THE UNIVERSITY



OF HONG KONG

Institute of Mathematical Research Department of Mathematics

# **HKU Workshop on Lie Theory and Related Topics**

Date: 12 May, 2025 (Monday)

Venue: Room 210, Run Run Shaw Building, The University of Hong Kong

10:00-10:50	<b>Eric Marberg (HKUST)</b> Title: Combinatorial positivity via type A crystals
11:00-11:50	<b>Ziquan Yang (CUHK)</b> Title: A simple intrinsic proof of the Tate conjecture for K3's of finite height
12:00-14:15	Lunch
14:15-15:05	<b>Wei Xiao (Shenzhen University)</b> Title: The product-generated structure of the parabolic category O
15:05-15:30	Tea Break
15:30-16:20	<b>Qingchao Yu (Shenzhen University)</b> Title: Shellability of the Admissible Set
16:30-17:20	<b>Quoc Ho (HKUST)</b> Title: On a braided monoidal Hall 2-category

# **Titles and Abstracts**

# Speaker: Quoc Ho (HKUST)

Title: On a braided monoidal Hall 2-category

Abstract: Hall algebras, appearing in many different forms and flavors, play an important role in representation theory. In this talk, I will describe a new version of the Hall algebra construction whose output is a (braided) monoidal 2-category. This construction can be thought of as a twice-categorified version of the usual Hall algebra story. The resulting object is interesting and rich already when the input category is the category of finite-dimensional vector spaces over a finite field. Focusing on this example, I will explain why such an object is desirable from the point of view of categorified representation theory and low-dimensional topology. This is joint work in progress with Jonte Goedicke, Yang Hu, and Walker Stern.

#### **Speaker: Eric Marberg (HKUST)**

Title: Combinatorial positivity via type A crystals

Abstract: Schur polynomials arise geometrically as cohomology representatives of Schubert varieties in the type A Grassmannian. When one studies K-theory instead of cohomology, these representatives are upgraded to functions known as symmetric Grothendieck polynomials. Many generating functions in combinatorics have nontrivial yet positive decompositions into Schur polynomials. The classical theory of type A crystals provides a graphical framework for proving this kind of Schur positivity. This talk will discuss a new category of "square root crystals" that can be used to establish stronger Grothendieck positivity properties. While the classical results extend to arbitrary Lie types, it is an open problem to determine if the same applies to our new "square root" setting.

#### Speaker: Wei Xiao (Shenzhen University)

Title: The product-generated structure of the parabolic category O

Abstract: This report presents a detailed analysis of a product-generated structure within the parabolic category O, with a specific focus on the Lie algebras of type A. Beginning with an introduction to the parabolic category O and its role in algebraic contexts, we uncover the characteristics and significance of this structure. As an application, we also show how to settle the problem of block decomposition for parabolic category O.

# Speaker: Ziquan Yang (CUHK)

Title: A simple intrinsic proof of the Tate conjecture for K3's of finite height

Abstract: In the past decade, a major triumph for the Tate conjecture (over finite fields) is its resolution for K3 surfaces. However, all known proofs rely crucially on the Kuga-Satake construction—effectively outsourcing certain difficulties to the theory of abelian varieties. There is an alternative approach which seeks to prove the conjecture by linking it to finiteness statements in arithmetic geometry, and using only the geometry of K3 surfaces. The spirit of this approach originated in Artin and Swinnerton-Dyer's proof for elliptic K3's and has recently been revitalized by Lieblich-Maulik-Snowden and Charles through the moduli theory of (twisted) sheaves. In particular, Charles gave an intrinsic proof assuming the Picard number is at least 2. In this talk, I will explain an intrinsic proof that finite height K3's a priori have an even geometric Picard number, and hence proves one step further in the second approach. If time permits, I will also discuss speculations for the supersingular case.

# Speaker: Qingchao Yu (Shenzhen University)

Title: Shellability of the Admissible Set

Abstract: A simplicial complex is said to be shellable if its facets can be ordered in such a way that each facet intersects the union of the previous facets in a pure and codimension-one manner. EL-shellability generalizes the notion of shellability for posets via edge-labeling. Admissible sets are significant in arithmetic geometry. Our main result is that the admissible set satisfies the dual EL-shellability condition. This property is related to the Cohen-Macaulayness of local models. This talk is based on my recent joint work with Xuhua He.