



Number Theory Seminar

A higher dimensional Waring's problem

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Abstract

In 1770, Waring proposed the following conjecture: for every positive integer k , there exists a positive integer s such that every positive integer can be expressed as a sum of at most s many k -th powers of positive integers. This conjecture was solved by Hilbert in 1909. In this talk, we consider the following higher dimensional analogue of Waring-Hilbert theorem: Let Γ be any finitely generated Zariski dense subgroup of $\mathrm{SL}_d(\mathbb{Z})$, possibly non-arithmetic, and \vec{v} be any primitive vector in \mathbb{Z}^d . Can one generate \mathbb{Z}^d , or a finite index subgroup of \mathbb{Z}^d if there is local obstruction, by taking only *boundedly* many additions or subtractions of vectors from $\Gamma \cdot \vec{v}$? We give an affirmative answer to this question for the case $d = 2$. The proof involves Hardy-Littlewood circle method, hyperbolic lattice point counting and homogeneous dynamics. This is joint work with Ben Kane, Daejun Kim and Ruixiang Zhang.

Date: December 16, 2025 (Tuesday)

Time: 1:00 – 2:00pm

Venue: Room 210, Run Run Shaw Bldg., HKU

All are welcome