



Colloquium

Asymptotics for 2D Allen-Cahn minimizers

Dr. Zhiyuan Geng

Department of Mathematics, Purdue University

Abstract

For the scalar two-phase (elliptic) Allen-Cahn equation, there is a rich literature on the celebrated De Giorgi conjecture, which reveals deep connections between diffuse interfaces and minimal surfaces/graphs. On the other hand, for three or more equally preferred phases, a vector-valued order parameter is required, and the resulting diffuse interfaces are expected to resemble weighted minimal partitions.

In this talk, I will present recent results on minimizers of a two-dimensional Allen-Cahn system with a multi-well potential. We describe the asymptotic behavior near the junction of three phases by analyzing the blow-up limit, which is a global minimizing solution converging at infinity to a Y-shaped minimal cone. We further establish an almost one-dimensional symmetry of the solution along the sharp interface. A key ingredient in our approach is the derivation of sharp upper and lower energy bounds via a novel slicing argument, which allows us to localize the diffuse interface within a small neighborhood of the sharp interface. As a consequence, we obtain a complete classification of global two-dimensional minimizers in terms of their blow-down limits at infinity. This is joint work with Nicholas Alikakos.

Date: January 26, 2026 (Monday)

Time: 4:30 – 6:00 pm

Venue: Room 210, Run Run Shaw Bldg., HKU