms and the Feynman-Kac formula. In this talk we present an efficient machine learning algorithm to approximate solutions of high-dimensional PDE and we also prove that deep artificial neural network (ANNs) do indeed overcome the curse of dimensionality in the case of a general class of semilinear parabolic PDEs. In the final part of the talk we present some recent mathematical results on the training of neural networks.

References:

[1] Becker, S., Jentzen, A., Müller, M. S., and von Wurstemberger, P., Learning the random variables in Monte Carlo simulations with stochastic gradient descent:

Machine learning for parametric PDEs and financial derivative pricing. Math. Financ. 34 (2023), no. 1, 90-150.

[2] Beck, C., Becker, S., Cheridito, P., Jentzen, A., and Neufeld, A., Deep splitting method for parabolic PDEs. SIAM J. Sci. Comput. 43 (2021), no. 5, A3135–A3154.

[3] Han, J., Jentzen, A., and E, W., Solving high-dimensional partial differential equations using deep learning. Proc. Natl. Acad. Sci. USA 115 (2018), no. 34, 8505–8510.

[4] Gallon, D., Jentzen, A., Lindner, F., Blow up phenomena for gradient descent optimization methods in the training of artificial neural networks. arXiv:2211.15641 (2023), 84 pages.

[5] Gonon, L., Graeber, R., and Jentzen, A., The necessity of depth for artificial neural networks to approximate certain classes of smooth and bounded functions without the curse of dimensionality. arXiv:2301.08284 (2022), 101 pages.

[6] Hutzenthaler, M., Jentzen, A., Kruse, T., and Nguyen, T. A., A proof that rectified deep neural networks overcome the curse of dimensionality in the numerical approximation of semilinear heat equations. Partial Differ. Equ. Appl. 1 (2020), no. 2, Paper no. 10, 34 pp.

[7] Jentzen, A. and Riekert, A., On the existence of global minima and convergence analyses for gradient descent methods in the training of deep neural networks. J. Mach. Learn. 1 (2022), no. 2, 141–246.

[8] Ibragimov, S., Jentzen, A., Riekert, A., Convergence to good non-optimal critical points in the training of neural networks: Gradient descent optimization with one random initialization overcomes all bad non-global local minima with high probability. arXiv:2212.13111 (2023), 98 pages.

All are welcome