Abstract

The ultimate goal of any numerical scheme for partial differential equations (PDEs) is to compute an approximation of user-prescribed accuracy at quasi-minimal computational time. Due to the incremental nature of an adaptive finite element method (FEM), the analysis of optimal rates of convergence should thus rather focus on rates with respect to the cumulative computational work than on rates with respect to the number of degrees of freedom. Moreover, the efficient numerical solution of nonlinear PDEs requires to combine adaptive mesh-refinement with an appropriate linearization scheme (e.g., the Kacanov method) and a norm-contractive algebraic solver (e.g., an optimal geometric multigrid method). In our talk, we show how these different contributions to the overall error can algorithmically be balanced in such a way that we can mathematically guarantee optimal complexity, i.e., optimal convergence rates with respect to the cumulative computational cost and hence time. In particular, we see that the heart of the matter is full linear convergence, i.e., contraction of an appropriate quasi-error independently of the algorithmic decision for either mesh-refinement or another linearization step or another algebraic solver step.