THE UNIVERSITY



OF HONG KONG

Institute of Mathematical Research Department of Mathematics



Self-organized criticality in planar statistical physics models

Prof. Pierre Nolin

Department of Mathematics, City University of Hong Kong

Abstract

Self-organized criticality (SOC) was first considered in statistical physics, in the late 1980s. It refers to the spontaneous arising, without any fine-tuning of a parameter, of a critical regime. This fascinating phenomenon can be used to explain the appearance of "complex structures" (in particular fractal shapes) in nature, and as such, it has important consequences. It has attracted a lot of attention and numerous works have been devoted to it, mostly in physics, but also on the rigorous mathematical side.

We discuss this phenomenon in connection with Bernoulli percolation, a model for random media introduced by Broadbent and Hammersley in 1957: for some parameter p, vertices of a given graph are independently occupied or vacant, with respective probabilities p and 1 - p. It is arguably one of the simplest models in statistical mechanics displaying a phase transition as the parameter p varies, i.e. a drastic change of behavior at some critical value p_c , and it has been widely studied. Percolation can be used to analyze various natural processes originating from physics, and we mostly focus on the celebrated Drossel-Schwabl forest-fire process. We explain how SOC arises in this case, with the phase transition of Bernoulli percolation playing a central role. In particular, a peculiar and striking phenomenon arises, that we called "near-critical avalanches".

This talk is based on joint works with Rob van den Berg (CWI, Amsterdam) and Wai-Kit Lam (NTU, Taipei).

Date: May 16, 2025 (Friday) Time: 3:00 – 4:00 pm Venue: Rm 210, Run Run Shaw Building, HKU