THE UNIVERSITY



OF HONG KONG

Institute of Mathematical Research Department of Mathematics

Seminar

Time-periodic measures, random periodic orbits, and the linear response for non-autonomous stochastic differential equations

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Abstract

We consider a class of dissipative stochastic differential equations (SDE's) with time-periodic coefficients in finite dimension, and the response of timeasymptotic probability measures induced by such SDE's to sufficiently regular, small perturbations of the underlying dynamics. Understanding such a response provides a systematic way to study changes of statistical observables in response to perturbations, and it is often very useful for sensitivity analysis, uncertainty quantification, and for improving probabilistic predictions of nonlinear dynamical systems, especially in high dimensions. Here, we are concerned with the linear response to small perturbations in the case when the time-asymptotic probability measures are time-periodic. First, we establish sufficient conditions for the existence of stable random time-periodic orbits generated by the underlying SDE. Ergodicity of time-periodic probability measures supported on these random periodic orbits is subsequently discussed. Then, we derive the socalled fluctuation-dissipation relations which allow to describe the linear response of statistical observables to small perturbations away from the timeperiodic ergodic regime in a manner which only exploits the unperturbed dynamics. The results are formulated in an abstract setting but they apply to problems ranging from aspects of climate modelling, to molecular dynamics, to the study of approximation capacity of neural networks and robustness of their estimates.

Date: April 28, 2025 (Monday)

Time: 2:00 – 3:00 pm

Venue: Room 210, Run Run Shaw Building, HKU

All are welcome